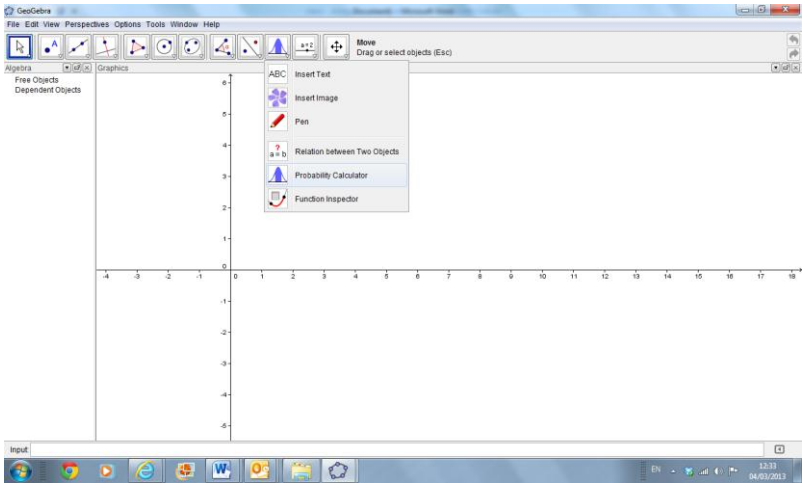
To use the Probability calculator in GeoGebra to demonstrate the Empirical Rule

The Empirical Rule states that for a Normal distribution approximately 68% of the values will be within 1 standard deviation of the mean, approximately 95% of the values will be within 2 standard deviations of the mean and approximately 99.7% of the values will be within 3 standard deviations of the mean. Hence this rule is sometimes known as the 68-95-99.7 rule.

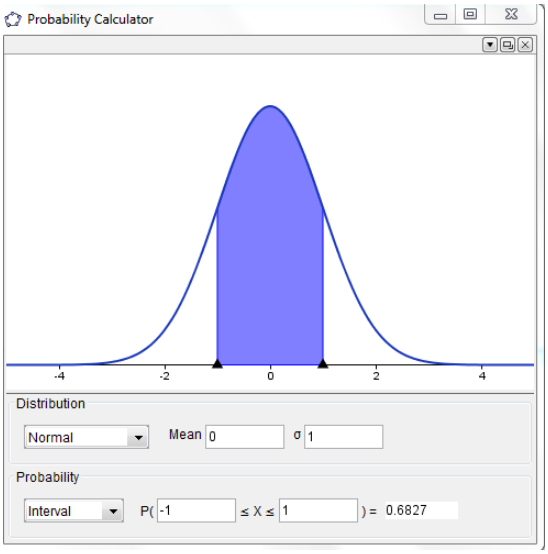
It is also necessary to note that 50% of the values will be above the mean and 50% of the values will be below the mean.

The next 8 questions are based on a situation where the mean score in an examination was 450 with a standard deviation of 100 and the distribution of the scores is Normal. An understanding of the Empirical Rule can enable students to verify the answers got in GeoGebra.

1. Open GeoGebra and in the third set of tools from the right of the toolbar click on the Probability Calculator tool  .

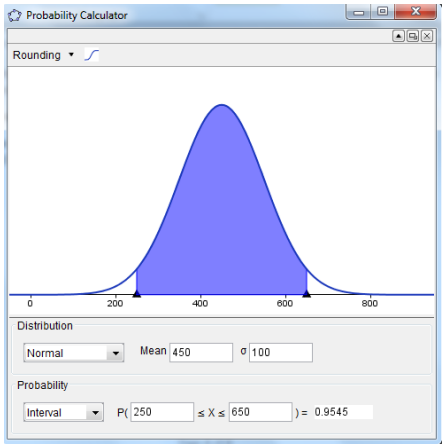


2. A new window opens.



Q1: What percentage of the students scored between 250 and 650?

a. Change the Probability to Interval by clicking on the Interval tool  .
b. Let μ equal to 450 and σ equal to 100. Remember to press Enter on your keyboard after you enter each number.
c. Insert 250 in the first box in P ($\blacksquare \leq X \leq \blacksquare$) and 650 in the second box.
d. Answer <u>0.9545 or 95.45%.</u>

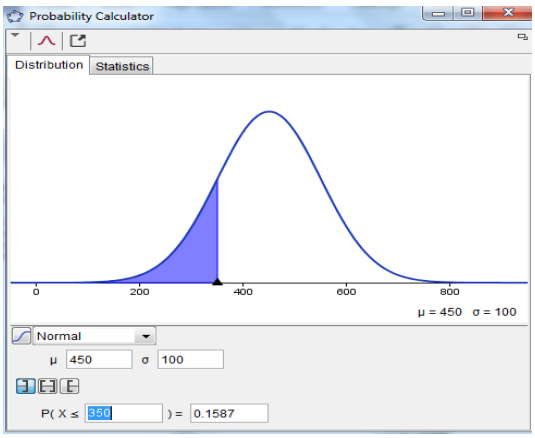


Reasoning: 250 is 2 standard deviations below the mean and 650 is 2 standard deviations above the mean. According to the **Empirical Rule** approximately 95% of the data will be within 2 standard deviations from the mean.

It should be noted that the Probability Calculator uses the normal distribution tables to work out the probabilities. This is why the Probability Calculator says 0.9545.

Q2: What percentage of the students scored less than or equal to 350?

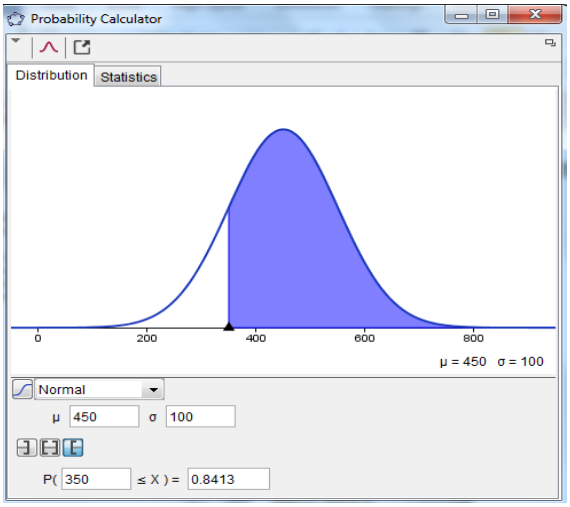
a. Change the Probability to Left Sided by clicking on the Left Sided tool  .
b. Let μ equal to 450 and σ equal to 100. Remember to press Enter on your keyboard after you enter each number.
c. Insert 350 in the box in P ($X \leq \blacksquare$).
d. Answer <u>0.1587 or 15.87%</u>.



Reasoning: According to the **Empirical Rule** approximately 68% of the data will be within 1 standard deviation from the mean. As 350 is 1 standard deviation below the mean the **Empirical Rule** states that approximately 34% of the data will be between 350 and 450. Hence approximately $50\% - 34\% = 16\%$ of the data will be 350 or less.

Q3: What percentage of the students scored greater than or equal to 350?

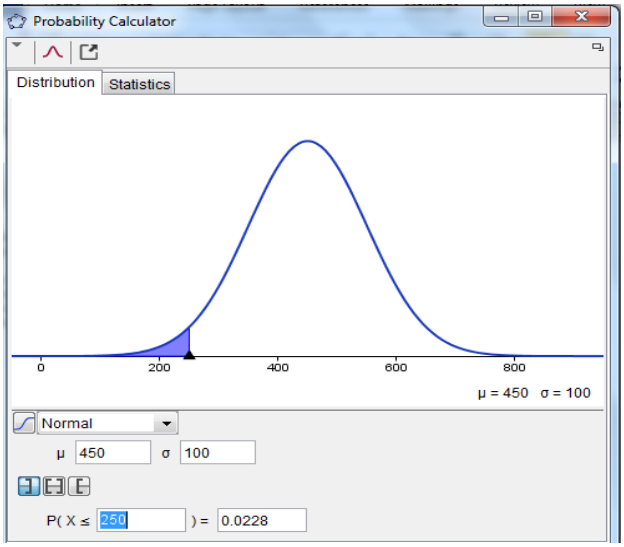
a. Change the Probability to Right Sided by clicking on the Right Sided tool .
b. Let μ equal to 450 and σ equal to 100. Remember to press Enter on your keyboard after you enter each number.
c. Insert 350 in the box in $P\ (\blacksquare \leq X)$.
d. Answer 0.8413 or 84.13%.



Reasoning: According to the **Empirical Rule** approximately 68% of the data will be within 1 standard deviation from the mean. As 350 is 1 standard deviation below the mean the **Empirical Rule** states that approximately 34% of the data will be between 350 and 450. Hence approximately $50\% + 34\% = 84\%$ of the data will be 350 or greater.

Q4: What percentage of the students scored less than or equal to 250?

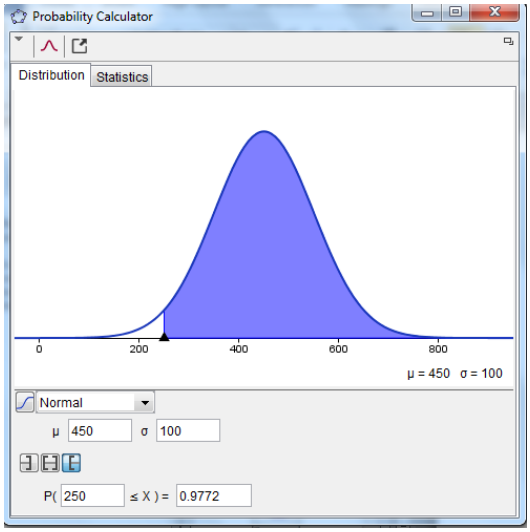
a. Change the Probability to Left Sided by clicking on the Left Sided tool .
b. Let μ equal to 450 and σ equal to 100. Remember to press Enter on your keyboard after you enter each number.
c. Insert 250 in the box in $P\ (X \leq \blacksquare)$.
d. Answer 0.0228 or 2.28%.



Reasoning: According to the **Empirical Rule** approximately 95% of the data will be within 2 standard deviations from the mean. As 250 is 2 standard deviations below the mean the **Empirical Rule** states that approximately 47.5% of the data will be between 250 and 450. Hence approximately $50\% - 47.5\% = 2.5\%$ of the data will be 250 or less. Another way of looking at this is 100%-95% is 5% and half of 5% is 2.5%

Q5: What percentage of the students scored greater than or equal to 250?

a. Change the Probability to Right Sided by clicking on the Right Sided tool .
b. Let μ equal to 450 and σ equal to 100. Remember to press Enter on your keyboard after you enter each number.
c. Insert 250 in the box in $P\ (\blacksquare \leq X)$.
d. Answer 0 .9772 or 97.72%.

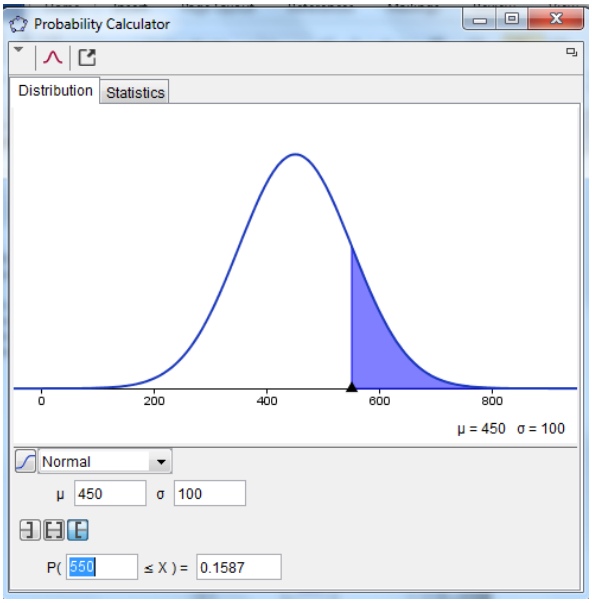


Reasoning: According to the **Empirical Rule** approximately 95% of the data will be within 2 standard deviations from the mean. As 250 is 2 standard deviations below the mean the **Empirical Rule** states that approximately 47.5% of the data will be between 250 and 450. Hence approximately

$50\% - 47.5\% = 2.25\%$ of the data will be to the left of 250 and 97.75% of the data will be 250 or greater

Q6: What percentage of the students scored greater than or equal to 550?

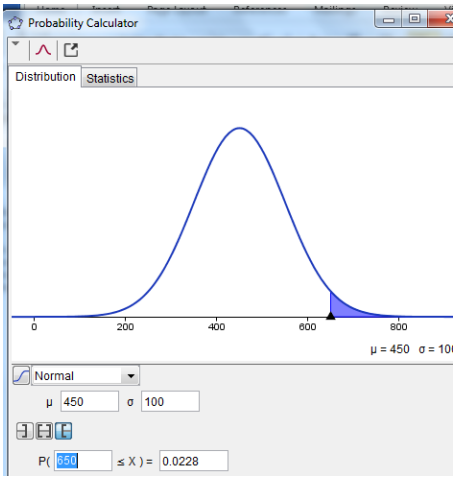
a. Change the Probability to Right Sided by clicking on the Right Sided tool  .
b. Let μ equal to 450 and σ equal to 100. Remember to press Enter on your keyboard after you enter each number.
c. Insert 550 in the box in $P(\blacksquare \leq X)$.
d. Answer 0.1587 or 15.87%.



Reasoning: According to the **Empirical Rule** approximately 68% of the data will be within 1 standard deviation from the mean. As 550 is 1 standard deviation from the mean the **Empirical Rule** states that approximately 34% of the data will be between 450 and 550. Hence approximately $50\% + 34\% = 84\%$ of the data will be below 550 and 16% of the data will be equal to or above 550.

Q7: What percentage of the students scored greater than or equal to 650?

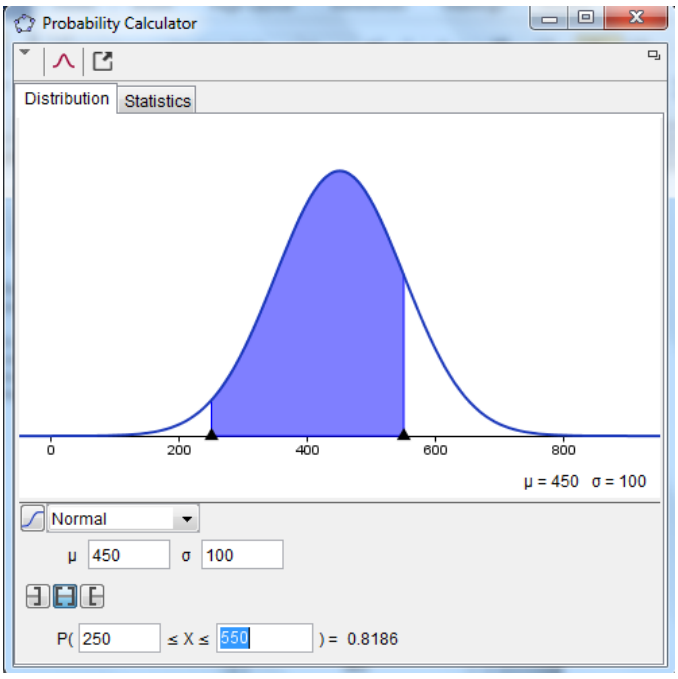
a. Change the Probability to Right Sided by clicking on the Right Sided tool  .
b. Let μ equal to 450 and σ equal to 100. Remember to press Enter on your keyboard after you enter each number.
c. Insert 650 in the box in $P(\blacksquare \leq X)$.
d. Answer 0.0228 or 2.28%.



Reasoning: According to the **Empirical Rule** approximately 95% of the data will be within 2 standard deviations from the mean. As 650 is 2 standard deviations from the mean the **Empirical Rule** states that approximately 47.5% of the data will be between 450 and 650. Hence approximately $50\% + 47.5\% = 97.5\%$ of the data will below 650 and 2.25% of the data will be equal to or above 650. Another way of looking at this is $100\% - 95\%$ is 5% and half of 5% is 2.5%.

Q8: What percentage of the students scored between 250 and 550?

a. Change the Probability Calculator to Interval by clicking on the Interval tool .
b. Let μ equal to 450 and σ equal to 100. Remember to press Enter on your keyboard after you enter each number.
c. Insert 250 in the first box in P (■ $\leq X \leq$ ■)and 550 in the second box.
d. Answer 0.8186 or 81.86%.



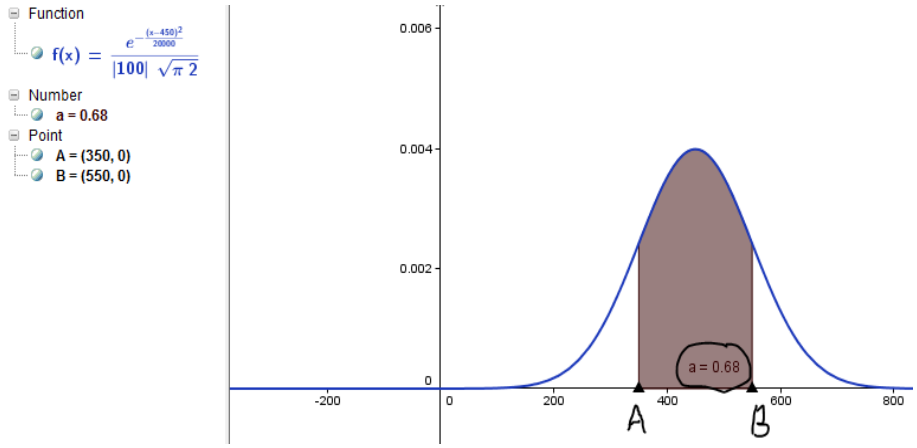
Reasoning: According to the **Empirical Rule** approximately 95% of the data will be within 2 standard deviations from the mean. As 250 is 2 standard deviations from the mean the **Empirical Rule** states that approximately 47.5% of the data will be between 250 and 450. As 550 is 1 standard deviation

from the mean the Empirical Rule states that approximately 34% of the data will be between 450 and 550. Hence approximately $47.5\% + 34\% = 81.5\%$ of the data will be between 250 and 550.

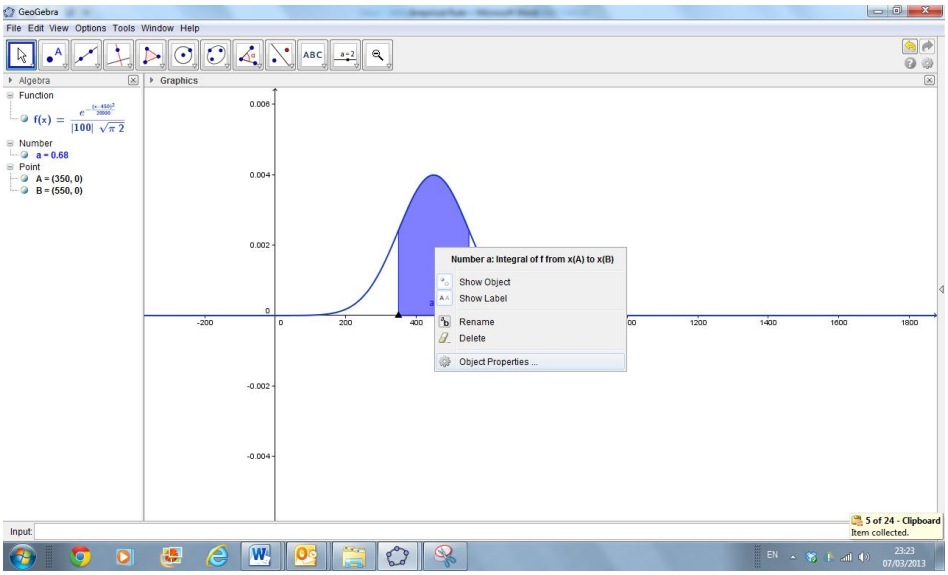
To insert the Probability Calculator into the Graphics View and change the colour of the Probability Calculator

a. Right click the Probability Calculator and choose Copy to Graphics View. (This may take a few seconds.)

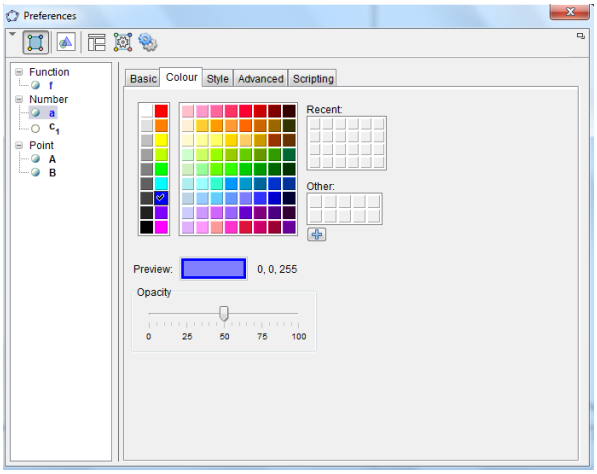
b. Note the probability is now stored in variable a. Variable a represents the Definite Integral between the two sliders underneath the coloured part of the graph. The positions of the sliders are represented by the *x* values of the points A and B.
A more precise value of the Integral can be found by going to Options, Rounding and choosing a higher number of decimal places.



c. Right click the coloured part of the diagram and choose Object Properties.

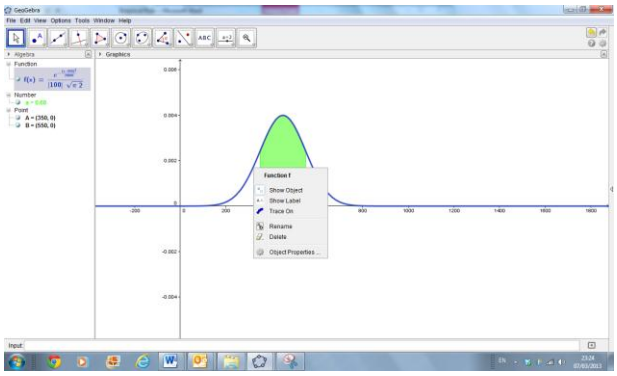


d. A new dialogue box appears. With the Colour tab open in this dialogue box pick the colour of your choice.

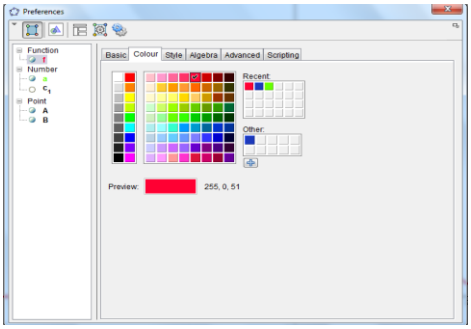


e. Click the X at the top of this dialogue box.

f. To change the colour of the function part of the graph right click on the function and choose Object Properties.

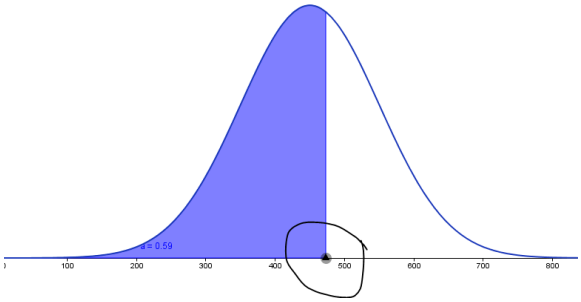


g. A new dialogue box appears. With the Colour tab open in this dialogue box pick the colour of your choice.



h. Click the X at the top of this dialogue box.

i. Note the slider can be moved when the Probability Calculator is in the Graphics View.



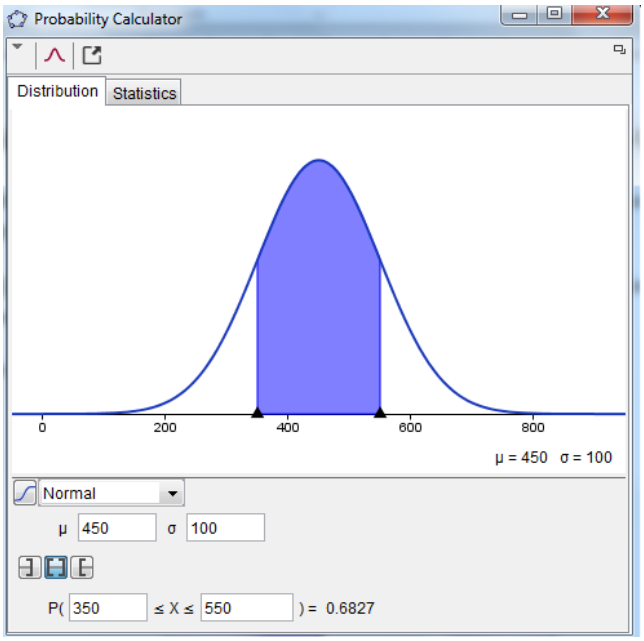
j. Using the Snipping Tool on your computer these coloured diagrams can be transferred to PowerPoint or Word.

Displaying More than One Region in the Graphics View

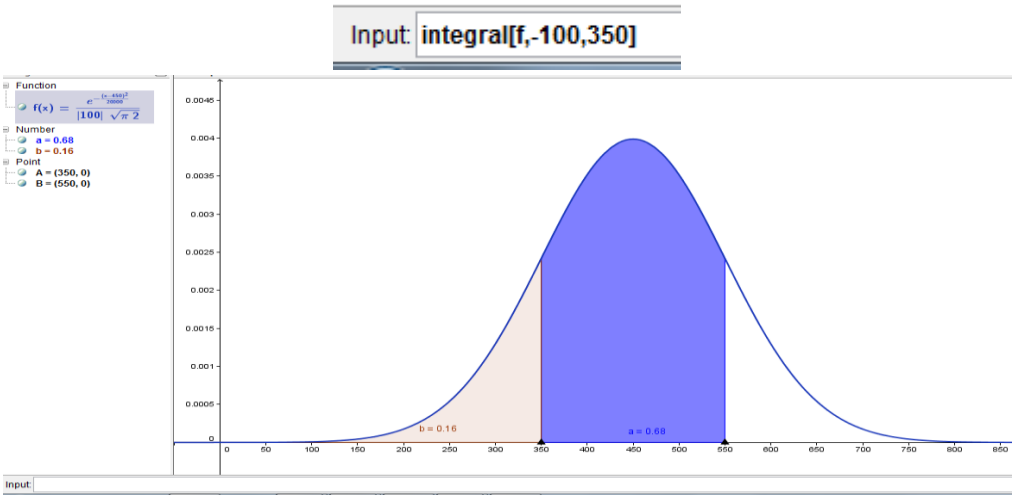
It can be difficult for some students to see where the 16% comes from in the question "What percentage of the students scored less than or equal to 350?"

Displaying the (i) middle 68%, (ii) the remaining 32% and finally (iii) the 16% we are interested in can help students see this more clearly.

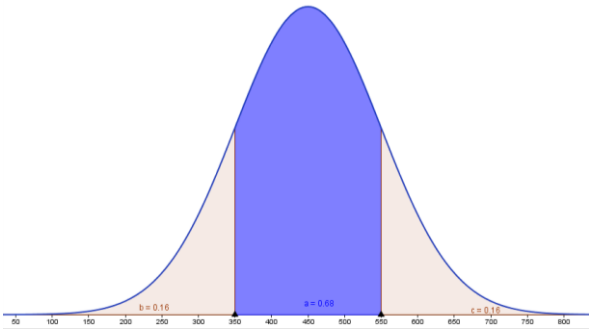
1. Let μ equal to 450 and σ equal to 100 set the Probability Calculator to display the region between 350 and 550.



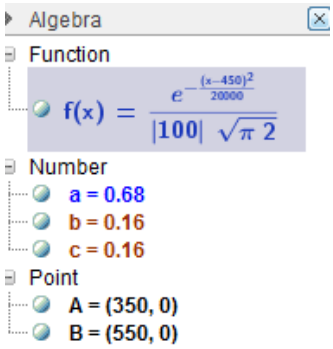
2. Insert the Probability Calculator into the Graphics View.
3. Type Integral[f,-100,350] into the Input bar. This will shade in the area between 0 and 350 (unfortunately GeoGebra doesn't accept "infinity" or "∞" for the normal function at the moment). This will be named b in the Algebra View.



4. Type Integral[f,550,100] into the Input bar. This will shade in the area between 550 and 1000 (unfortunately GeoGebra doesn't accept "infinity" or "∞" for the normal function at the moment). This will be named c in the Algebra View.



5. Use the  tool to tick and untick the blue dots beside a, b, and c in the Algebra View.

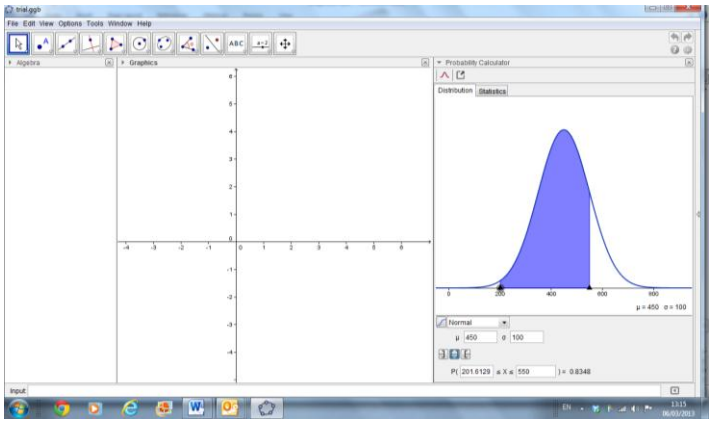


One good sequence could be to display (i) a, 68% (ii) b and c, the remaining 32% and finally (iii) b, the 16% we are interested in.

Move the Probability Calculator to the Main Window

Note: This is not the same as Copy to Graphics View. With Copy to Graphics View one cannot change the μ *and* σ values.

1. Click on  at the top of the Probability Calculator screen one can get the Probability Calculator to move into the main window. Note this is different from it appearing in the Graphics View.



2. Go to File and Save as. When you open this file the Probability Calculator will be in the main window.