For the lesson on 4th March 2016
At Rathdown School, TY class
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1. **Title of the Lesson:** To optimize the design of a rain gutter

2. **Brief description of the lesson:** How to maximize the volume of water flowing through the gutter

Aims of the Lesson:

- I'd like my students to appreciate that mathematics can be used to solve real world problems
- I'd like my students to become more creative when devising approaches and methods to solve problems
- I'd like to emphasise to students that a problem can have several equally valid solutions
- I'd like my students to experience meaningful mathematics i.e. that they see a need for what they are studying
- I'd like to build my students' enthusiasm for the subject by engaging them with stimulating activities
- I'd like my students to connect and review the concepts that we have studied already

Short-term goals

- That students will understand the concept of a prism
- That students will discover that varying the length of the base alters the area of the triangle
- That the students will realise that the volume decreases again having obtained the maximum

Learning Outcomes: As a result of studying this topic students will be able to:

- Understand that they can use experimentation when seeking a solution to an unfamiliar everyday Maths problem
- Appreciate that there may be several ways of achieving a satisfactory solution to a problem
- Recognise comfortably and confidently prism shapes used in everyday
- Recognise and understand that changing one variable or dimension of the shape will alter other dimensions

3. Background and Rationale

(a) The Leaving Certificate Mathematics syllabus requires the student to acquire strategic competence. This basically implies an ability to formulate, represent, and solve mathematical problems in both familiar and unfamiliar situations and scenarios.

The syllabus also seeks to ensure that students develop their capacity for logical thought and the ability to reflect on a problem and possible solutions. Following on reflection the students should be able to explain, justify and communicate effectively their reasoning and procedures used.

A further rationale for this sort of problem solving approach is the development of a habitual inclination to see mathematics as sensible, useful, and worthwhile. This habit should result in commitment to diligence and perseverance in dealing with mathematical problems which in turn will develop in the student a belief in their one efficacy.

(b) Students often perceive 3D shapes as 2D shapes due to a lack of hands on experience and as a result struggle to comprehend how changing a dimension or measurement would affect other measurements. There is often a major disconnect between geometrical shapes and figures that appear as drawings in the text books and the common place existence of such shapes in the world around us. When presented with a real life example of a problem that involves even familiar shapes with known formulas, students often find it difficult to recognise the essential components of the question and hence are not at all sure how to approach finding the solution.

(c) This lesson focusses on an everyday use of the prism shape that is most likely beyond the students' current experience but not beyond their capacity to work with, which is important. The students will be familiar with the formula for finding area of a triangle and for calculating the volume of a prism. This information will be applied to solving the problem of maximising the flow of water in a pipe. The students are familiar theoretically with functions and rates of change and graphing related variables. It is hoped that some of them may make the connection between this problem solving exercise and functions.

4. Research

The Leaving Certificate syllabus requires students to be competent in dealing with 2 dimensional and 3 dimensional shapes. It specifies solving problems that necessitate finding the volume of a prism that has a triangular base (Leaving Certificate syllabus, p. 33).

The new approach to Mathematics teaching and learning shifts the focus away from the teacher solving the problems while the students listen and watch and record the solution, to a focus on student activity and peer consultation. Mathematics is about problem solving. The students are encouraged to discover for themselves, using prior knowledge, problem solving techniques that will develop their skills and their belief in their own problem solving skills (Leaving Certificate syllabus, p.10).

5. About the Unit and the Lesson

The students will be presented with a real life problem that involves designing a channel to transport water as efficiently as possible. They will be given an A4 sheet of paper as the concrete material that will aid them in finding solutions. The use of this concrete material will facilitate their thinking moving to an abstract level as they experiment with the changing shape in 3 dimensions (Leaving Certificate syllabus, p.10)

6. Flow of the Unit:

Active Maths 3, Book 2, Chapter 12

Lesson		# of lesson periods
1	Right angled triangles and Pythagoras' theorem	2
2	Finding the length of a side in a right–angled triangle	1
3	Using trigonometry to solve practical problems	2
4	Area of triangle Area = $\frac{1}{2}ab \sin C$	2
5	The Sine Rule	2
6	The Cosine Rule	2
7	Nets of 3 dimensional shapes	2
8	Volume of prisms	2
9	To optimize the design of a rain gutter using one fold	2 x 35 min. (#9 = research lesson)
10	To optimize the design of a rain gutter using multiple folds	8

6. Flow of the Lesson

Teaching Activity	Points of Consideration
1. Introduction Using mathematics to solve real world problems. Using recent topics such as trigonometry, and volume of 3D shapes and nets, we will investigate the design of a rain gutter for a building which will give the maximum flow.	
2. Posing the Task Students will work in groups of 3 or 4. They will be given a piece of cardboard and asked to design a gutter by folding the cardboard in half. Each group will be given an A3 placemat, graph paper and A4 plain sheets. The task is to optimize the design of the gutter to achieve the maximum flow of water. Each group must prepare a graph to illustrate their results.	The cardboard can only be folded once to produce the gutter. After a 5 minute discussion the groups are to make a decision on the best way to fold the cardboard.
3. Anticipated Student Responses Does it matter which way the cardboard is folded? After a 5 minute discussion the groups are to make a decision on which way they plan to fold the cardboard. Does the shape of the gutter matter? How do we work out the volume of the gutter? Is the gutter open at the top? How do we graph the shape of a gutter? What things are we allowed to change? Do we need to measure anything?	The cardboard can be folded either landscape or portrait. Practical considerations should be used in the decision making. Installation and maintenance costs should be considered when making this initial decision. The gutter is a prism. The volume of a prism is the cross sectional area × height. The size of the cardboard can't be changed. The width of the opening is the only variable. Not necessarily. If your group has sufficient information to proceed then measuring is not required.
4. Comparing and Discussing Groups using Area $= \frac{1}{2}ab \sin C$ without measuring should complete the task before others. They should be encouraged to use angle variations as small as possible, to improve the accuracy of their graph. Groups that are measuring lengths of sides will be slower.	Accuracy of measurements What are the ideas to focus on during the discussion? What will indicate that students are benefiting from the discussion?
5. Summing up The lesson will be concluded with two or three groups, with different strategies and solutions being invited to present their solution and explain their thinking and activities to the class.	

7. Evaluation

Mathematics teachers familiar with the problem presented to the students will observe the groups as they investigate how to use the material provided to explore options for producing a solution to designing the required gutter. We will walk around the classroom, moving in around the groups avoiding paying more attention to one group than any other. We will not interfere or interact with the students as they engage in the problem-solving.

We will observe and record the steps taken by the groups as they plan their investigation in terms of deciding who does what. The two observing teachers will compile these records. The various concrete activities that are undertaken, using the cardboard, will be recorded in writing. Some of the investigations may be recorded using diagrams to represent the shapes created by the students. Notes will be taken of the students' discussions that are heard. Any questions posed by the students as they seek solutions will be noted also. If students ask for encouragement, for example 'are we on the right track?', we will give simple words of support but not add guidance unless absolutely necessary. We will encourage the students to keep trying using all the tools at their disposal in terms of their prior knowledge and current experience and materials.

We will particularly focus on how the students begin the process of solving the problem. What steps the group take to seek solutions, do they manipulate card first, or start talking about possible activities? Do they start writing down formulas, or drawing out pictures of what the gutter might look like? Do they engage in debate first or do they engage in concrete exploration first? What aspects of the problem do they focus on first? Do they start measuring first and then use formulae or do they label the cardboard with the measurements? Do they use examples of other problems they have perhaps solved before to find a solution? Do they work as a team or as three or four individuals who then pool their suggestions and findings? These problem solving activities and strategies will be recorded. At the conclusion, if time permits, we will select two or three groups to come and explain their problem solving technique and give us their design for the most economical cost effective and efficient gutter. We will collect any models of the gutter that they create along with any workings and/or graphs that they produce as part of their solution.

8. Board Plan

The task will be written across the top of the board. A PowerPoint presentation was prepared which outlined the task and this was left projected on the board as they started the task.

9. Post-lesson reflection

The students all engaged very enthusiastically and actively in exploring the details of the problem presented to them. They discussed the problem verbally and listened to each other. Some of them scribbled down formulas they considered might help. They manipulated the piece of card they had been provided with as a concrete tool to assist them finding a solution. It was clear that handling the card and experimenting with different shapes that reflected the shape of the required gutter made the problem accessible to all the students. The students measured the relevant dimensions of the shape that they decided would be necessary. By doing this, and manipulating the card they realised which dimensions could be varied. In the absence of the concrete material it is possible that this might not have been achieved so readily. One of the groups very quickly concluded that 90 degrees in the angle would produce that largest volume. This group was encouraged to experiment with different measures of the angle in order to produce the coordinates to graph the function. It was interesting the manner in which the students moved from receiving the information and being presented with the challenge to coming up with possible solutions. Group work was very much in evidence with all students keen to have an input, whether it was manipulating the shape, thinking of appropriate formulas and writing them down, carrying out the measurements required and recording them or discussing options. The accessibility of the various aspects of the problem and solution to all students was a very positive aspect of this lesson. The girls clearly appreciated that this was a real life challenge and commented on the value of having been able to use what they had learned in class to undertake a practical task. They appeared very pleased with their efforts and were very happy to display their work. Their thinking was very obvious as they communicated actively with each other. One of the groups decided that the efficiency of the gutter would also depend on the rate of flow of water in the gutter, not just the shape. This indicated thinking that went beyond the actual task which was to design the physical gutter.

The learning goals were achieved well. While divided into groups the girls worked together very effectively in devising their plan, executing the plan, agreeing on the overall strategies to be employed and the outcome for the most efficient gutter. The various tasks were undertaken efficiently with each student becoming actively engaged in the process. They used previous knowledge of volume, trigonometry and functions as planned and experienced how these topics in Mathematics can be used together when solving real problems. This was a very valuable aspect of this lesson as often in school students do not experience the overlap and simultaneous application of different procedures and mathematical tools.

Future lessons using a similar approach could alter the shape of the required container. The issue of cost could also be introduced where students are told the price per metre of material being used and they must produce a solution that is not only perfectly fit for efficient use but must also be cost effective.