The relationship between the volume of a cylinder and its height and radius

Problem solving lesson for Volume

3rd Year Higher Level

Teacher: Cara Shanahan

Lesson plan developed by:
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For the lesson on 26th January 2016
At Colaiste Dun Iascaigh, Cara Shanahan's Third Year Higher Level class
Teacher: Cara Shanahan
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1. **Title of the Lesson:** The relationship between the volume of a cylinder and its height and radius.

2. **Brief description of the lesson** To help students realise there is a relationship between the volume of the cylinder and the height, keeping the radius constant. Also there is a relationship between the volume of the cylinder and the radius, keeping the height constant.

3. **Aims of the Lesson:**

Long-range/thematic goals:

- I'd like to foster my students to become enthusiastic, independent learners
- I'd like my students to become more creative when devising approaches and methods to solve problems and to acknowledge that a problem can have several equally valid solutions
- I'd like my students to experience meaningful mathematics i.e. that they see a need for what they are studying
- I'd like my students to verbally and visually express their own ideas and concepts
- I'd like my students to connect and review the concepts that we have studied already

Short-term goals:

For students to understand:
- The relationship between the volume of a cylinder and its radius.
- To see that as the radius changes, the volume changes in proportion.
- Be able to comfortably use the volume of a cylinder formula.
- π is a constant and height, h, is a constant in this task.
- Scale factor – the effect of scaling.
- Estimation is a useful skill.

Learning Outcomes:

As a result of studying this topic students will be able to:
- Develop an understanding of the relationship between the volume and radius of a cylinder.
- See the relevance of leaving the answers in terms of π.
- Appreciate the value of estimation.
- Realise that reducing the number of variables can make solving a problem easier.

4. Background and Rationale

- The students have already met and used the formula in straightforward questions.
- Students have had difficulty with visualising how the volume changes, when the height or radius of the cylinder changes.
- Students have found substituting the correct variables into the formula difficult.
- In this lesson, students will have to realise that changing the radius of a cylinder have an effect on the volume.

Syllabus:
- Find the volume of cylinders.
- Modelling real world situations and solve a variety of problems (including multistep problems) involving volumes of cylinders.
- Perform calculations to solve problems involving the volume of cylinders.

The thematic focus of this lesson is that:
- The students appreciate that mathematics can be used to solve real world problems to make the topic relevant to their real lives.
- The students appreciate that mathematics can be used to communicate thinking effectively so that they become more comfortable using mathematical language.
- The students become more creative when devising approaches and methods to solve problems so they can solve different types of problems they meet in the future. Transferable skills.

5. Research

- Syllabus
- Textbooks
- Websites: fast.wistia.net, www.educationworld.com, ltodd.wikispaces, www.tes.com
- Resources: plastic pipes with diameter 5cm, 10cm and 15cm, all with height of 5cm and cat litter.

6. About the Unit and the Lesson

Page 24 of the 2016 Syllabus: Applied Measure

Topic	Description of topic Students learn about	Learning outcomes Students should be able to
3.4 Applied measure	Measure and time. 2D shapes and 3D solids, including nets of solids (two-dimensional representations of three-dimensional objects). Using nets to analyse figures and to distinguish between surface area and volume. Problems involving perimeter, surface area and volume. Modelling real-world situations and solve a variety of problems (including multi-step problems) involving surface areas, and volumes of cylinders **and prisms.** The circle and develop an understanding of the relationship between its circumference, diameter and π.	– calculate, interpret and apply units of measure and time – solve problems that involve calculating average speed, distance and time – investigate the nets of rectangular solids – find the volume of rectangular solids and cylinders – find the surface area of rectangular solids – identify the necessary information to solve a problem – select and use suitable strategies to find length of the perimeter and the area of the following plane figures: disc, triangle, rectangle, square, and figures made from combinations of these – draw and interpret scaled diagrams – **investigate nets of prisms (polygonal bases) cylinders and cones** – **solve problems involving surface area of triangular base prisms (right angle, isosceles, equilateral), cylinders and cones** – **solve problems involving curved surface area of cylinders, cones and spheres** – **perform calculations to solve problems involving the volume of rectangular solids, cylinders, cones, triangular base prisms (right angle, isosceles, equilateral), spheres and combinations of these**

7. Flow of the Unit:

Lesson		# of lesson periods
1	Units and formulae for perimeter, area and volume	4
2	Circle, circumference and π	3
3	Surface are and nets	3
4	Volume of 3D shapes	2
5	Problem solving involving surface area – prisms	2
6	Problem solving involving surface area – curved shapes	2
7	**Problem solving involving volume**	**Research Lesson**
8	Follow up Lesson and Extension	1
9	Combinations	2
TOTAL		19

8. Flow of the Lesson

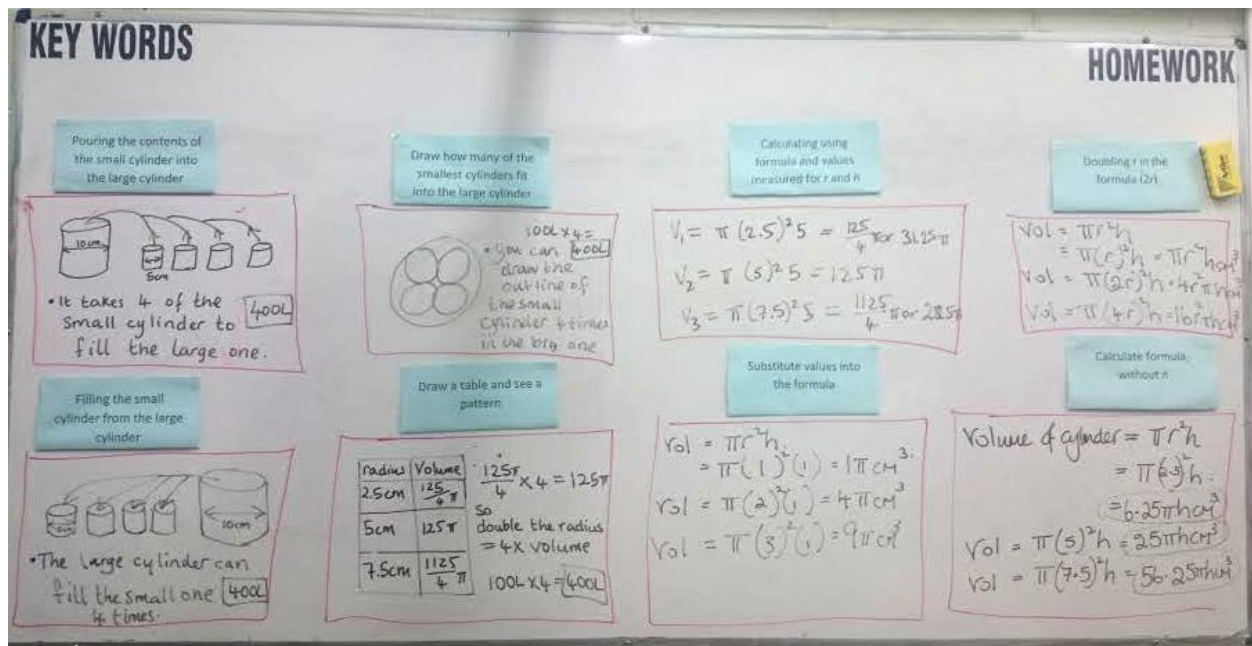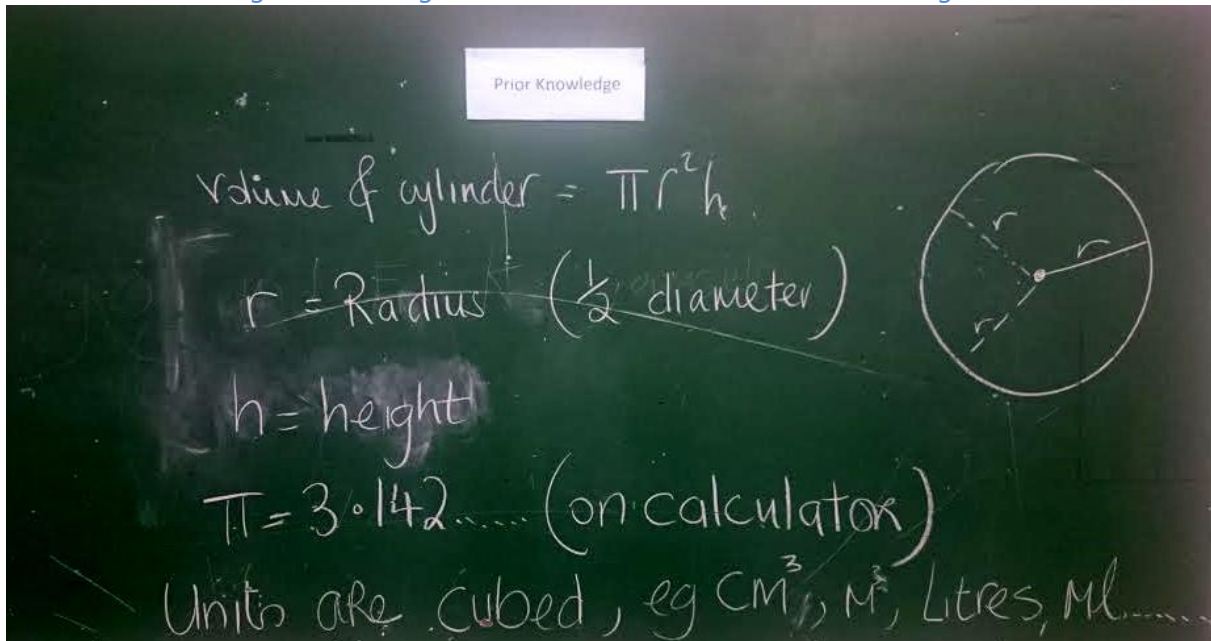| Teaching Activity	Points of Consideration
This column shows the major events and flow of the lesson.	This column shows additional moves, questions, or statements that the teacher may need to make to help students. This column identifies what the teacher should look for to determine whether to proceed, and what observers should look for to determine the effectiveness of the lesson.
1. Introduction (5 minutes) Discuss prior knowledge of volume of cylinders formula, and that π is a constant.	Get students to give formula and where to find it, What do the symbols and letters mean? Terms written beside each other means multiply. Terms: width, diameter, radius
2. Posing the Task (2 minutes) + (10 minutes) Your parents need to replace their hot water tank, to facilitate installing a solar panel. They cannot change the height of the tank but they can double the width of the tank to fit inside the hot press. If the previous tank could hold 100l, can we predict what the new tank will hold? You have 3 shapes in cylinders in front of you to help you explore this problem. **See Appendix 1**	Ask students are they clear about the instructions and do they have any questions. Tell the students they will have 10 minutes to solve the problem in as many ways as possible. They will need to present their data on a sheet (maybe on a table) and will be called to the board.
3. Anticipated Student Responses (20 minutes) • Students will double the volume = 200l • See how much cat litter physically fits in the smallest cylinder and use that to fill other cylinders. • Or fill biggest and see how many times that fills	Present on the board. Start with the most concrete solution, physically acting out the task. Trying different values instead of r Measure and calculate objects Algebraically manipulate formula

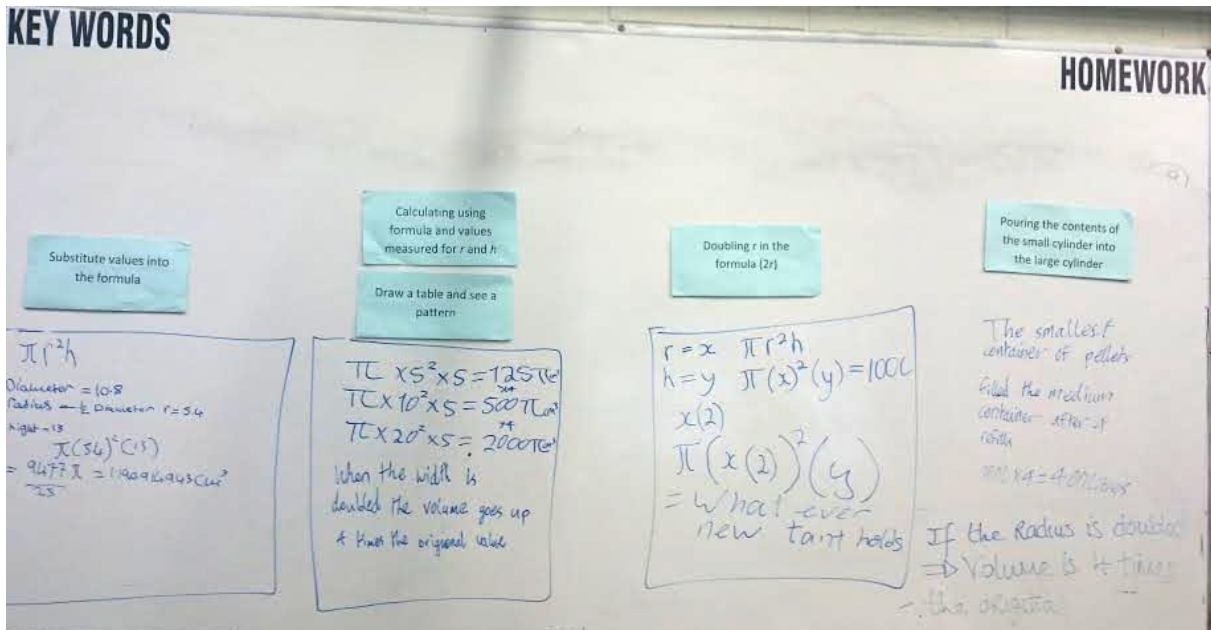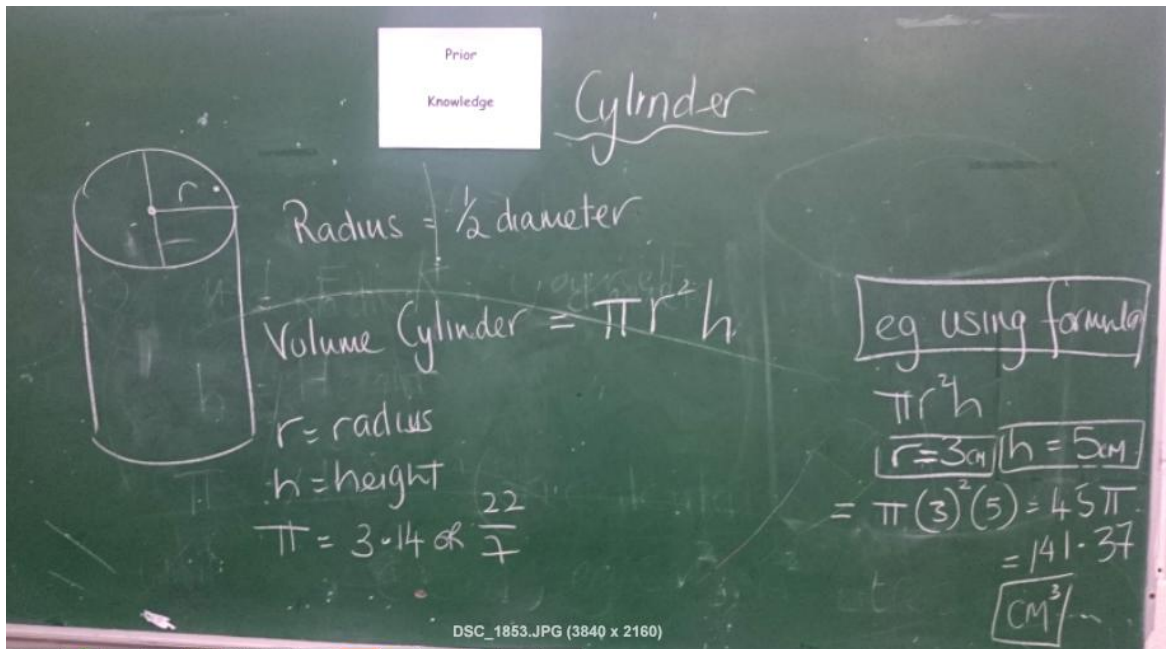|	
the smallest. - Draw how many of the smallest cylinder fit in the biggest cylinder. - Measure the diameter and height of the pipes and calculate using the formula. - Use values of cylinder 1 and 2 and substitute into formula. - Formula : $\pi(r)^2h$, $\pi(2r)^2h$ … - Try different values into the formula - Draw a table of results from their calculations and see a pattern. **See Appendix 2**	Answers could be given in many different forms – in terms of π, etc. See appendix for mathematical answers to potential student responses.
4. Comparing and Discussing Ask students to make connections between the different methods. "Can anyone see similarities between the strategies?" "Would you have any preference for a particular strategy? Why?" "Do you have a problem with any of the strategy? Why?" "Does it matter if you substitute a number for h? Why?" "What is the advantage of leaving the answers in terms of π?"	Focus on doubling the volume. Does this lead to an answer about the tank. Is doubling the radius that same as doubling the diameter?
5. Summing up (3 minutes) Make a statement, can we predict the volume for any cylinder when the radius is doubled and the height remains the same? Now make of note of what you have you learned today? Write down at least 2 of the strategies used above to help you with your homework. **Assessment**: What would happen if the width of the tank is tripled?	

9. Evaluation

- Observers have divided the room into 4 and will observe 6 – 7 students each.
- Observers will take note of student's behaviours, etc on the table we have designed for the lesson observation. **See Appendix 3**
- Student's worksheets will be collected and photographed also to be reflected on later. **See Appendix 4**
- A formal meeting will take place immediately after the lesson to discuss.
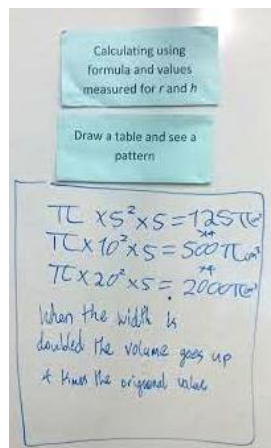
10. Board Plan

This section contains a diagram showing how work on the blackboard will be organized.

The Actual Board from the Lesson:

Most preferred answer by students:

11. Post-lesson reflection

Major Patterns and Tendencies:

- The student's most common mistake: multiplying by 2 = 200L
- Some students measured the cylinder and substituted the values into formula but they did not make any connection between the sizes of the cylinder and the relationship of 4 times the volume.
- The majority tried to substitute some values into the formula.
- A lot of students left their answers in terms of Pi.
- Most students did the supporting materials.
- Students did not draw a conclusion until they were prompted.

Key Observations:

- Photographs
- The formula was used by most students. The practical element was not utilised.
- By the end of the lesson, most students understood the concept, they were able to do the homework task.
- Students were engaged. They learned a lot from one another during the presentations.

Misconceptions and Insights:

- Some students struggled to see the tank was a cylinder shape.
- Students struggled with 2D and 3D. Some confused surface area and volume.
- Many expected the answer to be 200l because of doubling. They predicted that the radius and volume were directly proportional as opposed to proportional to the radius squared.
- Students didn't recognise the cut pipes as cylinders.
- There was confusion between the measuring jugs and the pipes.
- Majority did not use supporting materials, possibly due to nerves as so many adults in the room.
- They did not notice the ratio of the pipes 3:2:1
- One student used x and y as the variables, rather r and h.
- One student did complete the task and get the correct answer.

Did students achieve or not achieve the learning goals?

- One student achieved the goal by using substitution in the formula and confirmed by using the practical tools.
- Majority of students learned the most from the oral presentations. Many took down presentation 2 as it seemed the most logical to them.
- Students were able to do the homework task, therefore had gained some understanding from the lesson.
- Students realised that leaving the values in terms of Pi showed the scale factor of the fact that doubling the radius, increases the volume by 4.
- Students didn't show that they used estimation.

Revisions we would make to the lesson:

- In the previous lesson, explore cylinders in more detail, more everyday examples.
- During Prior Knowledge, ask for and show everyday examples of a cylinder.
- State the goal at the beginning of the lesson. The goal is to find out how much the new tank will hold.
- Show a clear picture of a hot water tank.
- The posing of the task needs longer.
- Read the question clearly and make clarifications.
- Simplify the props – just a bag of cat litter and 2 cylinders.
- Ideally, bring students up to present concrete practical approaches first.
- Emphasise the conclusion.
- Put student's name over the work to give student's ownership.

Implications for teaching

- Adopt more problem solving style questions in our class.
- Use as assessment for learning at the end of a topic.
- Realise the use of manipulates in problem solving.
- Relating to real life situations.
- Engaged more students using the problem solving methodology.
- Allowing students to present their work.
- Cross over between topics, how one lesson could teach so much.

Appendix 1: Worksheet and Props

Task   **Student Name: _____**

Your parents need to replace their hot water tank, to facilitate installing a solar panel. They cannot change the height of the tank but they

can double the width of the tank to fit inside the hot press. If the previous tank could hold 100l, can we predict what the new tank will hold?

Find **as many ways as possible** to answer the question

Resources

Homework and Reflection

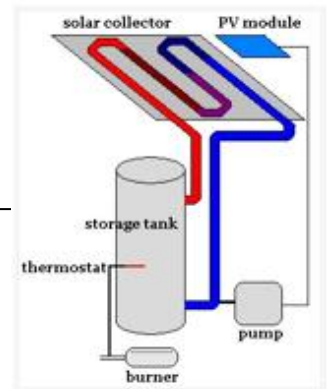## To discover: How the volume of a cylinder changes as the width of the cylinder is changed.

Name _____

Today I have learned

Original Task

Your parents need to replace their hot water tank, to facilitate installing a solar panel. They cannot change the height of the tank but they can double the width of the tank to fit inside the hot press. If the previous tank could hold 100l, can we predict what the new tank will hold?
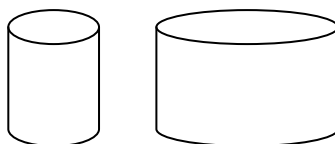
Homework:

What would happen if the width of the tank is tripled?

Appendix 2: Anticipated Student Responses

$$V = \pi r^2 h$$

1. Calculating using formula

"Widths" are 5cm, 10 cm and 15cm

Therefore, radii are 2.5cm, 5cm and 7.5cm

Height is always 5 cm

$$V = \pi(2.5)^2 5 = \frac{125}{4}\pi \text{ or } 31.25\pi \text{ or } 98.2$$

$$V = \pi(5)^2 5 = 125\pi \text{ or } 392.7$$

$$V = \pi(7.5)^2 5 = \frac{1125}{4}\pi \text{ or } 281.25\pi \text{ or } 883.6$$

Doubling the radius means that the volume is multiplied by 4

$$\frac{125}{4}\pi \times 4 = 125\pi$$

100l in old tank x 4 = 400l in new tank

Even if student multiplied π, they would see this relationship.

$$\frac{392.7}{98.2} \approx 4$$

2. Calculating using formula, leaving out height

Height is a constant in this instance

$$V = \pi(2.5)^2 h = 6.25\pi h$$

$$V = \pi(5)^2 h = 25\pi h$$

$$V = \pi(7.5)^2 h = 56.25\pi h$$

$$\frac{25}{6.25} = 4$$

Doubling the radius means that the volume is multiplied by 4

100l in old tank x 4 = 400l in new tank

3. Substituting values for the radius and height

$$V = \pi r^2 h$$

r = 1, h = 1

$$V = \pi(1)^2 1 = 1$$

r = 2, h = 1

$$V = \pi(2)^2 1 = 4$$

r = 3, h = 1

$$V = \pi(3)^2 1 = 9$$

r = 4, h = 1

$$V = \pi(4)^2 1 = 16$$

Doubling the radius means that the volume is multiplied by 4

100l in old tank x 4 = 400l in new tank

Students may see that squaring the radius is why the value increases ╪not 2 as they would have initially thought.

4. Doubling r, without values

$$V_1 = \pi r^2 h$$

$$V_2 = \pi(2r)^2 h = \pi 4 r^2 h$$

Doubling the radius means that the volume is multiplied by 4

100l in old tank x 4 = 400l in new tank

Students may see that squaring the radius is why the value increases ╪ not 2 as they would have initially thought.

Appendix 3: Student Observation Record

5. Beginning of Lesson:
Observe understanding of prior knowledge and of the task

	Student 1	Student 2	Student 3	Student 4	Student 5	Student 6	Student 7	Student 8
(i) Questions on the formula								
(ii) Misconceptions								
(iii) Wording of the task, e.g. "Width"								
(iv) Do students ask for measurements?								
(v) Questions asked by students								
Other Observations								

6. During Lesson:
Observe student engagement and note progress

	Student 1	Student 2	Student 3	Student 4	Student 5	Student 6	Student 7	Student 8
(i) Questions asked to teacher								
(ii) Questions asked to other group members								
(iii) Identify if and when a student used a practical approach How long did they spend? Did they go back?								
(iv) Identify if and when a student used a written approach How long did they spend?								
(v) Did students persist with the task? Or give up? Were prompts required?								
(vi) Rate student understanding of the practical element of the task. 1 = poor 2 = some understanding 3 = competent								
(vii) Rate student understanding of the written element of the task. 1 = poor 2 = some understanding 3 = competent								
Other observations								

7. During Discussion:
Observe student engagement.

	Student 1	Student 2	Student 3	Student 4	Student 5	Student 6	Student 7	Student 8
(i) Are the students attentive to what's happening on the board?								
(ii) Presenters: are clarifications or prompts needed to their board work?								
(iii) Did the presenting students' presentation and discussion promote their teaching and learning?								
(iv) During oral questioning at the end of the lesson, was there evidence of understanding and learning?								
Other observations								
Issues that need to be addressed in the next class								
Recommended changes to the lesson plan								

Appendix 4: Student's Work

100L = full tank
Can fit 2 × W
Can't change height
100L × 2 = 200L = 2 full tanks

1st tank - 100L
2nd tank is double the ~~witch~~
width of the 1st tank - 200L

$$\underline{1}00L + 100L = 200L$$

$$1ool × 2 = 200L$$

100L - 1st tank
? = 2nd tank
200L?

10cm

13cm   Capacity = 100L

10×2

13cm

20cm

13cm

$$\pi r^2 h = \pi(10)^2(13) = 1300\pi$$
$$= 4084.07$$

If you can't double the height
you have to double the width
therefore the diameter which is
10cm should be multiplied by 2.

$$\pi r^2 h = \pi(5)^2(13)$$
$$= 325\pi$$
$$= 1021.01$$

100L → double the width
(200L) the tank will be as
twice as big.

~~[crossed out]~~

if you double the width
the tank looks more
like a rectangle.
Then you have to multiply
the w×h×l to get the
volume.

Double the width get
the diameter of the
opening half it and
get the radius

then measure the
height and using the
$v = \pi r^2 h$ formula
get the new volume
of the tank.

Poor in water in the
tank until it's full.
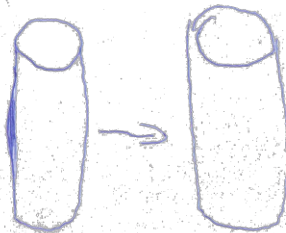Count the liters while
you pour it.

Shape

Find new volume

$V = \pi r^2 h$

Doubling the width
can hold 100L

Same be height but Double
width

Fill a container
width aved
than Fill a container
with double the width

example
Original width
2m
Original height
= 4m

after Doubling it

width = 4m
height stays the same
height = 4m

use formula

$V = \pi r^2 h$

$\pi (2)^2 (4) = 16\pi = 50.265$

Panel 1:

$r = x$
$h = y$

$\pi r^2 h$

$\pi (x)^2 (y)$

$= 100L$

$x(2)$

$\downarrow$

$\pi (x(2))^2 (y) =$ whatever new tank holds

Panel 2:

Vol of tank $= 100L = 100 cm^3$

Vol of Cylinder $= \pi r^2 h$

~~xxx~~ ~~V~~ $V = \pi (4)^2 (8.3)$

$= 417 cm^3$

$A = 2\pi r h$

$= 2\pi (4)(8.3)$

$= 208 cm^2$

$h\, 8.3$
x
$8cm$

Double Width: $V = \pi (8)^2 (8.3) = 1669 cm^3 = 4 \times$ the volume

Answer $= 400L$

Panel 3:

$\pi \times x^2 \times y = 100L$

$50mm$

$100mm$

$\pi \times 5^2 \times 5 = 125\pi$
$\times 4$

$\pi \times 10^2 \times 5 = 500\pi$
$\times 4$

$\pi \times 20^2 \times 5 = 2000\pi$
$\times 4$ ~~6000~~

$\pi \times 40^2 \times 5 = 8000\pi$

when the width is doubled the volume goes up 4 times. The new tank should hold 400L