Discovering the sum of an arithmetic sequence

Topic: Arithmetic Sequences

Year Group: Fifth Year Ordinary Level **Lesson Plan Taught:** 2 March 2016

Venue: Errigal College, Letterkenny, Co. Donegal.

Teacher: Janet Mc Geever

Lesson plan developed by: Janet McGeever

Eimear Logue

Deirdre Logue

(Errigal College)

Title of the Lesson

"Polly's Sum Selfie - Discovering the sum of an arithmetic sequence."

Brief description of the lesson

The lesson aims to allow students to discover how to develop a method of finding the sum of the terms in an arithmetic sequence.

Aims of the lesson

- To use real life problems as vehicles to motivate the use of algebra and algebraic thinking.
- To develop an understanding of arithmetic sequences and series.
- To connect and review the concepts that we have studied already.
- To become more creative at devising approaches and methods to solve problems.
- To foster students to become independent learners and thinkers.
- To emphasise that a problem can have several equally valid solutions.
- For students to understand the relationship between the pattern of a real life problems and the relevance to a mathematical class.
- To encourage students to discuss the mathematics they are studying and to learn from each other's viewpoints.

Learning Outcomes

- Students will model arithmetic sequences with manipulatives and on graph paper.
- Students will derive formulas for arithmetic sequence terms and partial sums of series.
- Students will communicate to others why the formulas work.
- Students will apply knowledge of partial sums and series to problem situations.

As a result of studying this topic students will be able to:

- Explain their findings.
- Explore patterns and formulate conjectures.
- Find the underlying formula written in words from which the data is derived (linear relationships).
- Show that relations have features that can be represented in a variety of ways.
- Appear in different representations: in tables, graphs, physical models, and formulae expressed in words.
- Use the representations to reason about the situation from which the relationship is derived and communicate their thinking to others.
- Discuss rate of change and the y-intercept; consider how these relate to the context from which the relationship is derived, and identify how they can appear in a table, in a graph and in algebraic form.

Background and Rationale

(Leaving Certificate Mathematics Syllabus, 2015, p36) Students need to know the following.

Topic	Description of topic *Students learn about*	Learning outcomes *Students should be able to*
4.1 (a) Generating arithmetic expressions from repeating patterns	Patterns and the rules that govern them; students construct an understanding of a relationship as that which involves a set of inputs, a set of outputs and a correspondence from each input to each output.	– use tables to represent a repeating-pattern situation – generalise and explain patterns and relationships in words and numbers – write arithmetic expressions for particular terms in a sequence
4.1 (b) Representing situations with tables, diagrams and graphs	Relations derived from some kind of context – familiar, everyday situations, imaginary contexts or arrangements of tiles or blocks. Students look at various patterns and make predictions about what comes next.	– use tables, diagrams and graphs as tools for representing and analysing linear patterns and relationships – develop and use their own generalising strategies and ideas and consider those of others – present and interpret solutions, explaining and justifying methods, inferences and reasoning
4.1 (c) Finding formulae	Ways to express a general relationship arising from a pattern or context.	– find the underlying formula written in words from which the data is derived (linear relationships)
4.1 (d) Examining algebraic relationships	Features of a linear relationship and how these features appear in the different representations. Constant rate of change. Proportional relationships.	– show that relations have features that can be represented in a variety of ways – distinguish those features that are especially useful to identify and point out how those features appear in different representations: in tables, graphs, physical models, and formulae expressed in words – use the representations to reason about the situation from which the relationship is derived and communicate their thinking to others – discuss rate of change and the y-intercept; consider how these relate to the context from which the relationship is derived, and identify how they can appear in a table, in a graph and in a formula – decide if two linear relationships have a common value – recognise problems involving direct proportion and identify the necessary information to solve them
4.1 (e) Relations without	Using graphs to represent	– explore graphs of motion

Relating a pattern to other aspects in the syllabus.

How to discover the nth term of the pattern.

Warm Up Conversation

Begin the lesson with a class discussion of patterns. Pre-assess their ability to think about the differences between patterns. Wear an article of clothing that day that has some sort of pattern on it. Point it out to the students and ask them questions to initiate discussion. What pattern do you see on my shirt, pants, etc.? How do you know that it is a pattern? Where do we find patterns in our classroom, homes, school, neighbourhood, students, etc.? Are there different kinds of patterns? Brainstorm these ideas and record them on the chalkboard.

Research

Algebra is a language used for representing and exploring mathematical relationships. The view of algebra expressed in current curriculum documents emphasizes multiple representations of relationships between quantities, and stresses the importance of focusing student attention on the mathematical analysis of change in these relationships. It is the relationships component of algebra that gives purpose and meaning to the language of algebra. Without a focus on relationships, the language of algebra loses its richness and is reduced to a set of grammatical rules and structures. Consequently, though students learn to manipulate algebraic expressions, they do not seem able to use them as tools for meaningful mathematical communication. The majority of students do not acquire any real sense of algebra and, early on in their learning of the subject, give up trying to understand algebra and resort to memorizing rules and procedures. Many students may find the rules of algebra arbitrary, because all too often they are unable to see the mathematical objects to which these rules are supposed to refer. It has been suggested that students be given meaningful experiences in algebra learning, involving the exploration of multiple representations of concepts (Borba & Confrey, 1996, pp. 319–320; Kieran & Sfard, 1999, p. 3). It has also been suggested that the traditional approach to teaching algebra, which typically starts with symbolic representation and decontextualized manipulation and moves on to visual and graphical representation and problem-based contexts, should be reversed (Borba & Confrey, 1996, pp. 319–320). Graphs, which are often treated as a mere add-on to algebra, could become the foundation of algebra teaching and learning (Kieran & Sfard, 1999, p. 3).

About the Unit and the Lesson

The Syllabus states that you must be able to use tables to represent a repeating-pattern situation, generalise and explain patterns and relationships in words and numbers, write arithmetic expressions for particular terms in a sequence. Use tables, diagrams and graphs as tools for representing and analysing linear, quadratic and exponential patterns and relations. Students must be able to develop and use their own generalising strategies and ideas and consider those of others present and interpret solutions, explaining and justifying methods, inferences and reasoning. Find the underlying formula algebraically from which the data is derived. Basically recognise patterns, try to find a general term for the pattern and try to find more terms of the pattern.

Flow of the Unit

Lesson		**# of lesson periods**
	Revision of JC pattern material - linear relationship between two variables - first difference	1 x 35 min.
1	• Graphing a linear pattern.	1 x 35 min.
2	• Using patterns to represent numbers	1 x 35 min.
3	• Discovering the formula for an arithmetic pattern	1 x 35 min.
4	• **Discovering the sum of an arithmetic pattern**	**3 x 35 min. (#1 = research lesson)**
5	• Introducing Sn formula.	1 x 35 min.
6	• Applied problems on arithmetic sequences.	1 x 35 min.

Flow of the Lesson

Teaching Activity	Points of Consideration
1. Introduction Verbally prior knowledge is tested through questioning of students. Asking students for all the different ways patterns had been represented in the previous lesson. The students are presented with a real life problem about the number of likes a picture got on Twitter and are asked to determine the type of pattern.	Students may need to be prompted by reminding them of previous problems they encountered relating to patterns. Do students understand that there is a pattern? Do students understand that pattern has a common difference?
2. Posing the Task Today we are going to show our understanding of linear patterns. The teacher gives the problem to the students and asks the students to determine the number of additional followers the picture attracted on the 10th minute.	Can students find the number of additional followers the picture has? Can students explain how they achieved this answer?
3. Anticipated Student Responses Most students should be able to find the number of additional followers the picture attracted on the 10th minute. Students may find it difficult to explain in words their mathematical reasoning.	Teacher will respond to each student answer stating and explaining if it is correct or incorrect.

4. Comparing and Discussing Teachers will check what students have done different methods. Solutions will be rated from 1 (easy) upwards. The students that completed the task using the easiest method will be invited to present their solutions on the board. The next easiest solutions will then be presented on the board and each solution up the hardest. All solutions will remain on the board for the duration of the lesson.	Do students provide different methods of finding the number of additional followers the picture attracted on the 10th minute. Are students confident explaining their reasoning? Students should be prompted to help them to explain their solutions.
5. Posing the Task The teacher asks the students 'How many followers in total does Polly have at exactly 6.10pm?	Can students find the number of additional followers the picture has? Can students explain how they achieved this answer?
6. Anticipated Student Responses Most students should be able to find the total of additional followers the Polly attracted on the 10th minute. Students may find it less difficult to explain in words their mathematical reasoning as they have already explained the previous question.	Teacher will respond to each student answer stating and explaining if it is correct or incorrect.

4. Comparing and Discussing Teachers will check what students have done different methods. Solutions will be rated from 1 (easy) upwards. The students that completed the task using the easiest method will be invited to present their solutions on the board. The next easiest solutions will then be presented on the board and each solution up the hardest. All solutions will remain on the board for the duration of the lesson. The teacher asks the students to explain their thinking in finding the total number of followers in the 10th minute.	Do students provide different methods of finding the total number of followers the Polly attracted on the 10th minute. Are students confident explaining their reasoning? Students should be prompted to help them to explain their solutions.
8. Summing up The teacher asks the students what they have learned in the lesson. The teacher highlights that there are many ways to solve a problem and that each solution is as good as the other. Each method results in the same solution, however the length of the solutions may vary. The teacher will ask students to describe each method and state the preferred method. The teacher praises the students for solving the problem themselves without teacher intervention.	Do students appreciate that they solved the problem themselves? Are students confident to apply their own logic to a mathematical problem? Can students see that there many ways of solving a problem?

Evaluation

We plan to observe the different methods the students come up with to solve the problem.

The local Regional Development Officer and Eimear Logue (Teacher) will observe the lesson.

Data will be recorded in observations sheets under the following headings: Prior Knowledge, Posing the problem, Problem Solving Section of the lesson, Student Feedback, Unexpected Outcomes, Expected outcomes.

Seating plans will be given to observers also.

Photos of student's work will be taken throughout the lesson.

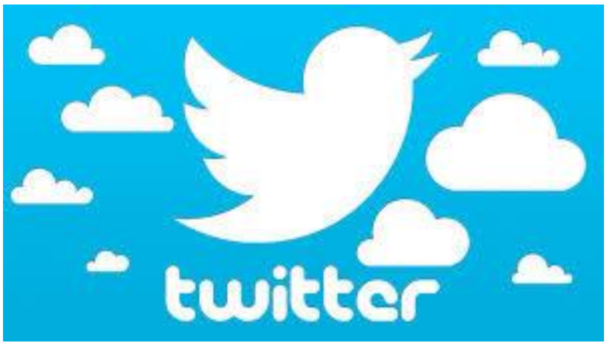
Questions the students ask will be recorded. Observers will look out for self-correction of student work.

We plan to focus on students misconceptions of the question. We will look out for students self-correcting their work.

Students' learning will be assessed through:

- Verbal responses
- Observation of student
- Student questioning
- Diverse methods of answering the question

Problem:



When Polly logged on to Twitter at 6pm she already had 1 follower. She uploaded a funny photo of herself. By 6.01pm Polly had gained an additional 3 followers. Amazingly by 6.02pm Polly had gained another additional 5 followers. This pattern continued indefinitely.

i) How many new followers did Polly gain between 6.09pm and 6.10pm?

ii) How many followers, in total, does Polly have at exactly 6.10pm?

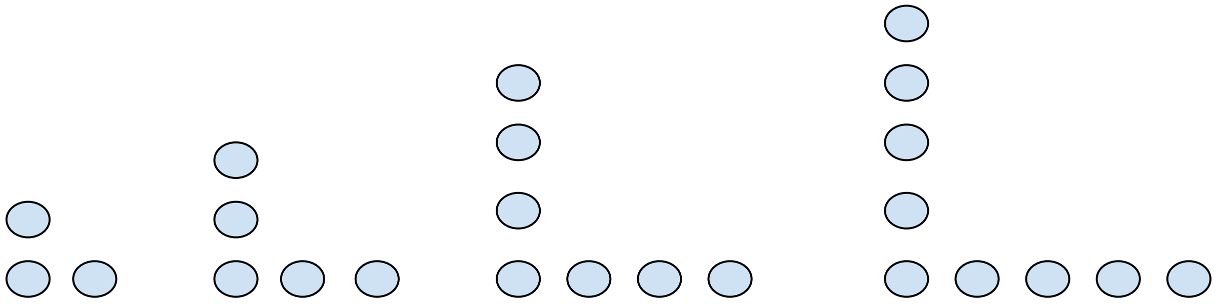
Anticipated Solutions

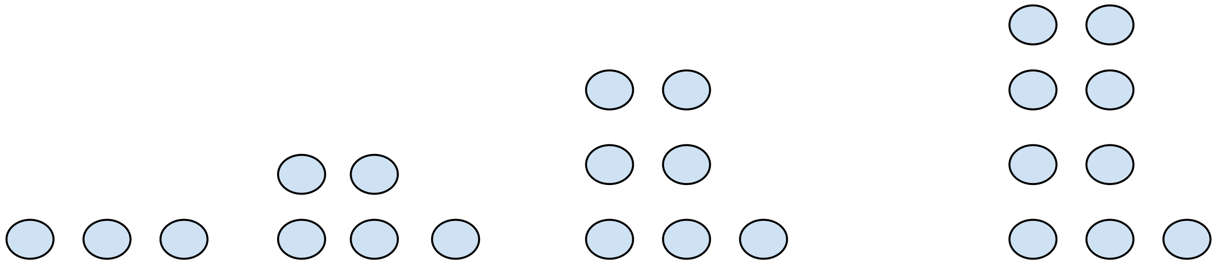
METHOD 1: (NUMBER PATTERN)

i) 3, 5, 7, 9, 11, 13, 15, 17, 19, 21

iii) 1 + 3 + 5 + 7 + 9 + 11 +13 +15 + 17 +19 + 21 =

METHOD 2: (VISUALISING THE PATTERN)





$$Tn = 2n + 1$$

METHOD 3: (DISCOVERING THE PATTERN IN THE FORMULA)

n = 1 $2(1) + 1$= 3

n = 2 $2(2) + 1$ = 5

n = 3 $2(3) + 1$ = 7

n = 4 $2(4) + 1$ = 9

n = 5 $2(5) + 1$ = 11

METHOD 4: (LOOKING AT INPUTS AND OUTPUTS)

x	2n + 1	f(x)
1	$2(1) + 1$	3
2	$2(2) + 1$	5
3	$2(3) + 1$	7
4	$2(4) + 1$	9
5	$2(5) + 1$	11

METHOD 5 (USING THE FORMULA):

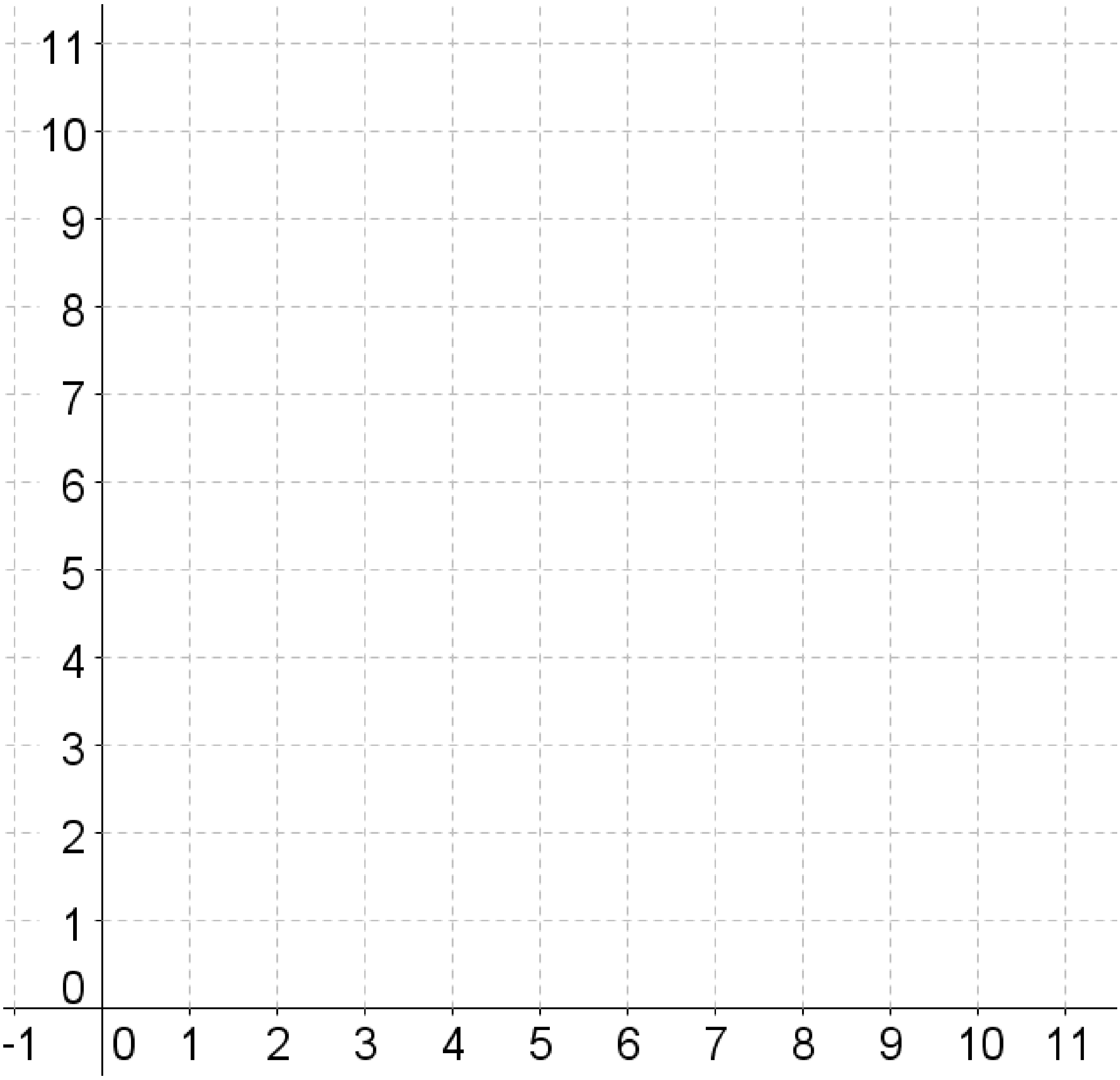
3, 5, 7, 9, 11, 13, 15

First Difference: +2, +2, +2, +2, +2, +2, +2

$$T_n = 2n + 1$$

METHOD 6: (GRAPHING THE PATTERN)

Graph y = 2x + 1



Post-lesson reflection

Misconceptions

1. $\frac{2}{9}$students found the sum of the first 10 terms to be 121. They then multiplied 121 by 3 to find the sum of the first 30 terms.
2. I will revisit the difference in thick lines and dotted lines when plotting.
3. Students were very confused around $T_{0.}$ I would spend more time on this in future.

General observations

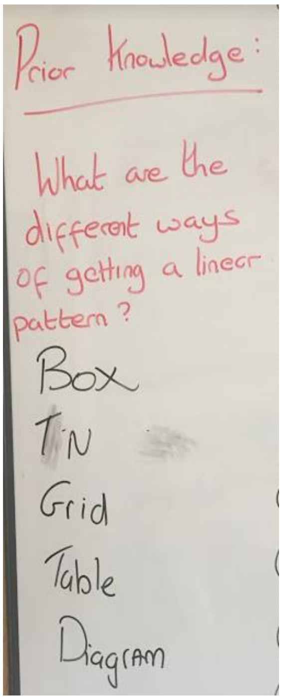
- I was happy that a student used the word "linear" to describe the pattern.
- Students spoke confidently about the first term and the difference but struggled to link these to a and d.
- A weak student came up with a summation table for the first 10 terms, which was pleasantly surprising.
- Explain what is meant by T0.
- Make a summation table for 30 terms as many students feel that 3(S10) = S30.
- Revisit the difference in thick lines and dotted lines when plotting.
- Practice questions like the like flowing:

  2(n-2)   and (n-2)2

Suggested Changes to the Lesson for Future use

- I would encourage the students to work further in their table rather than just working to the 10th pattern.
- The prior knowledge that the students came up with at the beginning of the lesson was key to approaching this task. I strongly encourage the revision of prior knowledge at the beginning of every class.

Photos from our Lesson



Prior Knowledge:

What are the different ways of getting a linear pattern?

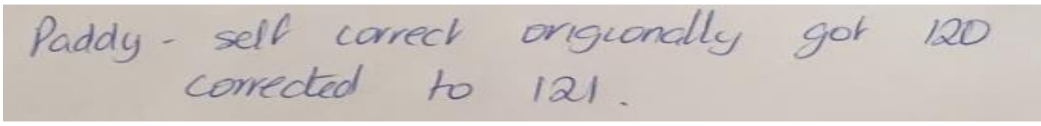
Box

TN

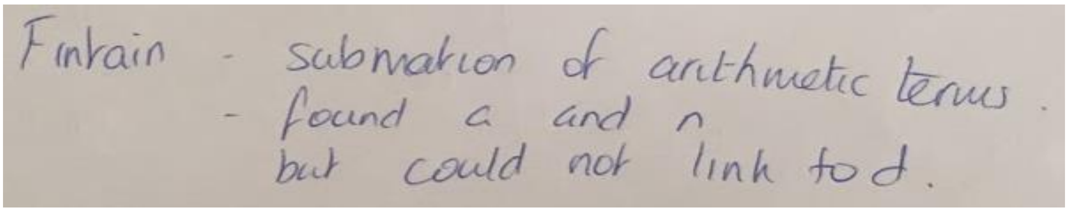
Grid

Table

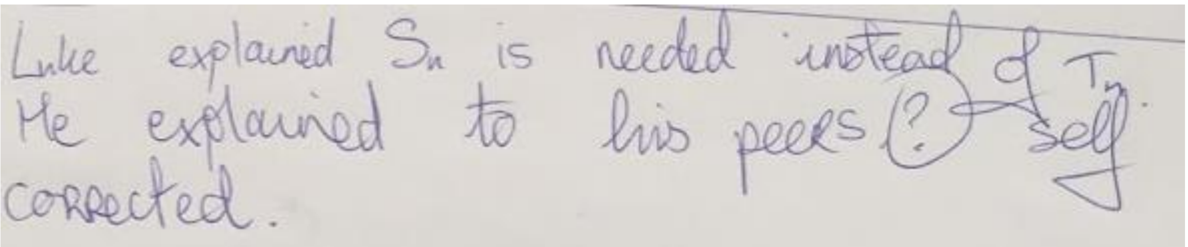
Diagram



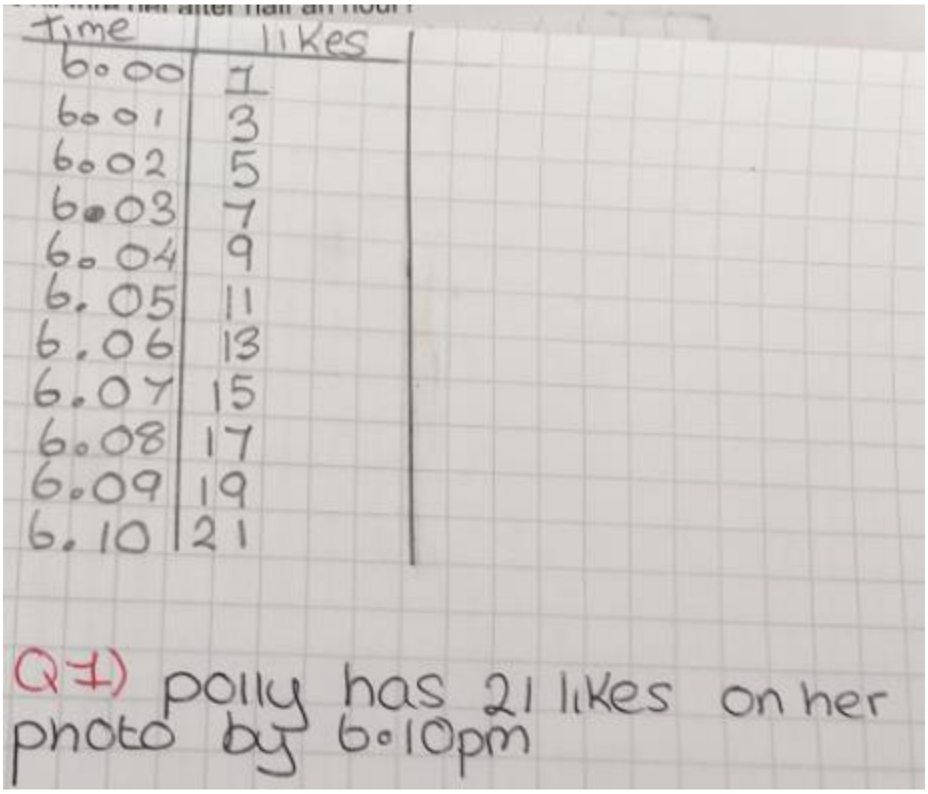
Paddy - self correct origionally got 120 corrected to 121.



Finrain - submation of arithmetic terms.
- found a and n but could not link to d.

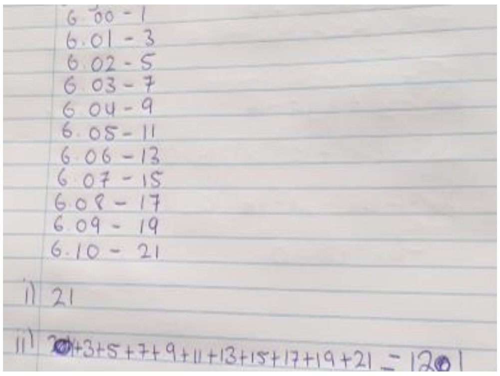


Luke explained S_n is needed instead of T_n. He explained to his peers (?) self corrected.



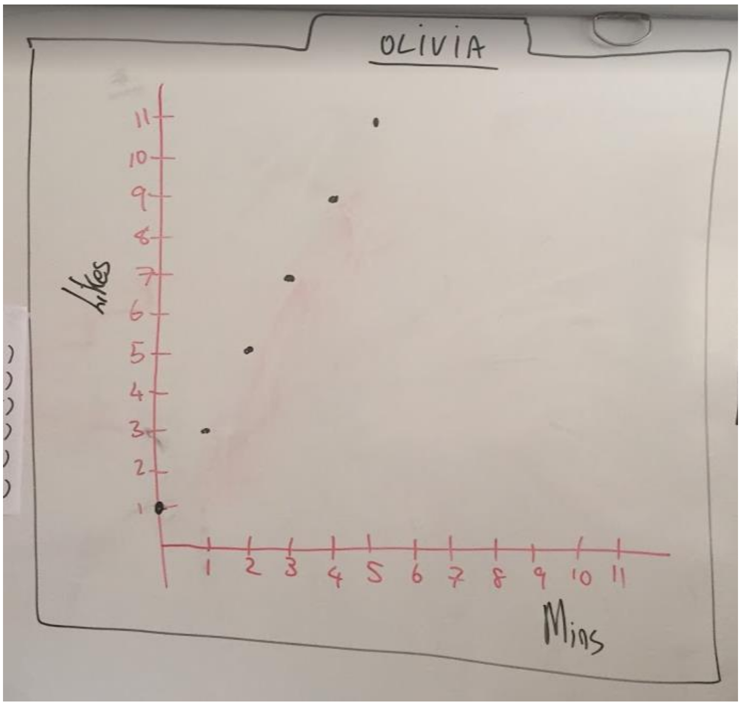
Time	likes
6.00	1
6.01	3
6.02	5
6.03	7
6.04	9
6.05	11
6.06	13
6.07	15
6.08	17
6.09	19
6.10	21

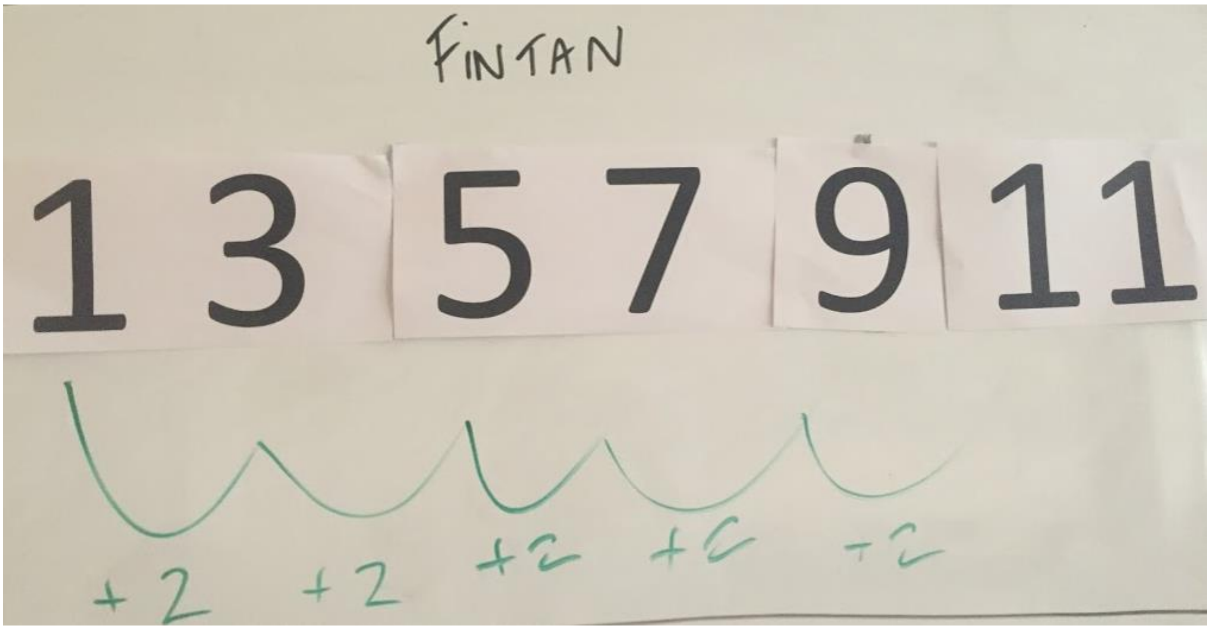
Q1) polly has 21 likes on her photo by 6.10pm

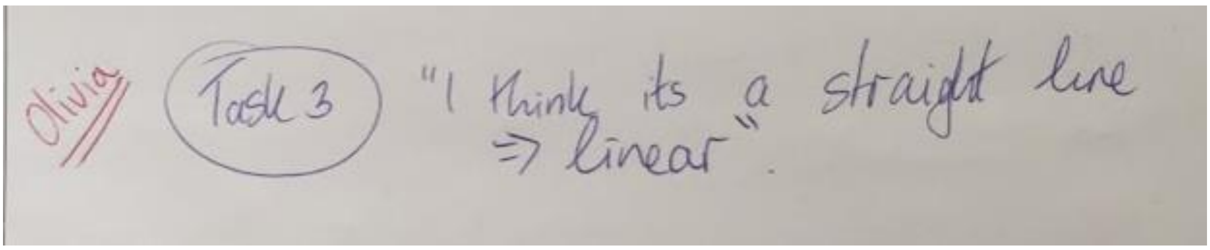

6.00 - 1
6.01 - 3
6.02 - 5
6.03 - 7
6.04 - 9
6.05 - 11
6.06 - 13
6.07 - 15
6.08 - 17
6.09 - 19
6.10 - 21

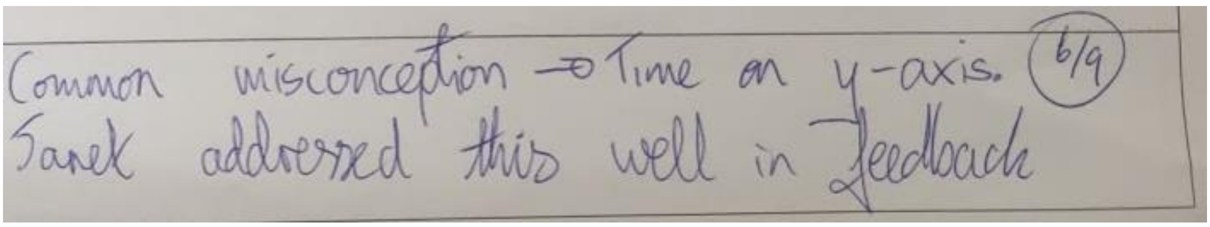
i) 21

ii) 1+3+5+7+9+11+13+15+17+19+21 = 121

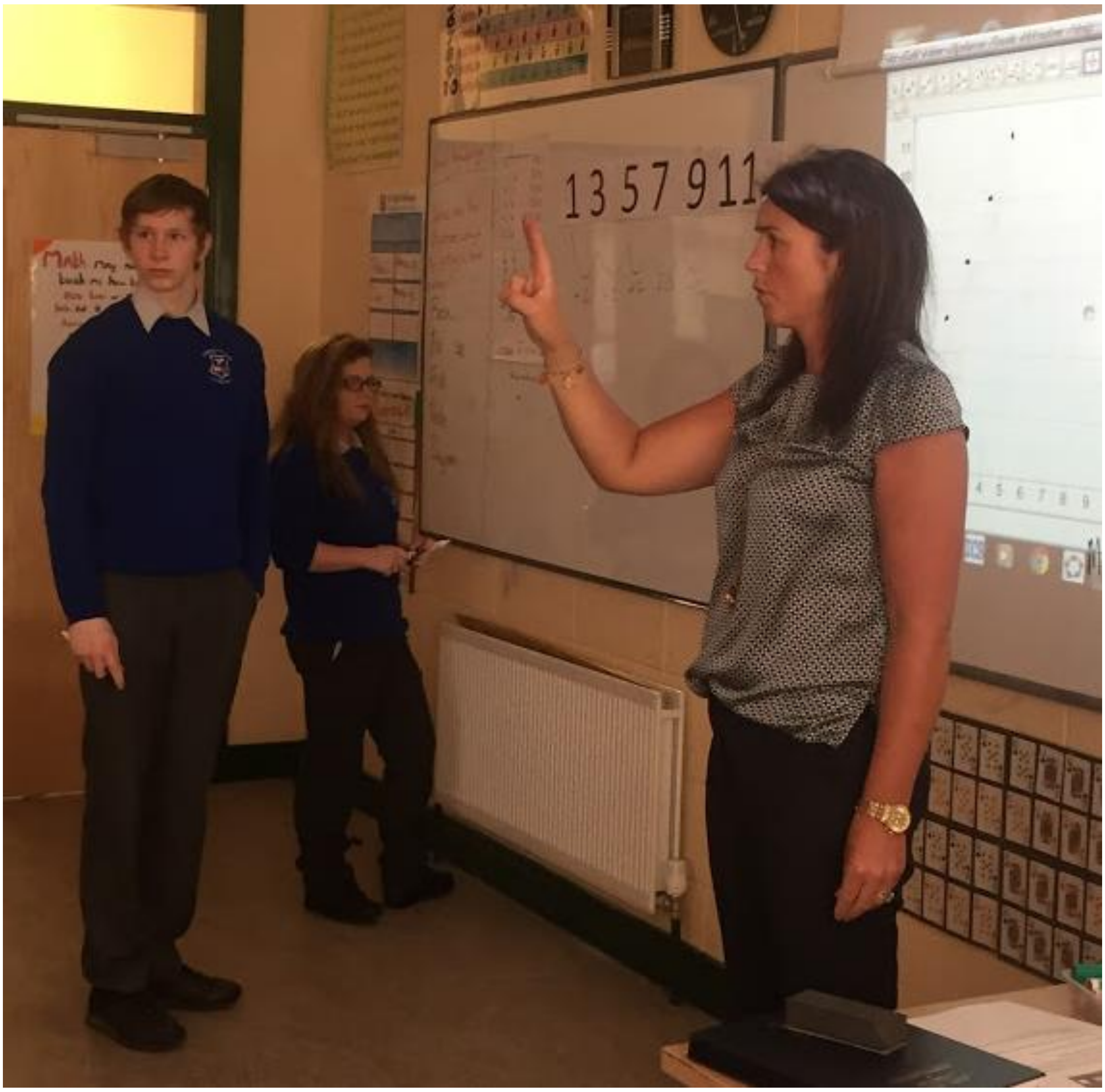




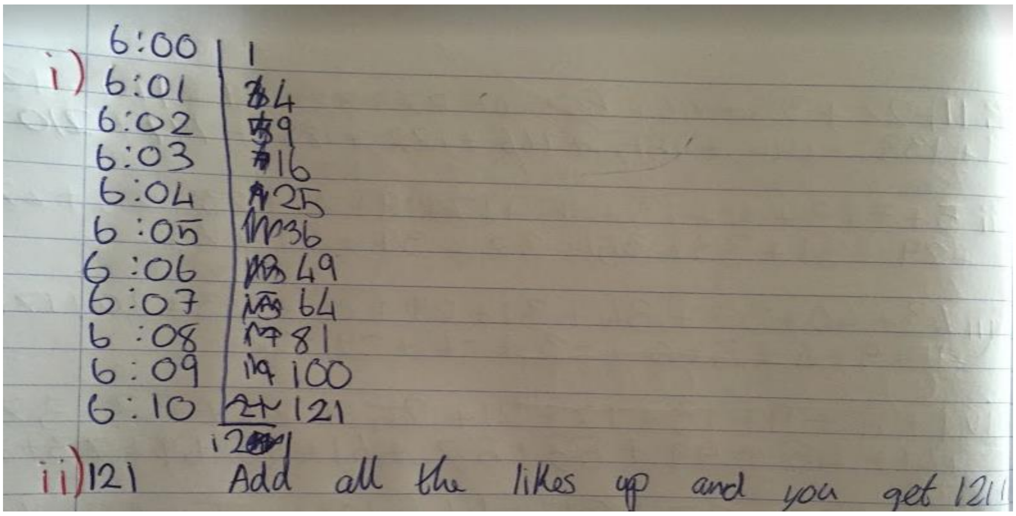

Olivia Task 3 "I think its a straight line ⇒ linear".


Common misconception → Time on y-axis. 6/9
Sarah addressed this well in feedback

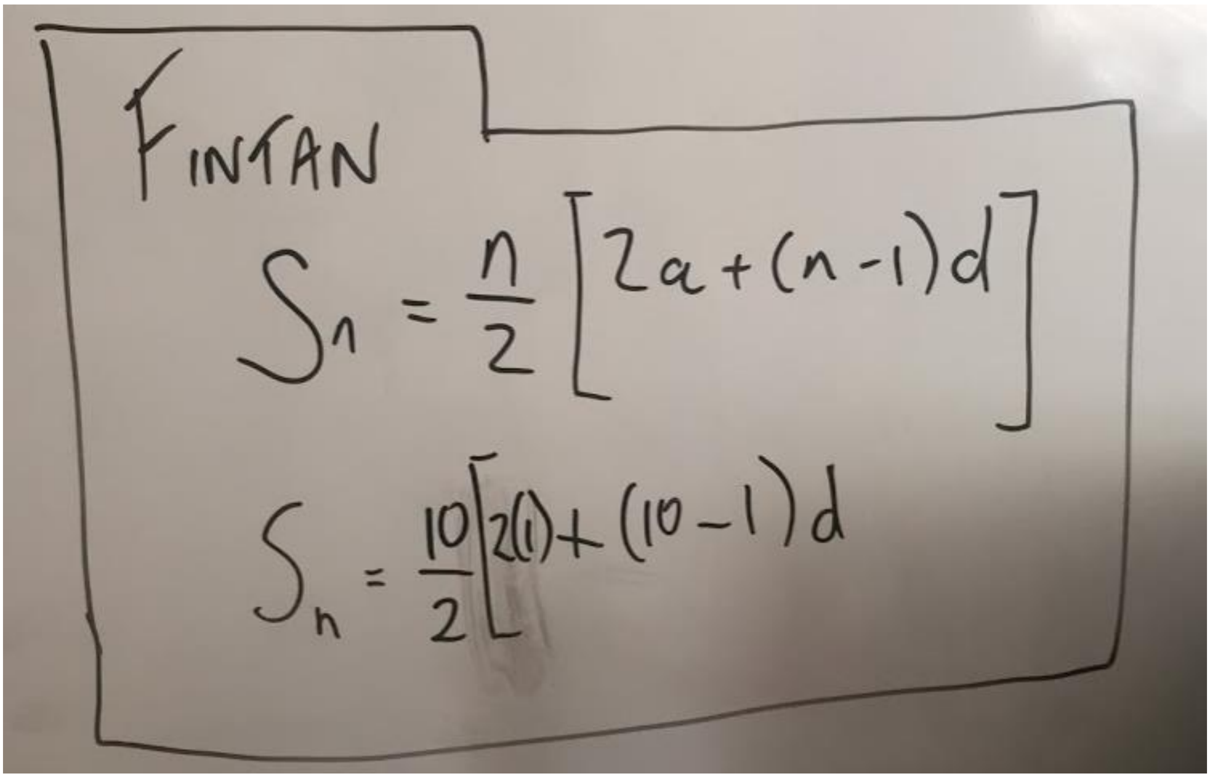
Students presenting their answers on the board while teacher discussing possible misconceptions.



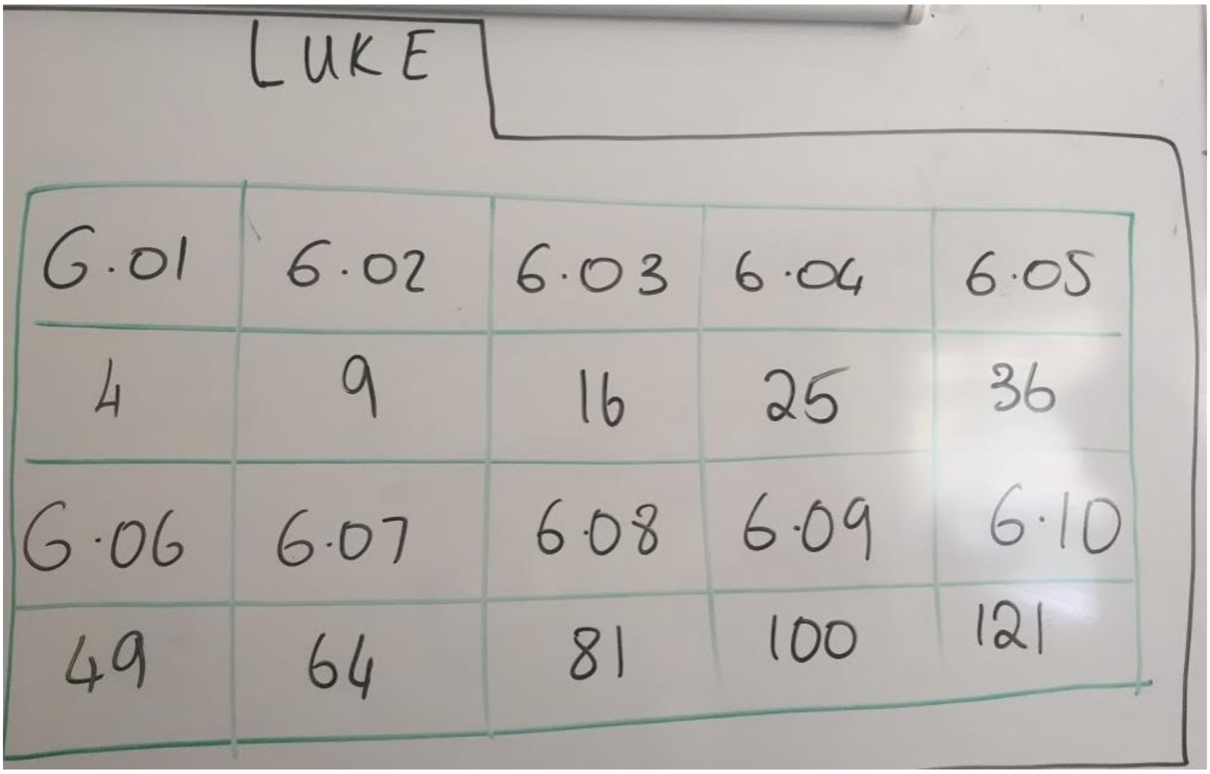
Here a student self-corrected their work. Initially the student created a pattern of new likes then changed to the summation of likes.

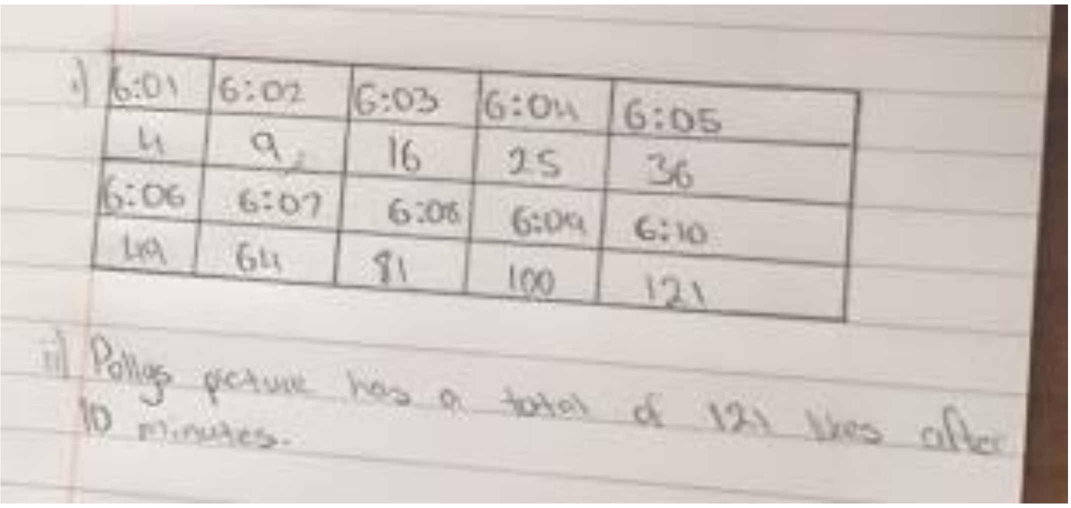


Fintan checked the log tables and discovered the formula for the sum of an arithmetic pattern. He was able to find a and n but struggled to find d.



Luke came up with a summation table for the first 10 terms of the sequence. We considered this to be the best answer to the problem.



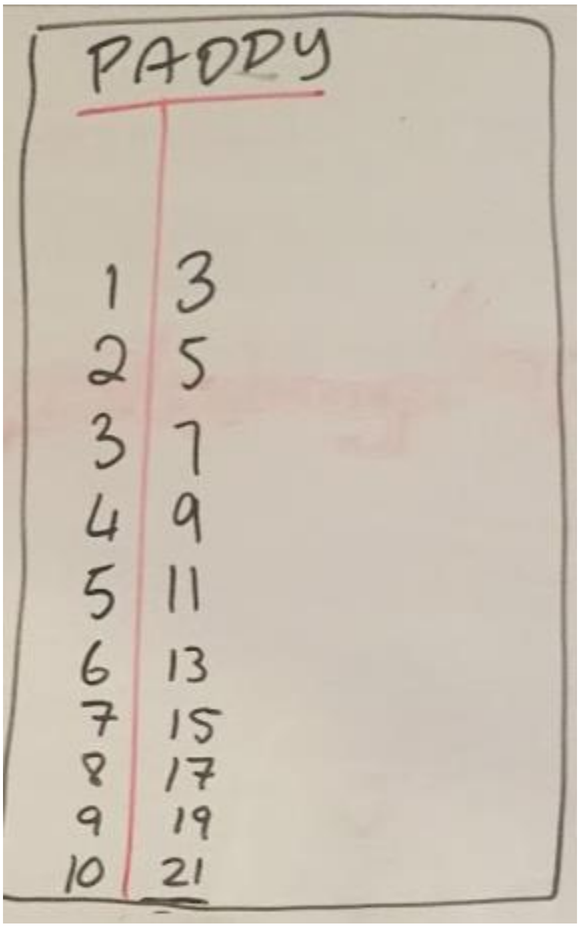


i)

6:01	6:02	6:03	6:04	6:05
4	9	16	25	36
6:06	6:07	6:08	6:09	6:10
49	64	81	100	121

ii) Pollys picture has a total of 121 likes after 10 minutes.

Paddy went with what we considered to be the most simple straight forward method.



PADDY

1	3
2	5
3	7
4	9
5	11
6	13
7	15
8	17
9	19
10	21