Unit and Lesson Plan for Grade 7 (13 and 14 Years Old Students)

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1. Title of the Unit: Ways of counting and mathematical expressions

2. Brief description of the Unit

This unit is designed for students to be able to represent, using mathematical expressions various ways of counting; understand what each number in the mathematical expression is representing; and use those mathematical expressions to solve problems

3. Goals of the Unit:

- Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.
- Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.

4. Students' prior learning

- Natural numbers: factors, multiples, indices, primes, HCF, LCM, Number properties, order of operations
- Integers: Number line, negatives, addition subtraction, division multiplication.
- Rational Numbers: Adding subtracting, Dividing, multiplying, fractions, ratio and proportion
- Decimals and percentages, Fractions and rounding
- Number patterns: arranging objects, linear sequences, general term, non-linear patterns, problems
- Algebra Introduction: Evaluating expressions, adding, subtracting multiplying terms, multiplying expressions
- Algebra: Linear Equations: Solving Linear equations, equations with brackets

5. Background and Rationale

This unit is designed for the students to understand the usefulness of mathematical expressions through solving a series of problems by themselves and comparing their solutions with several different approaches by their peers.

Since early elementary grades, students have been using mathematical expressions to express quantitative relationships in story problems with everyday contexts. From their experience they began to understand the usefulness of mathematical expression for determining what operation can be used to solve problems.

Building upon their prior learning, the students will experience communicating their own reasoning to others by expressing their own ways of counting in mathematical expressions. At the same time, other students will have opportunities to infer their peers' ways of thinking by interpreting their mathematical expressions. Throughout, students will be expected to justify the reasonableness of their own and others' counting strategies and mathematical expressions. These experiences are expected to foster their skill for

developing viable arguments and critiquing the reasoning of others while the process of writing and interpreting mathematical expressions helps students develop the ability to reason quantitatively and abstractly.

Students will also experience the value of mathematical expressions for determining the number of objects when counting one by one is too difficult, by recognizing the generalizability of the mathematical expressions they write to express their way of counting. Through these experience students are expected to find the usefulness of using mathematical expressions as a model for expressing mathematical situations. As a result it is expected that the students will grow in their ability to use mathematical expressions to analyze relationships mathematically and draw conclusions. This creates the foundation for student to eventually be able to represent, using variables, two quantities that change in relationship to one another.

6. About the Unit and the Lesson

A mini unit is designed based on the problems from the Japanese curriculum, *Mathematics International* published by Tokyo Shoseki.[1]

In the first lesson, the diagram shown as Figure 1 will be given for the students.

First, the students will work independently to come up a way of counting the total number of dots. Then each student will express his/her own ways of counting in a mathematical expression.

Second, through whole class discussion, the students will be given the opportunity to share their own mathematical expression to the class, and the class will given opportunities to infer the method of counting by interpreting students' mathematical expressions.

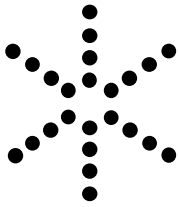


Figure 1.

Third, the students will be asked how many dots there will be if the dots in each arm of the star shape increases. This question is designed for students to explore the generalizability of mathematical expressions.

The dots in the diagrams from this lesson can be easily counted one-by-one, which makes it easy to verify the results obtained from mathematical expressions.

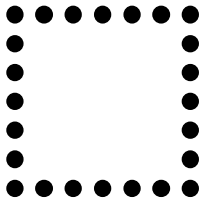


Figure 2.

In the second lesson, based on what students learned from lessons 1 about how to express in a mathematical expression their ways of counting dots, students are given opportunity to find the number of dots without counting one-by-one. The diagram shown as Figure 2 will be given to the students and they will be asked to find out the total number of dots. As an expanding learning students will be asked to find out the total number of the dots when each side of the diagram becomes 10 dots. Although students will be encouraged to find out the total number without drawing the diagram, doing so is still possible for any student who really needs it. This task provides a transition for students to see the possibility of finding the total number of dots in the absence of an actual diagram by using mathematical expressions. The

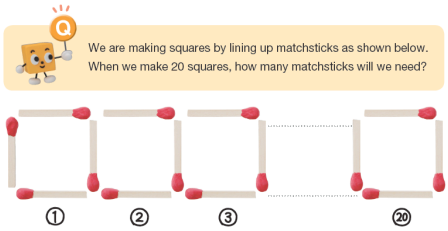


Figure 3.
Reprint of Mathematics International
Grade 7 p. 50

[1] http://www.globaledresources.com/products/books/mathematics-international/index-7-9.html

students are expected to recognize on their own that the numbers in their mathematical expressions may be generalized.

The third lesson is designed for students to apply all their learning from the previous problem solving lessons to a different situation. The stick problem, as shown as Figure 3, will be presented. The students will be asked to determine the number of sticks required to make a row of 20 adjacent squares. Students will explore a task in which the actual counting may not be easy so that they can utilize what they experienced in previous lessons about the generalizability of mathematical expressions. Although the arrangement of sticks will be shown as in Figure 3, the actual number of sticks will not be given to the students. Students are expected to come up with their own mathematical expressions to find the total number of sticks needed. In the whole class discussion, students will examine various mathematical expressions to see if they are reasonable.

Each of the three problem solving lessons described above will foster students' reasoning by challenging them to come up ways to find the number of objects in patterns, to express their ways of thinking in mathematical expressions, and to justify whether a mathematical expression is reasonable. At the same time, students will have opportunities to discuss the rules of arithmetic calculation (such as the use of parentheses and the order of operations).

7. Flow of the Unit

Lesson	Learning objective(s)	
1	Let's think about ways to show the number of dots. Students will try to express their ways of counting dots in mathematical expressions and also infer other students' ways of counting from their mathematical expressions.	40 min
2	Find the number of dots without counting one-by-one Students will be able use mathematical expressions to express the number of dots and use the expressions to find the total number of dots without counting one-by-one.	60 min
3	How many sticks are there all together? Students will express their ways of counting sticks in mathematical expressions and use their mathematical expressions to determine the number of sticks even when they cannot actually count the number of sticks.	60 min
4	Becoming fluent for using mathematical expressions to solving similar problems. (Exercise)	40 min

8. Demonstration Lesson Plans

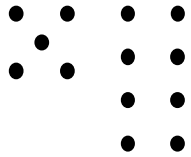
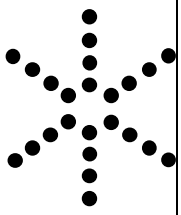
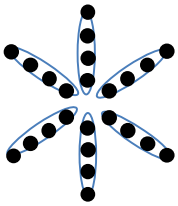
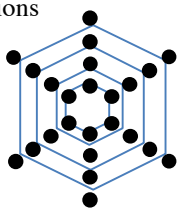
Lesson 1 (40 minutes): Let's think about ways to show the number of dots.

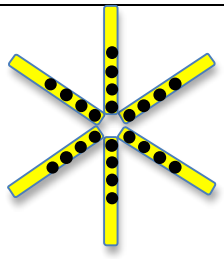
(1) Objectives

- Students will express their ways of counting dots in mathematical expressions and infer other students' ways of counting from their mathematical expressions.

(2) Flow of the lesson

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher's Support	Points of Evaluation
1. Introduction Understand how to express ways of counting in		Students understand what it

mathematical expression in simple cases. Using the two diagrams shown on the right to help students see how mathematical expressions can be used to show ways of counting the number of dots.  4+1=5 2x4=8 4x2=8 Help students remind that 2x4 and 4x2 are different ways to look at the diagram.	By briefly showing the diagrams one at a time, encourage students to share their ways of counting the number of dots in the diagrams. Remind multiplication is (number of group)x(number in each group)	means to show their way of counting using a mathematical expression and are ready for solving the problem
2. Posing the Problem Show the diagram on the right and ask the following question. Think about a way to count the number of dots in the picture shown on the right. Write a mathematical expression that describes your method of 	If students seem to not understand the task, share a few of the students' attempts at expressing their ways of counting in mathematical expressions as examples.	Students understand that the task is to express ways of counting with mathematical expressions.
3. Anticipated Student Responses 1) Making groups of dots in the diagram. e.g. 4+4+4+4+4+4=24  2) Making equal groups to use multiplications e.g. 6x4=24  4x6=24 3) Decompose 24 in arbitrary ways and show them in mathematical expressions. e.g. 10+10+4=24, 12x2=24	Let each student write his/her way of counting and mathematical expression in his/her note. Using a seating chart to note each student's way of counting and mathematical expressions to prepare for organizing the whole class discussion.	Each student comes up with at least one mathematical expression to express how to count the number of dots.
4. Comparing and Discussing For each mathematical expression, 1) Ask one of the students who came up with a mathematical expression to show it to the class, 2) Let other students infer how the student counted the dots by interpreting the mathematical expression, and 3) let the student who came up with the mathematical expression justify if the other student's inference is correct. Repeat the above so students understand a variety of ways to count the number of dots by making equal groups.	By providing an opportunity to infer other students' way of counting, help students see that a mathematical expression can communicate a way of thinking. Help them recall their prior learning, such as the meanings of operations and the order of operations to examine if they can express their ways of counting correctly.	Students can use their prior learning such as the meanings of operations and the order of operations to express correctly their ways of counting.
5. Expanding the learning Using a mathematical expression developed when there were 4 dots on each arm of the star shape, determine the	Encourage students to find the total number of the dots	Students try to use a mathematical

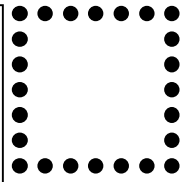
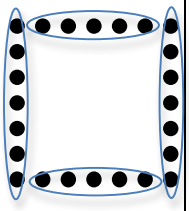
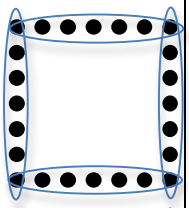
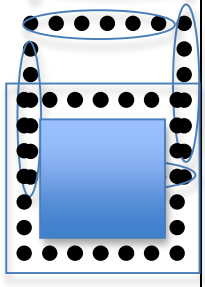
total numbers of dots when there are 10 dots on each arm of the shape. Ask a few students if they can use their mathematical expressions and how. 	without drawing all the dots. Help student understand that arbitrary expressions for 25 are not useful.	expression to find the total number of the dots.
5. Summing up Help each student identify the learning from the class and record it in their notes. • Mathematical expression can be used to show ways of counting the number of the dots. • Mathematical expression may be used to find the number of dots even without seeing the actual diagram.		Each student summarizes their learning and records it in their notes.

Lesson 2 (60 minutes): Find the number of dots without counting one-by-one

(1) Objectives

- Students will express their ways of counting dots in mathematical expressions and infer other students' ways of counting from their mathematical expressions.
- Students will use the mathematical expressions they developed while counting the dots when there are 7 dots on each side of the square to determine the total number of dots when the number of dots on each side is changed.

(2) Flow of the lesson

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher's Support	Points of Evaluation
1. Introduction Let a few students read their journal reflections from the Day 1 and help the class to recall what they learned from Day 1.	Select a few exemplary journal reflections from Day 1 note before the class.	Students are ready for the new problem.
2. Posing the Problem Showing the diagram on the right, ask the following question. Think about ways to count the number of dots in the picture shown on the right. For each way of counting, write a mathematical expression that describes each of your methods of 	Help students see the characteristics of the shape. There are 7 dots in each side of the square.	Students understand that the task is to express ways of counting in mathematical expression. Students understand 4x7 is not correct.
3. Anticipated Student Responses 1) Making groups of dots in the diagram. e.g. 7+5+7+5=24, 2x7+2x5=24, or 2x(7+5)  2) Making equal groups to use multiplications. e.g. 4x7-4=24  4x5+4=24 or 4x(7-2)+4=24 4x6=24 or 4x(7-1)=24  3) Apply the idea of finding the area of a square. 7x7-5x5=24	Let each student write their ways of counting and mathematical expressions in their notes. Using a seating chart to note each student's way of counting and mathematical expressions to prepare for organizing the whole class discussion.	Each student comes up with at least one mathematical expression to express how to count the number of dots.

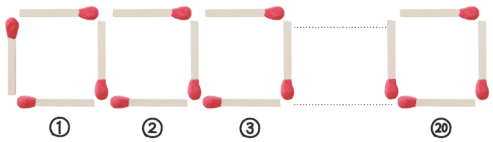
4. Comparing and Discussing For each mathematical expression, 1) Ask one of the students who come up the mathematical expression to show to the class, 2) Let other students infer how the student count the number of dots by interpreting the mathematical expression, and 3) let the student who come up the mathematical expression to justify if the other students inference is correct. Repeat the above to understand variety of ways to count the number of dots by making equal groups. Discuss what each number in the mathematical expression is representing. e.g.) • 4 in the mathematical expressions represent the number of sides of the shape because it is a square. • 6 is 1 take away from 7 and 5 is 2 take away from 7.	By providing opportunity to infer other students' way of counting, help students see that mathematical expression can be a communication tool for understand others' thinking. Help them recall their prior learning such as the meanings of operations and the order of operations and meanings and examine if they can express their ways of counting correctly.	If students be able to use their prior learning such as the meanings of operations and the order of operations and meanings to express their ways of counting correctly in mathematical expressions.
5. Expanding the learning Using a mathematical expression developed when there were 7 dots on each side of the square, determine the total number of dots when there are 10 dots on each side of the square. Ask a few students if they can find the total number without drawing the shape.	Encourage students to see which number or numbers in the mathematical expression would be different.	Students try to use a mathematical expression to find the total number of the dots.
5. Summing up Helping each student highlights the learning from the class and records them on their note. • Mathematical expressions for finding the number of dots for the first problem can be used to find the number of dots in the second problem even without seeing the actual diagram.		Each student summarizes their learning and records it in their notes.

Lesson 3 (60 minutes): How many sticks are there all together?

(1) Objectives

- Students will express their ways of counting sticks in mathematical expressions and infer other students' ways of counting from their mathematical expressions.
- Students will determine the number of sticks when the number of squares is 20 using the mathematical expressions they developed while counting cases of small number of squares.
- Students will confirm that they can use their mathematical expressions to determine the number of sticks even when they cannot actually count the number of sticks because the expressions are based on their ways of counting.

(2) Flow of the lesson

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher's Support	Points of Evaluation
1. Introduction Let a few students read their journal reflections from the Day 1and help the class to recall what they learned from the Day 2.	Select a few exemplary journal reflections from Day 1 note before the class.	If students are ready for the new problem.
2. Posing the Problem Show the diagram below and ask the following question. We are making squares by lining up matchsticks as shown below. When we make 20 squares, how many matchsticks will we need?  	If students do not understand the situation, guide them to see how the number of sticks increases as the number of squares increase. Using animation to show how the number of squares will increase.	Students understand the situation.
3. Anticipated Student Responses By arranging sticks or drawing a diagram, think about ways to count the sticks more easily. (a) Notices that the number of sticks increases by 3 when 1 square is added and writes a mathematical expression. • $4 + 19 \times 3 = 61$ • $4 + (20-1) \times 3 = 61$ (b) Each square has 4 sides, so calculate 20×4. Then, since the sides between two adjacent squares are double counted, subtract the number of overlapping sides, 19 (or $20 - 1$), from the product. • $20 \times 4 - 19 = 61$ • $20 \times 4 - (20 - 1) = 61$ (c) Since there are 20 sticks at the top and 20 sticks at the bottom, 2×20. The number of vertical sticks is $20 + 1$. • $2 \times 20 + 21 = 61$ • $2 \times 20 + (20 + 1) = 61$. By using a drawing of a diagram as the number of squares is increased, explore the number of sticks in relationship to the number of squares.	If students cannot think about ways of counting without drawing, encourage them to use an example case, such as the case with 8 squares, to come up with a way to count the total number of sticks, and express it in a mathematical expression. Then use it to think about which number in the mathematical expression will be different if the number of squares is 20. Using seating chart to note each student's way of counting and mathematical expressions to prepare for organizing the whole class discussion.	Each student comes up with a mathematical expression to express how to count the number of sticks.

(d) Notices that the number of sticks increases by 3 when 1 square is added and writes a mathematical expression. • $4 + 19 \times 3 = 61$ • $4 + (20\text{-}1) \times 3 = 61$ (b) Notices that the number of sticks is increasing by 3 but cannot write a mathematical expression. Misconception e. g. 20x4=120		
4. Comparing and Discussing By using a drawing of the diagram as an example (e.g. diagram with 4 squares), ask students to share how to count the number of sticks. • Tries to figure out ways of counting represented by mathematical expressions. Let students share their idea for finding the number of sticks when there are 20 squares. • Using the strategies discussed when there were 4 squares, students will examine how many sticks will be used when there are 20 squares.	For the students who do not understand the way of counting of 20 squares, the whole class discussion begins by letting them represented by a mathematical expression by looking at the case of 4 squares.	Do students understand ways of counting the number of stick using the example.
5. Expanding the learning Using a mathematical expression developed when there were when there are 20 squares, determine the total numbers of sticks when there are 70 squares.	Encourage students to see which number in the mathematical expression would be different.	Students try to use the mathematical expression to find the total number of sticks.
5. Summing up Helping each student highlights the learning from the class and records them on their note. • The same mathematical expressions for finding the number of sticks may be used to find the number of sticks for various numbers of squares even without seeing the actual diagram.		Each student summarizes their learning and records it in their notes.