Teaching & Learning Plans

Applications of Geometric Sequences and Series

Junior Certificate Syllabus
Leaving Certificate Syllabus

Project Maths
Tionscadal Mata
Development Team

The Teaching & Learning Plans are structured as follows:

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Aims outline what the lesson, or series of lessons, hopes to achieve.

Prior Knowledge points to relevant knowledge students may already have and also to knowledge which may be necessary in order to support them in accessing this new topic.

Learning Outcomes outline what a student will be able to do, know and understand having completed the topic.

Relationship to Syllabus refers to the relevant section of either the Junior and/or Leaving Certificate Syllabus.

Resources Required lists the resources which will be needed in the teaching and learning of a particular topic.

Introducing the topic (in some plans only) outlines an approach to introducing the topic.

Lesson Interaction is set out under four sub-headings:

i. **Student Learning Tasks – Teacher Input:** This section focuses on possible lines of inquiry and gives details of the key student tasks and teacher questions which move the lesson forward.

ii. **Student Activities – Possible Responses:** Gives details of possible student reactions and responses and possible misconceptions students may have.

iii. **Teacher's Support and Actions:** Gives details of teacher actions designed to support and scaffold student learning.

iv. **Assessing the Learning:** Suggests questions a teacher might ask to evaluate whether the goals/learning outcomes are being/have been achieved. This evaluation will inform and direct the teaching and learning activities of the next class(es).

Student Activities linked to the lesson(s) are provided at the end of each plan.

Teaching & Learning Plans: Applications of Geometric Sequences and Series

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Aims

- To generate and be able to apply the compound interest formula
- To investigate the effects of compounding over different periods
- To introduce the idea of a reducing balance and depreciation

Prior Knowledge

Indices, simple interest calculations and calculating percentages of P using P x 1.05 etc, (see Appendix, page 19).

Learning Outcomes

On completion of this Teaching and Learning Plan students should be able to:

- Calculate the compound interest over a number of periods
- Explore the compound interest formula (page 30 of the *Formulae and Tables* book)
- Use the calculator with this formula
- Use the compound interest formula to find the value of different variables
- Convert from monthly rates to annual rates and vice versa
- Explain what is meant by a reducing balance
- Explain the effects of a reducing balance on interest paid on loans
- Gain an understanding of depreciation

Relationship to Junior Certificate Syllabus		
Topic	**Description of topic**	**Learning outcomes**
3.3 Applied arithmetic	Solving problems involving, e.g., mobile phone tariffs, currency transactions, shopping, VAT and meter readings. Making value for money calculations and judgments. Using ratio and proportionality.	– solve problems that involve finding profit or loss, % profit or loss (on the cost price), discount, % discount, selling price, compound interest for not more than 3 years, income tax (standard rate only), net pay (including other deductions of specified amounts) – **solve problems that involve cost price, selling price, loss, discount, mark up (profit as a % of cost price), margin (profit as a % of selling price) compound interest, income tax and net pay (including other deductions)**

Relationship to Leaving Certificate Syllabus			
Students learn about	**Students working at FL should be able to**	**In addition, students working at OL should be able to**	**In addition, students working at HL should be able to**
3.3 Arithmetic	– check a result by considering whether it is of the right order of magnitude and by working the problem backwards; round off a result – make and justify estimates and approximations of calculations; calculate percentage error and tolerance – calculate average rates of change (with respect to time) – solve problems involving • finding depreciation (reducing balance method) • costing: materials, labour and wastage • metric system; change of units; everyday imperial units (conversion factors provided for imperial units) – estimate of the world around them, e.g. how many books in a library	– accumulate error (by addition or subtraction only) – solve problems that involve calculating cost price, selling price, loss, discount, mark up (profit as a % of cost price), margin (profit as a % of selling price), compound interest, depreciation (reducing balance method), income tax and net pay (including other deductions)	– use *present value* when solving problems involving loan repayments and investments

Resources Required

Calculator, Copy of *Formulae and Tables*

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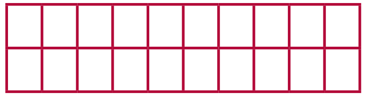
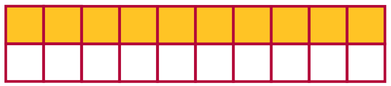
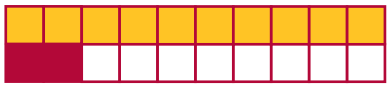
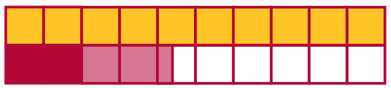
Teacher Reflections

Lesson Interaction

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
Section A: Student Activity 1 Investigating Compound Interest			
» We are now going to look at what happens when an amount or quantity is increased repeatedly by the same percentage.		» Distribute **Section A: Student Activity 1**.	
» Using your squared paper or white boards, draw 20 identical boxes.		» Draw an example of the boxes on the white board. 	
» Imagine you are paid €100 per day and you get a 20% pay rise and then another 20% pay rise.			
» Letting each box represent €10, shade in €100.	• **€100** 		
» Then using a different colour, shade in 20% and write down the amount it represents altogether.	• **€120** 		
» Now, using a third colour, add 20% to the entire shaded area. How much does the shaded area now represent?	• **€144** 	» Observe what students are writing. Assist them as required.	

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Teacher Reflections

<table>
<tr><th>Student Learning Tasks: Teacher Input</th><th>Student Activities: Possible Responses</th><th>Teacher's Support and Actions</th><th>Assessing the Learning</th></tr>
<tr><td>» How much was the first 20% worth?</td><td>• €20</td><td></td><td></td></tr>
<tr><td>» How much was the second 20% worth?</td><td>• €24</td><td></td><td></td></tr>
<tr><td>» How is it that one 20% is worth €20 and the next 20% is worth €24?</td><td>• The first 20% was of a smaller amount than the second 20%.
• Because the starting amount for the first 20% was less than that for the second 20%.</td><td>» Give students time to discuss what is happening.</td><td>» Can students understand the concept of different starting amounts?</td></tr>
<tr><td>» Now we are going to complete a table to show how the total value increases if we repeatedly increase by 20%.</td><td>» Students fill in the table:
<table>
<tr><th>Days (time elapsed)</th><th>Amount</th><th>Increase by %</th><th>Total decimal</th><th>Pattern/Total amount of money received per day</th></tr>
<tr><td>0</td><td>100</td><td>0%</td><td>1</td><td>100</td></tr>
<tr><td>1</td><td>120</td><td>20%</td><td>1.2</td><td>100 x 1.2</td></tr>
<tr><td>2</td><td>144</td><td>20%</td><td>1.44</td><td>100 x 1.2 x 1.2</td></tr>
<tr><td>3</td><td>172.80</td><td>20%</td><td>1.728</td><td>100 x 1.2 x 1.2 x 1.2</td></tr>
<tr><td>4</td><td>207.36</td><td>20%</td><td>2.0736</td><td>100 x 1.2 x 1.2 x 1.2 x 1.2</td></tr>
<tr><td>5</td><td>248.832</td><td>20%</td><td>2.48832</td><td>100 x 1.2 x 1.2 x 1.2 x 1.2 x 1.2</td></tr>
</table></td><td>» Ask students to come to the board to fill in the following table.
<table>
<tr><th>Days (time elapsed)</th><th>Amount</th><th>Increase by %</th><th>Total decimal</th><th>Pattern/ Total amount of money received per day</th></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
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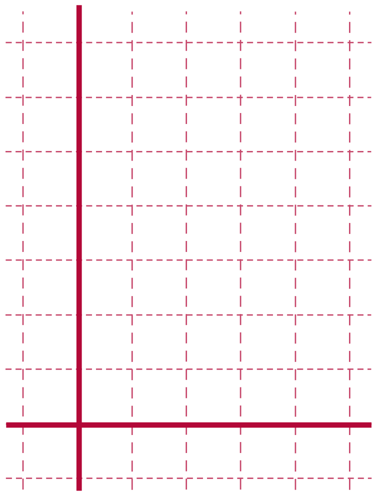
</table></td><td></td></tr>
</table>

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
» If we were to graph the data from the table, what would it look like?	• It increases at the same rate so is it linear?		» Do students understand the concept of dependent and independent variables?
» Let's graph the information and see what it looks like.	• The amount it increases by doesn't stay the same so it isn't linear.	» Ask a student to come to the board to fill in the blank graph below. 	
» What two variables will we put on the graph?	• Time and amount of money.		
» On which axis will we put them?	• Time goes on the horizontal axis because that is going to happen anyway. • As time is the independent variable, it goes on the horizontal axis.		

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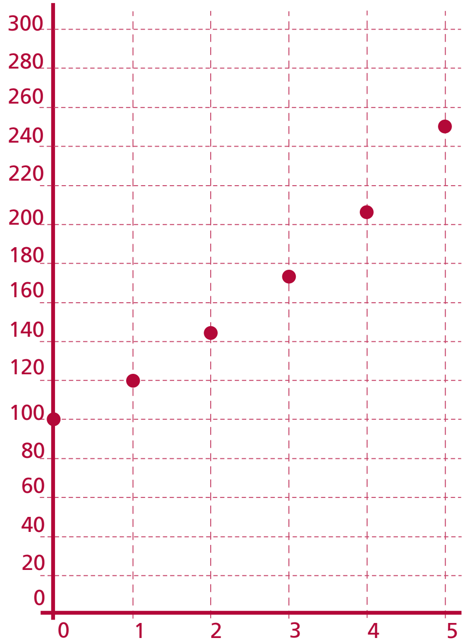
Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
	» On white boards/copies/**Student Activity 1**, students draw the graph. 	» Circulate to monitor students' progress.	
» Is the relationship between time and amount of money linear? » Is there anything else we could discover using the pattern from the table? **Note:** The points are not joined up as the relationship with time and amount is discrete.	• It isn't a straight line so the relationship between time and amount is not linear. • Could we get a formula relating to amount and time? • We could work out a rule that would give the amount irrespective of the time.	» Engage students in discussing the pattern.	» Can students make the connection between a linear relationship and a constant rate of change?

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Teacher Reflections

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Teacher Reflections

<table>
<tr><th>Student Learning Tasks: Teacher Input</th><th>Student Activities: Possible Responses</th><th>Teacher's Support and Actions</th><th>Assessing the Learning</th></tr>
<tr>
<td>» Describe what is happening to the money as the time increases.</td>
<td>• The amount increases by 20% everyday.
• We add 20% to the amount each day.
• The start amount each day is increasing so the amount it goes up by also increases.
• To calculate the amount at the end of each day, we multiply each day's starting amount by 1.2.</td>
<td>»
<table>
<tr><th>Days (time elapsed)</th><th>Amount</th><th>Increase by %</th><th>Total decimal</th><th>Pattern/ Total amount of money received per day</th></tr>
<tr><td>0</td><td>100</td><td>0%</td><td>1</td><td>100</td></tr>
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</table>
</td>
<td></td>
</tr>
<tr>
<td>» If we wanted to find out how much money we would have at the end of day 10, would we have to extend the table or graph?</td>
<td>• That would work but would take a long time.
• Looking at the last column, we seem to be multiplying by the 1.2 the same number of times as to reflect the number of days that pass.
• For day 10, could we multiply 100 by 1.2 ten times.</td>
<td>» Give students time to discuss this themselves.</td>
<td rowspan="2">» Can students apply the use of indices correctly?</td>
</tr>
<tr>
<td>» Write down what this looks like in figures.</td>
<td>• 100 x 1.2^{10}
• 100 x 1.2 x 1.2 x 1.2 x 1.2 x 1.2 x 1.2 x 1.2 x 1.2 x 1.2 x 1.2
• 100 x 1.2 x 10</td>
<td>» Write 100 x 1.2^{10} on the board. Get students to compare it with values on the graph and in the table.

Note: If students write 100 x 1.2 x 10, this needs to be discussed and the misconception rectified.</td>
</tr>
</table>

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning

Section B: Student Activity 2
Discovering the formula

Student Learning Tasks: Teacher Input

» We are going to discover a general rule which will apply to all cases similar to the one we encountered in **Section A**.

» Working in pairs, do Question 1 from **Section B: Student Activity 2**.

Student Activities: Possible Responses

»

Table 1

Method 1		Method 2	
Principal (P)	5,000	i =	0.04
Interest for the 1st year (4% of 5,000)	200	$(1 + i)$ =	1.04
Final Value (end year 1)	5,200	Calculate the value of (end year 1) P x $(1 + i)$ Answer →	5,000 x 1.04 5,200
Interest for the 2nd year	208	$(1 + i)$ =	1.04
Final Value (end year 2)	5,408	Calculate the value of (end year 2) P x $(1 + i)$ Answer →	5,200 x 1.04 5,408
Interest for the 3rd year	216.32	$(1 + i)$ =	1.04
Final Value (end year 3)	5,624.32	Calculate the value of (end year 3) P x $(1 + i)$ Answer →	5,408 x 1.04 5,624.32

Teacher's Support and Actions

» Distribute **Section B: Student Activity 2**.

Note: i is the interest rate expressed as a decimal.

» Circulate to monitor progress. Facilitate discussion if there are difficulties.

» Ask a student to fill in the answers on the board as others call them out.

» Allow students to discuss their answers.

Assessing the Learning

» Are students using the terms *principal*, *interest* and *rate* as they are presenting their answers?

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Teacher Reflections

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Student Learning Tasks: Teacher Input

» Looking at the table, which of the methods is closest to what we used in **Section A**?

» Work in groups and using the same idea as in **Section A**, try and get the general rule or formula for this instance. You may use the table on the board.

» Write out the entire sum with the final answer.

» Now write it out in words, explaining what each term means.

» Each explanation is correct but it might be easier if we all used the same terminology and abbreviations. Let's look at page 30 of the *Formulae and Tables* book.

Student Activities: Possible Responses

» Students complete table.

- Method 2

Years (time elapsed)	Amount	Increase by %	Total decimal	Pattern/ Total amount of money received per year
0	5,000	4%	1	5,000
1	5,200	4%	1.04	5,000 x 1.04
2	5,408	4%	1.0816	5,000 x 1.04 x 1.04
3	5,624.32	4%	1.124864	5,000 x 1.04 x 1.04 x 1.04

- €5,000 x 1.04 x 1.04 x 1.04
- €5,000 x $(1.04)^3$
- €5,000 x $(1.04)^3$ = €5,624.32

- The start amount multiplied by the rate to the power of the number of years gives the answer.

Teacher's Support and Actions

» Draw the following table on the board to remind students of **Section A**.

Years (time elapsed)	Amount	Increase by %	Total decimal	Pattern/ Total amount of money received per year
0	5,000	4%		
1		4%		
2		4%		
3		4%		

» Write €5,000 x $(1.04)^3$ = €5,624.32 on the board.

» Ask a number of students to give their answers verbally and explain their reasoning.

» Fill in the table on the board as per column 2.

Assessing the Learning

» Can students see the similarity between what was done in **Section A** and this question?

» Can students verbalise what they have discovered?

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Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
» Now write out the general rule for **Section A** in the same order as the formula in the *Formulae and Tables* book. » Using €5,000 x $(1.04)^3$ = €5,624.32 and the formula in the tables, rewrite the equation to look like the original formula.	• $F = P(1 + i)^t$ • €5,624.32 = 5,000 $(1+0.04)^3$	• Write the formula from the tables on the board: $F = P(1 + i)^t$	

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
Section C: Student Activity 3 Investigating the Compounding Period			
» Is it useful to be able to predict how much money you will have, **Section C: Student Activity 3**, in the future if you have a savings account for a specific reason?	• It is, because you need to know how long to save for. • You might decide to save more each week/month to get the full amount faster.		
» What do you need to know to work this out?	• The rate of interest.		
» If all we need to know is the rate of interest, what would the rate of interest be after 1 year at 1% per month.	• Would it be 12% if it's 1% per month? • Would it be $(1 + 0.1)^{12}$?		
» Using the formula we discovered in the last exercise, see what €100 amounts to after 12 months at 1% per month.	» Students use the formula: $F = P(1 + i)^t$ • $F = €100(1 + 0.1)^{12}$ • $F = €112.68$	» Ask a student to write the formula on the board: $F = P(1 + i)^t$	» Can the students recognise that the 1% per month compounded is actually 12.68% over the whole year and not 12%?
» Why did we get €12.68 interest?	• If we added the interest each month, we start with a higher amount when the next round of interest was calculated.	» Write Annual Equivalent Rate on the board. Then write the abbreviated version of **AER**. Ask students to bring in advertisements from the papers with AERs on them.	
» Is 12% per annum compounded once for 1 year the same as 1% per month compounded 12 times?	• No.		
» The monthly rate is 1% which equates to an Annual Equivalent Rate (AER) of 12.68%.			

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Teacher Reflections

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<tr><th>Student Learning Tasks: Teacher Input</th><th>Student Activities: Possible Responses</th><th>Teacher's Support and Actions</th><th>Assessing the Learning</th></tr>
<tr>
<td>
» So what would 1.5% compounded per month per year be?

» What would the general formula be to find an annual rate if given the monthly rate?

» If you were given the annual rate AER what would be the monthly rate?

» Let's see if we can use what we've learned already to calculate the principal which will yield a particular amount at the end of a given time period.

» Working in pairs, do Student Activity 3, Questions 1-3.

» Can we find out the rate of interest needed if we know the principal is €7,000 and the final amount is €10,000 after 4 years.

» Now do Student Activity 3, Question 4.
</td>
<td>
• $(1.015)^{12} = 1.1956$ => the AER is 19.56%

• $(1 + i)^{12}$

• $(1.015)^{12} = 1.1956$
$i = 0.1956$
19.56 AER
$0.1956 + 1 = (1 + i)^{12}$
$(1.1956)^{1/12} = 1.015$
$1.015 - 1 = 1.5\%$

» Students work on Student Activity 3.

<table><tr><th>P</th><th>F</th><th>$(1 + i)$</th><th>t</th></tr><tr><td>?</td><td>10,000</td><td>1.07</td><td>10</td></tr></table>
» Students work out the rate of interest.
<table><tr><th>P</th><th>F</th><th>$(1 + i)$</th><th>t</th></tr><tr><td>7,000</td><td>10,000</td><td>?</td><td>10</td></tr></table>
» Students complete Student Activity 3, Question 4.
</td>
<td>
» Ask similar questions and get class to work out the AER.

» Distribute Section C: Student Activity 3.

» Ask a student to fill in the relevant values. Allow discussion to take place.
<table><tr><th>P</th><th>F</th><th>$(1 + i)$</th><th>t</th></tr><tr><td>5,083.49</td><td></td><td></td><td></td></tr></table>
» Write the formula $F = P(1 + i)^t$ on the board.

» Again, ask a student to do out the answer on the board.
<table><tr><th>P</th><th>F</th><th>$(1 + i)$</th><th>t</th></tr><tr><td></td><td></td><td>9.32%</td><td></td></tr></table>
» Again, ask a student to do out the answer on the board. [If students have difficulty with getting the n^{th} root, this should be revised.]
</td>
<td>
» Can students verbalise how to convert a monthly rate to an AER. i.e. $(1+i)^{12}$ and vice versa?

» Are students able to fill in the boxes without difficulty?

» Can they distinguish correctly between P and F?

» Are students comfortable getting n^{th} roots?
</td>
</tr>
</table>

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Teacher Reflections

<table>
<tr><th>Student Learning Tasks: Teacher Input</th><th>Student Activities: Possible Responses</th><th>Teacher's Support and Actions</th><th>Assessing the Learning</th></tr>
<tr><td colspan="4">Section D: Student Activity 4 Depreciation</td></tr>
<tr><td>» An item was being produced for €16 twenty years ago. Due to technological innovations, the production cost was reduced by 10% ten years ago and reduced again by 10% this year. How much does it now cost to produce?</td><td>• 16 x 0.1 = 1.6
€16 - 1.6 = €14.40

• 14.40 x .01 = 1.44
€14.40 - 1.44 = €12.96</td><td></td><td></td></tr>
<tr><td>» Could we use the formula we have been using to calculate this?</td><td>• Yes, because you are dealing with the same things, final amount, principal, rate of interest.</td><td></td><td></td></tr>
<tr><td>» Let's look at the change in the cost over each of the intervals using a table and our formula $P(1 + i)^t$. However, if you think about it, if something is depreciating should I add or subtract i?</td><td>• We should subtract i as the final amount is getting smaller.</td><td>» Draw a blank table on the board. Ask students to copy it into their copy books and fill in the relevant figures.
<table>
<tr><th>Time in years</th><th>Interval</th><th>Pattern of the amount of money at the end of each year.</th><th>Total amount</th></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
</table></td><td>» Can students generate an expression for depreciation and understand that we use $F = P(1 - i)^t$?</td></tr>
</table>

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<table>
<tr><th>Student Learning Tasks: Teacher Input</th><th>Student Activities: Possible Responses</th><th>Teacher's Support and Actions</th><th>Assessing the Learning</th></tr>
<tr><td></td><td>

- Table

<table>
<tr><th>Time in years</th><th>Interval</th><th>Pattern of the amount of money at the end of each year.</th><th>Total amount</th></tr>
<tr><td>0</td><td></td><td>16</td><td>16</td></tr>
<tr><td>10</td><td>1</td><td>$F = P(1 - i)^t$
$F = 16(1 - 0.1)^1$
$F = 16(.9)^1$
$F = 14.4$</td><td>14.4</td></tr>
<tr><td>20</td><td>2</td><td>$F = P(1 - i)^t$
$F = 16(1 - 0.1)^2$
$F = 16(0.9)^2$
$F = 12.96$ or $F = 14.4(1 - 0.1)$</td><td>12.96</td></tr>
</table>

- Formula
$F = P(1 - i)^t$
$F = 16(1 - 0.1)^1$
$F = 16(.9)^1$
$F = 14.4$

$F = P(1 - i)^t$
$F = 16(1 - 0.1)^2$
$F = 16(0.9)^2$
$F = 12.96$

</td><td></td><td>» Can students verbalise why we use $F = P(1 - i)^t$ instead of $F = P(1 + i)^t$ for depreciation?</td></tr>
<tr><td>» Can we see from the table if the relationship between total amount and time is linear or would we need to draw a graph?
» It is obvious from the table that the relationship between amount and time is not linear. Using the figures you have, draw a graph with time and amount.</td><td>• If the rate of change is constant, isn't the relationship linear?
• If the amount goes down uniformally, wouldn't the relationship be linear?
• A graph might be helpful as well.</td><td>» Allow students time to discuss.

Note: this misconception needs to be fully explored.</td><td></td></tr>
</table>

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Teacher Reflections

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Teacher Reflections

Student Learning Tasks: Teacher Input

» Ask the students to indicate the appropiate axes for time and amount. Get them to explain their reasoning.

» Ask students to calculate the amount for $T = 3$, 4 and 5.

» What do we see?

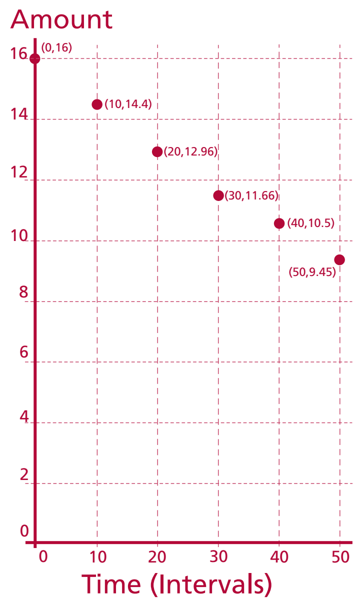
» Is the graph increasing or decreasing?

» Is the decrease/ change getting bigger or smaller?

» If the relationship is not linear, what kind of relationship might it be?

» How can we decide which it is?

Student Activities: Possible Responses



- It's a curve
- It's not linear
- The graph is decreasing
- It's going down
- The decrease/change is getting smaller
- It could be quadratic or exponential
- If it's quadratic, isn't the 2nd difference constant?
- If it's exponential, the 2nd change isn't the same
- Exponential relationship has change in a ratio

Teacher's Support and Actions

» Put the table up and ask a student to go through the changes

Time	Interval	Pattern of the amount of money at the end of each year.	Amount
0		16	16
10	1	$F = 16\,(1 - 0.1)$	14.4
20	2	$F = 16\,(1 - 0.1)^2$	12.96
30	3	$F = 16\,(1 - 0.1)^3$	11.66
40	4	$F = 16\,(1 - 0.1)^4$	10.50
50	5	$F = 16\,(1 - 0.1)^5$	9.45

Note: Discuss and expand any misconceptions regarding linear and exponential relationships and reinforce any correct ideas that are offered.

Assessing the Learning

» Do students understand the concept of dependent and independent variables? Are they aware of how they should be plotted?

» Can the students relate the rate of change with the type of relationship?

<table>
<tr><th>Student Learning Tasks: Teacher Input</th><th>Student Activities: Possible Responses</th><th>Teacher's Support and Actions</th><th>Assessing the Learning</th></tr>
<tr>
<td>» Continuing the pattern, what would the next three amounts be?

» What are we doing to get each amount?

» Looking at the changes, and your graph, can we decide on the relationship?</td>
<td>• 11.66, 10.50 and 9.45.

• We are using the formula $F = P\,(1 - i)^t$

• It's an exponential relationship.</td>
<td>» Get a student to write the answers on the board.
<table>
<tr><th>Time</th><th>Interval</th><th>Pattern of the amount of money at the end of each year.</th><th>Amount</th></tr>
<tr><td>0</td><td></td><td>16</td><td>16</td></tr>
<tr><td>10</td><td>1</td><td>$F = 16\,(1 - 0.1)$</td><td>14.4</td></tr>
<tr><td>20</td><td>2</td><td>$F = 16\,(1 - 0.1)^2$</td><td>12.96</td></tr>
<tr><td>30</td><td>3</td><td>$F = 16\,(1 - 0.1)^3$</td><td>11.66</td></tr>
<tr><td>40</td><td>4</td><td>$F = 16\,(1 - 0.1)^4$</td><td>10.50</td></tr>
<tr><td>50</td><td>5</td><td>$F = 16\,(1 - 0.1)^5$</td><td>9.45</td></tr>
</table>
</td>
<td></td>
</tr>
<tr>
<td>• Complete Section D Student Activity 4.</td>
<td>» Students should try Section D: Student Activity 4, compare answers around the class and have a discussion about why the answers are not all agreeing</td>
<td>» Distribute Section D: Student Activity 4.</td>
<td></td>
</tr>
</table>

Project Maths Development Team

Teacher Reflections

Project Maths
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Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning

Section E: Student Activity 5 Reducing Balance

Student Learning Tasks: Teacher Input

» Working in pairs, do **Section E: Student Activity 5, Questions 1-5**.

Student Activities: Possible Responses

	David			Michael		
Time	Monthly Total	Interest	Payment	Monthly Total	Interest	Payment
0	€600.00	€9.00	€100.00	€600.00	€9.00	€60.00
1	€509.00	€7.64	€100.00	€549.00	€8.24	€60.00
2	€416.64	€6.25	€100.00	€497.24	€7.46	€60.00
3	€322.88	€4.84	€100.00	€444.69	€6.67	€60.00
4	€227.73	€3.42	€60.00	€391.36	€5.87	€100.00
5	€171.14	€2.57	€60.00	€297.23	€4.46	€100.00
6	€113.71	€1.71	€60.00	€201.72	€3.03	€100.00
7	€55.42	€0.83	€56.25	€104.72	€1.57	€100.00
8				€6.29	€0.09	€6.38
9						
10						

Teacher's Support and Actions

» Distribute **Section E: Student Activity 5.**

» Get a student to put the answers on the board.

Student Learning Tasks: Teacher Input

» Do David and Michael pay the same amount?

Student Activities: Possible Responses

- It's not the same
- Michael ends up paying back more
- They both pay back the original €600 but Michael's interest is higher
- David's amount decreases faster
- David's is paid back a week sooner

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Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
» Now do **Section E: Student Activity 5**, questions 6 and 7.	• Students complete **Section E: Student Activity 5**, questions 6 and 7. • Students present their graphs to the class and discuss their findings.	» Suggested further investigation: • Investigate the effect of two people taking out the same loan, making equal repayments, but being charged a different rate. • Compare the balances and the interest being paid throughout the term of the loan. (E.g. A loan of €900, with both people paying €100 per month, but person 1 is charged an annual interest rate of 10%, while person 2 is charged an interest rate of 8%.)	» Can students verbalise their reasoning?

Appendix

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Revision of Prior Knowledge Required

The teacher may use some or all of the following activities in preparing this topic. This document covers the following:

1. The terms used
2. Interest rate as r/100
3. Adding (or subtracting) the decimal rate to/from the unit
4. Multiplying indices
5. Calculating simple interest and checking it
6. Basic calculator skills.

Terms used:

1. John put €200 into the bank for 1 year and got 10% interest during that year. At the end of the year he had €220. This means that he had gained €20 on his investment. Match John's figures to each of the words in the table below.

Principal	Interest rate	Final Value	No. of years	Interest

2. Mary put €600 into the bank for 2 years at 9% per annum and at the end of the 2 years she had €712.86 in total. Complete the table below to illustrate this.

Principal	Interest rate	Final Value	No. of years	Interest

3. The following figures represent a certain amount of money put into a bank for a certain number of years at a certain interest rate. Using all of the words in the table above, write out a few sentences which would explain all of these figures. Figures: €472.05, 4 yrs, €300, 12%, €172.05 ______________________

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Appendix (continued)

Project Maths Tionscadal Mata Development Team

4. Complete the table below. **Note:** p.a. means per annum (per year)

Name	Principal	Interest rate % (p.a.)	Final Value	No. of years	Interest
Anne	€1,000.00	6%	€1,338.23	5	
Michael	€1,000.00	7%		9	€838.46
Dominic		8%	€5,038.85	3	€1,038.85
Joseph		14%	€12,370.79	5	€5,945.79
Eileen	€580.00	7%	€870.42	6	

Interest rate as r/100

A percentage is a fraction having a denominator of 100. Therefore 9% means 9/100 which is 0.09 as a decimal. Therefore an annual interest rate of 9% could be written as a decimal as 0.09.

The data below shows some percentages. Convert these to decimals.	
1. 6%	
2. 3%	
3. 5%	
4. 4.5%	
5. 12%	
6. 18%	
7. 3¾ %	

The data below shows some decimals. Convert these to percentages.	
8. 0.07	
9. 0.08	
10. 0.16	
11. 0.2	
12. 0.075	
13. 0.0125	
This is the "i" in the *Formulae and Tables* book.	

Adding/subtracting the decimal rate to/from the unit

For the work which follows the decimal rate is added/subtracted to/from the unit. The unit is 100% i.e. 100/100 = 1). For example if the annual rate is 7% then the decimal rate is 0.07 and the unit added to this is 1.07. In the table below some of the figures are missing. Complete the table.

Decimal Rate (i)	i added to the unit ($1 + i$)	Decimal Rate (i)	i subtracted from the unit ($1 - i$)
1. 0.02		1. 0.05	
2. 0.09		2. 0.07	
3. 0.17		3. 0.13	
4. 0.03		4. 0.19	
5. 0.3		5. 0.2	
6. 0.25		6. 0.085	
	7. 1.05		7. 0.08
	8. 1.035		8. 0.09
	9. 1.1		9. 0.88
	10. 1.01		10. 0.94

Appendix (continued)

Multiplying indices

On page 21 *Formulae and Tables* book it says $a^p\ a^q = a^{p+q}$ at the top of the page

An example of this would be $k^5 \times k^4 = k^9$ (since 5 + 4 is 9)

Complete the following table, without the use of a calculator. Leave your answer in index form:

Before multiplying	After multiplying	Before multiplying	After multiplying
1. $a^7 \times a^3$		6. $1.02^3 \times 1.02^3$	
2. $8^3 \times 8^2$		7. $1.14^5 \times 1.14^2$	
3. $8.2^5 \times 8.2^2$		8. $1.06^4 \times 1.06$	
4. $(6.4)^5\ (6.4)^2$		9. $(1.08)^5\ (1.08)$	
5. $1.3^2 \times 1.3^5$		10. 1.07×1.07^5	

Calculating simple interest and checking it

1 Patrick puts €400 into the bank for 1 year and gets an annual interest rate of 4%. At the end of the year he asks the bank how much money he has in total and how much interest he earned. Fill out the table below to see what figures the bank might give him.

Method 1		Method 2	
Principal		If the annual rate is 4% then fill in the value of i	$i =$
Interest for the year (calculate 4% of €400)		Fill in the value of the unit	$(1 + i) =$
Final value		Calculate the value of P x $(1+i)$ (i.e. final value)	

Did both methods give the same final value? ____________

Patrick had a final value of € ___________ and earned €___________ in interest.

2. Kathleen puts €200 into the bank for 1 year and gets an interest rate of 8% during that year. Use method 1 and method 2 to work out how much she had in the bank and how much interest she earned during the year.

Method 1		Method 2	
Principal		If the annual rate is 4% then fill in the value of i	$i =$
Interest for the year (calculate 4% of €400)		Fill in the value of the unit	$(1 + i) =$
Final value		Calculate the value of P x $(1 + i)$ (i.e. final value)	

Did the both methods give the same final value? ____________

Kathleen had a final value of € ___________ and earned €___________ in interest.

Appendix (continued)

3. Raul puts €900 into the bank for 1 year and gets an interest rate of 7% during that year. Use method 2 to work out how much he had in the bank at the end of the year. Then find out how much interest he earned during the year.

Checking Understanding

Write out, in your own words, the meaning of each word and term in the table below.

Word or Term	Explanation
Principal	
Final Value	
Interest	
Annual Interest Rate	
i	
$1 + i$	
p.a.	

Basic Calculator Skills

Evaluate the following using your calculator:

(a) $7^4 =$	(h) $10\,(3)^4 =$
(b) $4.5^4 =$	(i) $100\,(6)^3 =$
(c) $1.8^5 =$	(j) $1{,}000\,(2.5)^3 =$
(d) $1.06^6 =$	(k) $300\,(1.03)^6 =$
(e) $1.325^5 =$	(l) $2{,}000(\,1.025)^5 =$
(f) $(3/2)^7 =$	(m) $250\,(1.16)^4 =$
(g) $(1/2\,)^4 =$	(n) $400\,(1.08\,)^4 =$

In the following table, fill in the correct figures:

(a) $3^2 = 9$	(i) $5^3 = 125$
(b) $\sqrt[2]{9} = 3$	(j) $\sqrt[3]{x} = 5$
(c) $2^4 = x$	(k) $(3.2)^2 = 10.24$
(d) $\sqrt[4]{16} = x$	(l) $\sqrt{x} = 3.2$
(e) $2^5 = 32$	(m) $(3.6)^3 = 46.656$
(f) $\sqrt[5]{32} = x$	(n) $3.6 \times 3.6 \times x = 46.656$
(g) $2^6 = x$	(o) $(1.02\,)^8 = ?$
(h) $\sqrt[x]{64} = 2$	

Section A: Student Activity 1

Project Maths Development Team

Investigating Compound Interest

1. If each block represents €10, shade in €100.
2. Then, using another colour, add 20% to the original shaded area.
3. Finally, using a third colour, add 20% of the entire shaded area.
4. What is the value of the second shaded area?______________________________
5. What is the value of the third shaded area?________________________________
6. Why do they not have the same amount?________________________________
7. Complete the following table and investigate the patterns which appear.

Time/day	Amount	Increase by	Total decimal	Pattern/Total Amount of money received per day
0		0%		100
	€120	20%	1.2	100 x 1.2

8. Can you find a way of getting the value for day 10 without having to do the table to day 10?______________________________

____

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9. Use the diagram on the right to graph time against amount. Is the relationship linear?

Section B: Student Activity 2

Project Maths Tionscadal Mata Development Team

Discovering the Formula

Mary received a gift of €5,000. She is hoping to buy a car costing €6,000 with her savings in 3 years time. She intends to invest the €5,000 until then. The bank is offering her interest of 4% p.a. She leaves it in for the 3 years. Will she be able to afford to buy the car?

Method 1		Method 2	
Principal (P)		i =	
Interest for the 1st year (4% of 5,000)		$(1 + i)$ =	
Final Value (end year 1)		Calculate the value of (end of year 1) using P x $(1 + i)$ Answer →	
Interest for the 2nd year		$(1 + i)$ =	
Final Value (end year 2)		Calculate the value of (end of year 2) using P x $(1 + i)$ Answer →	
Interest for the 3rd year		$(1 + i)$ =	
Final Value (end year 3)		Calculate the value of (end of year 3) using P x $(1 + i)$ Answer →	

1. Does this amount to the proof of the formula? Discuss. ______________________
____
____
____

2. Write a sentence explaining whether or not Mary can afford the car from her bank savings. ____
____
____

3. Write the value of the interest for the 1st, 2nd and 3rd years in the boxes below.

Interest for 1st year	Interest for 2nd year	Interest for 3rd year

4. Fill in the appropriate values:
   a. Principal = ________________________________
   b. Total interest = ____________________________
   c. Final value = ______________________________

5. Using the formula from the tables, write out the entire equation in the same order. ____
____
____

Section C: Student Activity 3

Investigating the Compounding Period

John wants to have €10,000 saved in 10 years time to pay for his child's education. The bank is offering him an annual interest rate of 7%. How much money would he need to put in now in order to have €10,000 in 10 years time? Round off to the nearest €100.

1. Fill as many variables as you can into the table below

P	F	$(1 + i)$	t

2. Use the boxes below to find the value of P

$$F = P(1 + i)^t$$

$\square = P\,\square^{\square}$ ← Fill in the variables from the table above

$\square = P\,\square$ ← Use your calculator to evaluate $(1 + i)^t$

$\dfrac{\square}{\square} = P$ ← Divide both sides by the value of $(1 + i)^t$

3. How much, to the nearest euro, should John deposit in the bank?___________

Section C: Student Activity 3 (continued)

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4. The formula for compound interest is $F = P(1 + i)^t$

   Using the relevant n^{th} root, find the rate of interest applied to get the final sum from the principal sum. Remember that you are working with money so rounding to the nearest cent has occurred. The rates should be written to one decimal place (AER) for this exercise.

   Example:

F	P	F/P	$(1 + i)$	t	Rate
15,918.12	15,000	15,918.12/15,000= 1.061208	$\sqrt[3]{1.061208} = 1.05$	3	5%

   Complete the following:

	F	P	F/P	$(1 + i)$	t	Rate
i	11,576.25	10,000	1.157625		3	
ii	8,268.75	7,500	1.1025		2	
iii	1,215.51	1,000	1.21551		4	
iv	5,970.26	5,000	1.194052		6	

Section D: Student Activity 4

Depreciation

1. The selling price €S of a car after t years can be expressed as follows:
   $S = 25000\ (0.9)^t$
   a. What is the current selling price of the car?
   b. What will be the selling price of the car after 3 years?

2. Mary has gone on a diet. Her weight w kg after t weeks is given by
   $w = 50(2)^{-0.01t}$
   a. Find her current weight.
   b. Find her weight after 10 weeks.

3. The current value of a vehicle is €18,000 and it depreciates by 25% every year.
   a. Express the value of the vehicle after t years in terms of t.
   b. What kind of a function is obtained in part (a)?
   c. What is the percentage change in the value of the vehicle after 2 years?

4. Conor makes a New Year's resolution and plans to keep fit for the coming year. His target is to decrease his current weight of 80 kg by 1% each week in the coming months. If Conor reaches his target each week,
   a. Express his weight after t weeks in terms of t.
   b. Find his weight after 4 weeks.
   c. Find the percentage change in his weight after 4 weeks.

Section E: Student Activity 5

Reducing Balance

David and Michael are going on the school tour this year. They are each taking out a loan of €600, which they hope to pay off over the next year. Their bank is charging a monthly interest rate of 1.5% on loans. David says that with his part-time work at present he will be able to pay €100 for the first 4 months but will only be able to pay off €60 a month after that. Michael says that he can only afford to pay €60 for the first 4 months and then €100 after that. Michael reckons that they are both paying the same amount for the loan. Why? ___________________________

____

____

Note: This problem is posed based on the following criteria: (a) A loan is taken out (b) After 1 month interest is added on (c) The person then makes his/her monthly repayment. This process is then repeated until the loan is fully paid off.

	David			Michael		
Time	Monthly Total	Interest	Payment	Monthly Total	Interest	Payment
0						
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						

1. What do David and Michael have in common at the beginning of the loan period? ____
2. Calculate the first 3 months transactions for each. (How much, in total, had they each paid back after 3 months?) David ___________ Michael ___________
3. What is the total interest paid by each? David __________ Michael ___________
4. Based on your answers to the first 3 questions, when would you recommend making the higher payments and why? ____________________________________

   ____
5. Is Michael's assumption that they will eventually pay back the same amount valid? ____

Section E: Student Activity 5 (continued)

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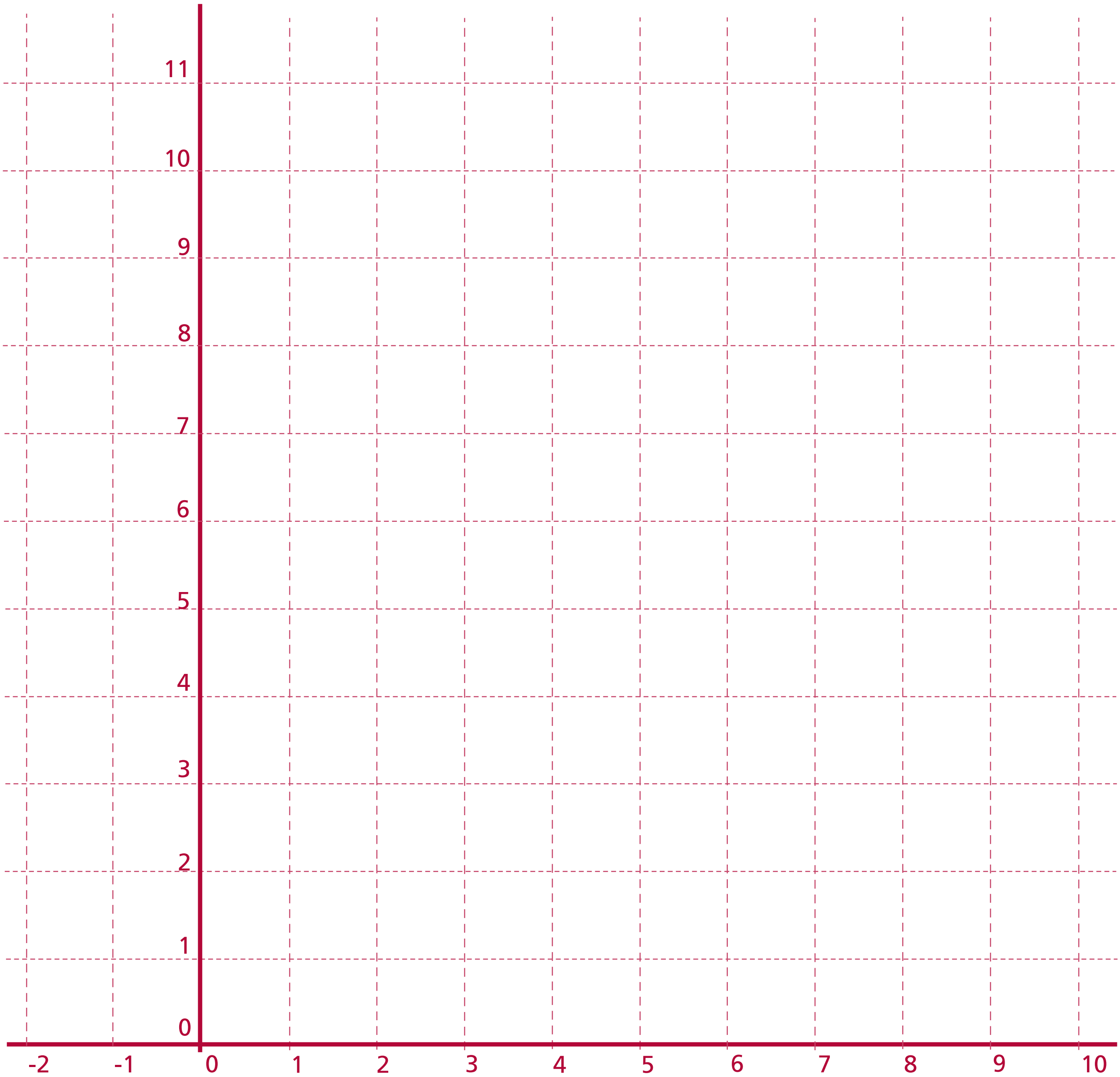

6. Using the above, plot the amount of interest added each month to both David's and Michael's account.

7. Looking at the graph, who will pay the most interest overall? ______________