Teaching & Learning Plans

Geometric Sequences

Leaving Certificate Syllabus

Project Maths
Tionscadal Mata
Development Team

The Teaching & Learning Plans are structured as follows:

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Aims outline what the lesson, or series of lessons, hopes to achieve.

Prior Knowledge points to relevant knowledge students may already have and also to knowledge which may be necessary in order to support them in accessing this new topic.

Learning Outcomes outline what a student will be able to do, know and understand having completed the topic.

Relationship to Syllabus refers to the relevant section of either the Junior and/or Leaving Certificate Syllabus.

Resources Required lists the resources which will be needed in the teaching and learning of a particular topic.

Introducing the topic (in some plans only) outlines an approach to introducing the topic.

Lesson Interaction is set out under four sub-headings:

i. **Student Learning Tasks – Teacher Input:** This section focuses on possible lines of inquiry and gives details of the key student tasks and teacher questions which move the lesson forward.

ii. **Student Activities – Possible Responses:** Gives details of possible student reactions and responses and possible misconceptions students may have.

iii. **Teacher's Support and Actions:** Gives details of teacher actions designed to support and scaffold student learning.

iv. **Assessing the Learning:** Suggests questions a teacher might ask to evaluate whether the goals/learning outcomes are being/have been achieved. This evaluation will inform and direct the teaching and learning activities of the next class(es).

Student Activities linked to the lesson(s) are provided at the end of each plan.

Teaching & Learning Plan: Leaving Certificate Syllabus

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Aims

- To understand the concept of geometric sequences
- To enable students recognise a geometric sequence (geometric progression)
- To enable students apply their knowledge of geometric sequences to everyday applications
- To use and manipulate the appropriate formula
- To enable students find "a", the first term and, "r" the common ratio, when given two terms of a geometric sequence

Prior Knowledge

It is envisaged that in advance of tackling this Teaching and Learning Plan, the students will understand and be able to carry out operations in relation to:

- the concept of pattern
- basic number systems
- the concept of a sequence
- basic graphs in the co-ordinate plane
- simultaneous equations with 2 unknowns
- the concept of T_n as the general term of a geometric sequence
- indices
- proof by induction (If covering the section on proving the formula for S_n by induction.)
- students should also have completed the following Teaching and Learning Plans: Arithmetic Sequences and Arithmetic Series

Learning Outcomes

Having completed this Teaching and Learning Plan the students will:

- recognise geometric sequences in everyday applications
- recognise sequences that are not geometric
- apply their knowledge of geometric sequences to everyday life situations
- apply the relevant formula in both theoretical and relevant applications
- calculate the value of a the first term, r the common ratio and T_n the general term of a geometric sequence from information given about the sequence.

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Relationship to Leaving Certificate Syllabus

Students learn about	Students working at FL should be able to	In addition, students working at OL should be able to	In addition, students working at HL should be able to
3.1 Number systems			– investigate geometric sequences and series

Resources Required

- Whiteboard and markers or blackboard and chalk.
- Student Activities that accompany this Teaching and Learning Plan.

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Teacher Reflections

Lesson Interaction

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
Section 1: To introduce geometric sequences (also known as geometric progressions) (GPs) and gain an understanding of the formula $T_n = ar^{n-1}$			
» What is a sequence? » Sequences can be finite or infinite. » Can you give me examples of sequences? » Are all sequences arithmetic? » Can you give me examples of sequences that are not arithmetic?	• A sequence is a list of terms (or numbers) arranged in a definite order. There is a rule by which each term after the first may be found. • {2, 4, 6, 8, 10,…} {7, 5, 3, 1, -1,…} • No • {2, 4, 8, 16,…} {1, 3, 9, 27,…} {1, 2, 4, 7, 11,…}	» Write each of the students' suggestions on the board and write yes or no beside it. » Write each sequence mentioned on the board. Indicate on the board whether each sequence is arithmetic or not.	» Can all students recognise a sequence? » Can students come up with examples of geometric and non-geometric sequences? » Can the students identify when sequences are not arithmetic?
» Do the first three questions in **Section A: Student Activity 1** and use the results of these to complete Question 4.		» Distribute **Section A: Student Activity 1**. » Teacher circulates and offers support where needed.	

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
» What did the first three questions have in common? » Sequences generated in this manner are known as a geometric sequence. » What did we call the first term when dealing with sequences previously? » The number that we multiply the first term by to get the second term is known as r, the common ratio. » In terms of a and r what is T_1? » In terms of a and r what is T_2? » In terms of a and r what is T_3? » In terms of a and r what is T_4? » In terms of a and r what is T_n? » This formula can be found on page 22 of the formula and tables booklet. It can be used to find the general term of any geometric sequence. » What type of number is n? Why?	• An initial term. Each term apart from the first term was found by multiplying the preceding term by the common ratio. • a • a • ar • ar^2 • ar^3 • ar^{n-1} • Natural number, because we can only have whole positive numbers to represent the position of a term in a sequence.	» Develop the following rules on the board leading to the formula for T_n for a geometric sequence: $T_1 = a$ $T_2 = ar$ $T_3 = T_2\,(r) = ar\,(r) = ar^2$ $T_4 = T_3\,(r) = ar^2\,(r) = ar^3$ $T_5 = T_4\,(r) = ar^3\,(r) = ar^4$ $T_n = T_{n-1}\,(r) = ar^{n-2}\,(r) = ar^{n-1}$ » If, having completed the discussion, the students are unsure of the common features of the first three Questions in **Section A: Student Activity 1** they should be asked to revisit the questions and identify the value of a and the value of r in each question.	» Do students understand the characteristics of a geometric sequence? » Are students conversant with the relevant notation? » Do students understand the purpose of the T_n formula?

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Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
» What name can also be given to the type of pattern produced in a geometric sequence? » Do geometric sequences form an exponential pattern? » When r is a proper fraction, we have exponential decay rather than growth? » Give me an example of a geometric sequence that represents exponential decay?	• Exponential • Yes • {8, 4, 2, 1, ½, ¼ ....}		» Do students understand that geometric sequences form an exponential pattern? » Do students understand that when r is a proper fraction it represents exponential decay rather than growth?
» Complete the remaining exercises in **Section A: Student Activity 1.**		» The students may need some help in applying the formula $T_n = ar^{n-1}$ in the different circumstances presented by the problems in Section **A: Student Activity 1**. » At this stage focus on developing the students' understanding of geometric sequences rather than addressing specific questions.	» Can the students use the formula $T_n = ar^{n-1}$ to determine the values of a, r and n in the situations presented by the problems in the **Section A: Activity Sheet 1**?

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Teacher Reflections

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Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
Wrap up and Homework		» Write the formula and the meaning of a, r and n on the board.	
» What do a, r and n stand for in relation to geometric sequences?	» a is the first term, r is the common ratio, the term we multiply the preceding term by to get the next term. n is the term number – the position of the term in the sequence.		
» What is the formula for the general term of a geometric sequence?	» $T_n = ar^{n-1}$		
» Complete the following question on **Section A: Student Activity 1**.		» Depending on the progress of the students to date on the activity select appropriate questions for homework.	

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
Section 2: To further explore the concepts of geometric sequences including challenging student exercises			
» What is a geometric sequence? » What is the formula for the n^{th} term of a geometric sequence? » What do a, r and n represent in this formula? » Working in pairs list as many applications of geometric sequences as possible.	• A geometirc sequence is a sequence in which the ratio of consecutive terms is fixed. • $T_n = ar^{n-1}$ • a is the first term, r is the common ratio and n is the position of the term in the sequence. • Doubling, trebling etc. the amount you are saving. • Compound interest • Exponential growth • Depreciation • Exponential decay	» If students are unable to suggest examples of geometric sequences direct them to examples used in **Section A: Student Activity 1**. » The outcomes of the students' deliberations, even if some of the proposed applications are incorrect, they should lead to interesting discussions and provide opportunities for shared learning.	» Are students able to decide whether a sequence is geometric or not?
» Do the activities contained in **Section B: Student Activity 2**.		» Distribute **Section B: Student Activity 2**. » If the technology is available the video shown in the following link may be shown to supplement question 1. http://www.youtube.com/watch?v=rlZ-uhf9wUk » Students should be encouraged to discuss the problems contained in the Activity Sheet with their classmates. » As the class works through the questions the teacher will need to offer assistance both to individuals and the class as a whole with aspects that cause difficulties.	» Are students able to apply their knowledge of geometric sequences to the problems outlined in **Section B: Student Activity 2**?

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
» Given T_n the n^{th} term of a geometric sequence is given by $T_n = ar^{n-1}$ can you spot any relationship between $\frac{T_{n+1}}{T_n}$? » This is characteristic of all geometric sequences. » Is $\frac{T_{n+1}}{T_n} = \frac{T_{n+2}}{T_{n+1}}$? **Note:** You need a clear understanding of these formulas as they are not provided in the formula and tables booklet.	• $T_{n+1} = ar^n$ $T_n = ar^{n-1}$ $\frac{T_{n+1}}{T_n} = \frac{T_{n+2}}{T_{n+1}}$ • Yes • $\frac{T_{n+2}}{T_{n+1}} = \frac{ar^{n+1}}{ar^n} = r$ and $\frac{T_{n+1}}{T_n} = \frac{ar^n}{ar^{n-1}} = r$ Therefore they are equal in all cases.	» Through discussion develop the following on the board: $\frac{T_{n+1}}{T_n} = \frac{T_{n+2}}{T_{n+1}}$ » Develop on the board. $\frac{T_{n+1}}{T_n} = \frac{ar^n}{ar^{n-1}} = r$ $\frac{T_{n+2}}{T_{n+1}} = \frac{ar^{n+1}}{ar^n} = r$ » Therefore $\frac{T_{n+1}}{T_n} = \frac{T_{n+2}}{T_{n+1}}$ for all geometric sequences. » The teacher then shows that ratios of the type $\frac{T_2}{T_1} = \frac{T_3}{T_2}$ hold for all geometric sequences and in general $\frac{T_{n+1}}{T_n} = \frac{T_{n+2}}{T_{n+1}}$ is also always true.	» Do students accept that $\frac{T_n}{T_{n-1}} = r$ is characteristic of all geometric sequence? » Do students recognise that $\frac{T_{n+1}}{T_n} = \frac{T_{n+2}}{T_{n+1}}$ is true for all geometric sequences?

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Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
» Complete the following exercises in **Section C: Student Activity 3**.		» Distribute **Section C: Student Activity 3**. » If time does not permit all questions to be attempted, the teacher needs to balance the selection between the more theoretical and application-type questions. » Encourage students to show calculations at all stages and bring their answers back to the context of the problem being addressed.	
Wrap up and Homework » Working in pairs: each member should devise a problem on geometric sequences and ask their partner to solve it.			» Can students devise their own test questions, which illustrate and explain the features of geometric sequences?

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Teacher Reflections

Section A: Student Activity 1

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1. Nigel saves €2 in the first week of the New Year, if he doubles the amount he saves every week after that, how much will he save in the 2nd, 3rd, 4th and 5th week of the year?

   a. Complete the following table and explain your reasoning:

Week No.	1st Week	2nd Week	3rd Week	4th Week	5th Week
Amount Saved					

2. The price of an item trebles on the first day of every month. It costs €2.00 on the 1st January, calculate its cost on the first day of the following four months

   a. Complete the following table and explain how you got your calculations:

Date	1st February	1st March	1st April	1st May
Cost				

3. A town had a population of 100. In January 2010, A new factory was being built in the town and consequently, the local authority expected that the town's population would increase by 20% year-on-year for the next 5 years.

   a. Explain clearly how the town's population in January 2011 is calculated?

   b. Complete the following table and explain your reasoning:

Date	January 2011	January 2012	January 2013	January 2014	January 2015
Population					

4. Examine the completed tables from the previous three questions.

   a. Do the numbers you calculated follow any particular pattern in each table?

   b. Can you generate a formula to calculate the entries in each of the tables?

5. Assuming the patterns in Questions 1 and 2 (above) continue, complete the table below. It is not necessary to evaluate the powers.

Question 1	Amount saved in the 50th month	Amount saved in the 100th month
Question 2	Population in January 2020	Population in January 2040

For question 1 above what does T_{50} mean?

Section A: Student Activity 1 (continued)

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6. Find T_1, T_2, T_3, T_5, T_{10} and T_{100} of a geometric sequence where a = 3 and r =2. It is not necessary to evaluate the powers.

T_1	T_2	T_3	T_5	T_{10}	T_{100}

7. Find T_1, T_2, T_3, T_5, T_{10} and T_{100} of a geometric sequence where a = 1024 and R= ½.

T_1	T_2	T_3	T_5	T_{10}	T_{20}

8. A ball is dropped from a height of 4 metres. The ball bounces to 75% of its previous height after each sequential bounce. How high does the ball bounce on the 1st 2nd, 3rd, 4th and 5th bounce correct to 2 decimal places?

   a. Complete the following table.

1st Bounce	2nd Bounce	3rd Bounce	4th Bounce	5th Bounce

   b. Represent the information on a graph.

9. Seven houses each contain seven cats. Each cat kills seven mice. Each mouse had eaten seven ears of grain. Each ear of grain would have produced seven hekats of wheat. (The hekat was an ancient Egyptian volume unit used to measure grain, bread, and beer.)
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   a. Complete the table below and describe the type of sequence formed by numbers.

Houses	Cats	Mice	Ears of Grain	Hekats

   b. Give the general formula that could be used to generate these numbers.

10. A local club lottery offers a standard first prize of €50 and there is one draw each week. If there is no winner the prize doubles for the following week. This pattern continues until the first prize is won. How much would the first prize be worth if there was no winner for (a) six consecutive weeks and (b) six consecutive months? Show your calculations.

11. If €1,000 is invested at 5% compound interest (assume for this problem the interest accrues on the anniversary of lodgement date.) How much will be in the account after 4 years? Show your calculations.

Section A: Student Activity 1 (continued)

12. When a coin is tossed there are two possible outcomes, if it is tossed twice there are four possible outcomes and so on. How many outcomes are there if the coin is tossed three times? Now complete the table below, (show your calculations):

No. of Tosses	10	15	20	30
No. Of Outcomes				

13. An antique costing €4,000 appreciates each year by 6% of the value it was at the beginning of that year. Find its value after 11 years. Show your calculations.

14. The value of a car depreciates at an annual rate of 5%. If the car initially cost €25,000, find:

   a. its value at the end of the first year (show your calculations)

   b. its value after 8 years? (show your calculations)

15. A frog wants to hop a total distance of 1 metre. The first hop is 1/2 metre in length, the second is 1/4 metre and the third is 1/8 metre. If this pattern continues:

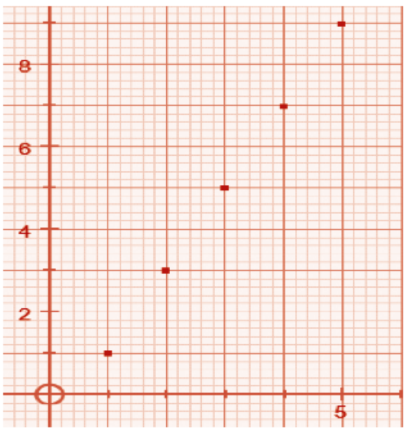
   a. what will be the length of the 9th hop?

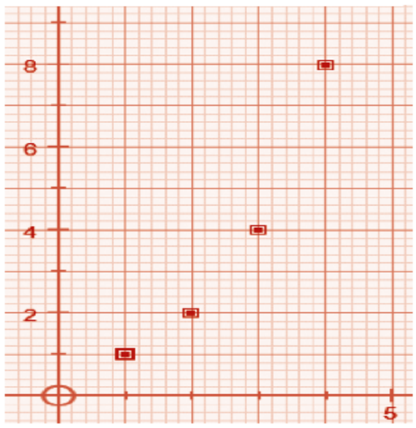
   b. how many hops will it take for the frog to travel a total distance of 1m? (Think carefully before you answer this one.)

16. The number of pupils in a school increases at the rate of 5% per annum. When the school opened there were 200 pupils on the roll. How many pupils will there be in the school on the 10th anniversary of the opening?

17. A novice swimmer can complete the 50m breaststroke in 36s. He follows a programme designed to increasing improve his performance by 4% each year. How fast will he complete the 50m breaststroke after six years of the programme?

18. Tabulate the y co-ordinate in each case for the graphs to the right and decide in each case if the y co-ordinates form a geometric sequence. Give a reason for your answer in each case.





Section B: Student Activity 2

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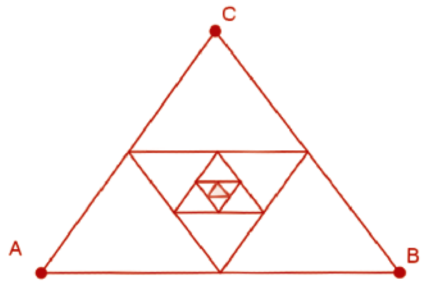
(Calculations must be shown in all cases.)

1. There is a legend that tells how an Indian king promised the inventor of the chessboard a grain of rice for the first square of a chessboard, 2 for the second, 4 for the third, 8 for the fourth square etc. (Note a chessboard has 64 squares). http://www.expert-chess-strategies.com/who-invented-chess.html
   a. What weight of rice was on the first square if one grain of rice weighs 20mg?
   b. What weight of rice was on the second square if one grain of rice weighs 20mg?
   c. What weight of rice was on the fourth square if one grain of rice weighs 20mg?
   d. What weight of rice was on the tenth square if one grain of rice weighs 20mg?
   e. What weight of was in the last square of the board, if one grain of rice weighs 20mg?
   f. Does the weight of rice on consecutive squares of the cheeseboard constitute a geometric sequence? Explain why.

2. As I was going to St. Ives, I met a man with seven wives. Every wife had seven sacks, and every sack had seven cats, every cat had seven kittens.
   © http://www.daviddarling.info/encyclopedia/S/St_Ives_problem.html
   a. How many wives did I meet?
   b. How many sacks did I meet?
   c. How many cats did I meet?
   d. How many kittens did I meet?
   e. Do these numbers constitute a geometric sequence? Explain why.

3. If the perimeter of the equiangular triangle ABC is x cm. The mid points of the sides are joined to form a smaller triangle and the process goes on as in the diagram. If the perimeter of the 6th triangle is 3cm. Find the value of x.



4. If 1,024 tennis players enter a tournament. How many matches must be played in the 6th round if in the tournament the winner of each match moves to the next round and the loser of each match is removed from the tournament?

5. The first week of July was the busiest week of the year at the Deluxe Hotel. It had 200 guests that week. In the weeks immediately following this, the bookings decreased by 5% each week. How many guests did the hotel have in the 10th week following its busiest week?

Section B: Student Activity 2 (continued)

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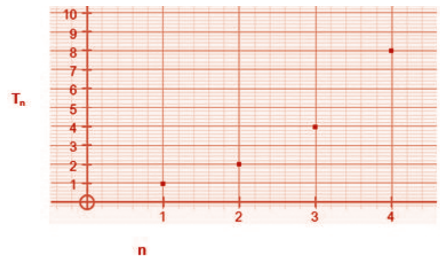
6. In a geometric sequences when will the value of terms decrease? When will they increase?

7. A culture of bacteria doubles every 2 hours. If there are 400 bacteria in the culture on a given day, how many bacteria will there be at the same time the following day?

8. If the graph opposite represents the first 4 terms in a geometric sequence, create a story that could be represented by the graph. What values have a and r in the sequence?



9. If a, the first term of a geometric sequence is equal to 10 and r, the common ratio is ½, draw a graphical representation of this geometric sequence.

10. An employee has a starting salary of €20,000 and can choose from two salary options:
    Option 1: A salary increase by 5% each year.
    Option 2: A guaranteed increase of €1,000 each year.
    a. Which option is initially more beneficial?
    b. Which option is more beneficial after 10 years of employment?
    c. Explain your reasoning.

11. A line segment is divided into six parts forming a geometric sequence. If the shortest length is 3cm and the longest is 96cm, find the length of the line segment. Show your calculations.

12. A local authority decided to control a pest infection in an area by trapping the pests, it was initially estimated that there were 12,000 of these pests in the area and the daily rate of trapping is 1% of the number that exist at the beginning of that particular day.
    a. What is the expected number of pests that will be trapped on the first day?
    b. What is the expected number of pest that will exist after the 1^{st} day?
    c. What is the expected number trapped on the 10^{th} day?
    d. What is the expected number of pests that will exist after the 10^{th} day of trapping?
    e. What is the total number of pests trapped after 5 days?
    f. What is the population of the pests in this area after 5 days?

Section C: Student Activity 3

(Calculations must be shown in all cases.)

1. Assuming this pattern continues, determine whether the following are geometric sequences?

   a. 75, -25, 5, b. 4, 6, 8, 10, c. b^6, b^5c, b^4c^2,

2. If the first term of a geometric sequence is $\frac{1}{5}$ and the fifth term of the same sequence is $\frac{81}{80}$, find:

   a. r ,the common ratio

   b. T_{10}, the 10th term

3. The first term of a geometric sequence is 6 and the common ratio is 3. Find T_n, the general term.

4. Find r, the common ratio and T_n, the general term T_n for the following geometric sequences:

   a. 64, -96, 144, -216,

   b. 2, 6, 18, 54,

   c. $\frac{2}{3}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{12}$,

   d. xy, x^2y, x^3y, x^4y,

5. Insert 3 numbers between 2 and 18 such that the sequence formed is geometric.

6. Originally Holly and Saoirse were the only employees in a firm. Holly earns €80,000 and the Saoirse earns €10,000. As the firm expands two new positions are created with a pay scale between the two existing staff such that all four yearly salaries will be in geometric sequence. Find the monthly salaries of the two new employees. Show your calculations.

7. In a geometric sequence $T_5 = 3$ and $T_8 = \frac{3}{8}$, find the common ratio and the first term.

8. If $x - 4$, $x + 2$ and $3x + 6$ form a geometric sequence, find x.

Section C: Student Activity 3 (continued)

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9. Three numbers form an arithmetic sequence. The first term minus the third term is 8. When the 1st, 2nd and 3rd terms are increased by 3, 5, and 8 respectively, the resulting numbers form a geometric sequence. Find the common difference of the arithmetic sequence and find the first three terms of the geometric sequence. Is it true in general that if the terms of a geometric sequence are increased by a fixed amount the resulting sequence is also geometric.

10. In a firm, all employees have the same starting salary and it increases by a fixed percentage each year. Show the salary pattern of any employee over the first four years of his or her employment. If Joan earns €40,000 and she is in her third year of service and Sally earns €50,000 and is in her fourth year of service. Find the starting salary for employees in this firm.

11. Three numbers of a geometric sequence are $\frac{16}{5}$, y, $\frac{5}{16}$. Find values for y.

12. A four year old car is valued at €16,038.00. A year later the same car is valued at €14,434.20. If the value of the car continues to depreciate at a fixed annual rate. Find the original value of the car correct to the nearest euro.

