Teaching & Learning Plans

Integers

Junior Certificate Syllabus

Project Maths
Tionscadal Mata

Development Team

The Teaching & Learning Plans are structured as follows:

Aims outline what the lesson, or series of lessons, hopes to achieve.

Prior Knowledge points to relevant knowledge students may already have and also to knowledge which may be necessary in order to support them in accessing this new topic.

Learning Outcomes outline what a student will be able to do, know and understand having completed the topic.

Relationship to Syllabus refers to the relevant section of either the Junior and/or Leaving Certificate Syllabus.

Resources Required lists the resources which will be needed in the teaching and learning of a particular topic.

Introducing the topic (in some plans only) outlines an approach to introducing the topic.

Lesson Interaction is set out under four sub-headings:

i.   **Student Learning Tasks – Teacher Input:** This section focuses on possible lines of inquiry and gives details of the key student tasks and teacher questions which move the lesson forward.

ii. **Student Activities – Possible Responses:** Gives details of possible student reactions and responses and possible misconceptions students may have.

iii. **Teacher's Support and Actions:** Gives details of teacher actions designed to support and scaffold student learning.

iv. **Assessing the Learning:** Suggests questions a teacher might ask to evaluate whether the goals/learning outcomes are being/have been achieved. This evaluation will inform and direct the teaching and learning activities of the next class(es).

Student Activities linked to the lesson(s) are provided at the end of each plan.

Teaching & Learning Plan: Integers

Aims

- To familiarise students with addition, subtraction, multiplication and division of integers
- To engage students in activities that will help their mental arithmetic using integers
- To engage students with the everyday uses of integers

Prior Knowledge

Prior knowledge and experience of handling addition, subtraction, multiplication and division of natural numbers

- addition and subtraction of natural numbers from primary school
- positive and negative numbers on a number line
- addition of simple positive and negative numbers with the aid of a number line

Note: The notation encountered by students at primary level is different to the standard notation used at post-primary level. For example, $^+5 + {}^-7$, is read as positive 5 and negative 7. It may be an idea to start students with this notation initially when dealing with integers. For example $^-6 + {}^-7$ (read as negative 6 plus negative 7) gives $^-13$ (read as negative 13).

Learning Outcomes

As a result of studying this topic, students will be able to:

- investigate the properties of arithmetic, commutative, associative and distributive properties and the relationships between operations including inverse operations
- appreciate the order of operations, including brackets
- investigate models such as the number line to illustrate the operations of addition, subtraction, multiplication and division in Z
- explore some of the laws that govern these operations and use mathematical models to reinforce the algorithms they commonly use

Catering for Learner Diversity

In class, the needs of all students, whatever their level of ability level, are equally important. In daily classroom teaching, teachers can cater for different abilities by providing students with different activities and assignments graded according to levels of difficulty so that students can work on exercises that match their progress in learning.

Less able students, may engage with the activities in a relatively straightforward way while the more able students should engage in more open–ended and challenging activities. Selecting and assigning activities appropriate to a student's ability will cultivate and sustain his/her interest in learning. In interacting with the whole class, teachers can employ effective and inclusive questioning. Questions can be pitched at different levels and can move from basic questioning to ones which are of a higher order nature.

Relationship to Junior Certificate Syllabus

| Topic Number	Description of topic *Students learn about*	Learning outcomes *Students should be able to*
1.6 Number Systems Z: the set of integers, including 0.	The binary operations of addition, subtraction, multiplication, and division and the relationships between these operations, beginning with whole numbers and integers. They explore some of the laws that govern these operations and use mathematical models to reinforce the algorithms they commonly use.	• investigate models such as decomposition, skip counting, arranging items in arrays and accumulating groups of equal size to make sense of the operations of addition, subtraction, multiplication and division, in N where the answer is in N • investigate the properties of arithmetic: commutative, associative and distributive laws and the relationships between them including their inverse operations • appreciate the order of operations, including the use of brackets • investigate models such as the number line to illustrate the operations of addition, subtraction, multiplication and division in Z

Introducing the Topic

Students should be familiar with the concept of directed numbers from primary school. The number line with directed numbers on it will reinforce concepts learned at the primary level.

Opposite Integers

-4   -3   -2   -1   0   1   2   3   4

←Negative Direction   Positive Direction→
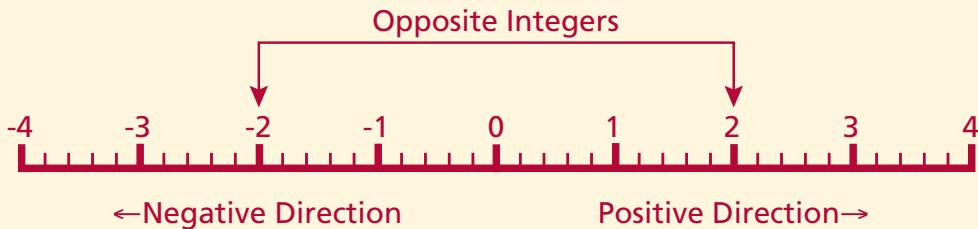
Real Life Context

The following examples could be used to explore real life contexts.
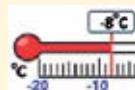
- Temperature
- Money e.g. Owing €10 being the same as -10
- Golf scores
- Heights above and below sea level
- Counter on a DVD player or rewinding live TV
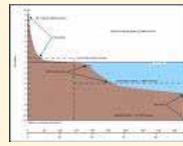
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Lesson Interaction

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning	Teacher Reflections
Section A: Number Line				
» In this lesson we will be investigating positive and negative whole numbers. » Where have you used, or seen the use of, negative whole numbers?	• Temperature. • Golf scores. • Rewinding live TV. • Depth below sea level. • Credit Union book displaying deposits or borrowings.	» If the students have no suggestions, ask a few leading questions such as "What was the lowest temperature last winter?" or "Does anybody know what was the lowest score Rory McIlroy shot?"	» Did students come up with several varied suggestions?	
» We will now begin by looking at a number line. » Working in pairs, draw a number line from -12 to +12 on a blank sheet of squared paper. » Does the number line begin at -12 and finish at +12?	• We marked in from -12 to 12 but the number line keeps going on in both directions.	» Ask a student to draw a number line on the board going from -12 to +12. » Check the number lines drawn by the students to see if they have put in the arrows at each end. » Check to ensure that the positive and negative numbers are in the correct places.	» Do students reaslise that number lines extend indefinitely in both directions? » Are students putting negative numbers to the left of the zero and positive numbers to the right of zero?	

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
Section B: Ordering			
» Is -3 greater than 3? Can you give me some everyday examples to verify your answers?	• -3°C is lower than 3°C. • Owing €3 is worse than having €3.		» Can students come up with varied, real life examples?
» Do you understand the difference between the symbols > and < ? » How do you remember which is which? » Working on your own, do **Section B: Student Activity 1**.	• Greater than or less than • The open side is always facing the biggest number.	» Draw the symbols > and < on the board. » Ask some students to put examples on the board. » Allow for discussion on the best way to remember which is which. » Distribute **Section B: Student Activity 1**. » As students are filling in **Student Activity 1**, circulate to monitor progress.	» Can students use the > and < signs without too much hesitancy?

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
Section C: Addition and Subtraction			
» Starting at zero on your number line, go 3 places in the positive direction. What number are you now at?	• +3	» Encourage the use of number lines.	» Are students using the number line to get the answers?
» In which direction did you move?	• To the right	» Take answers from different students and ask them to show how they arrived at their answer using the number line on the board.	
» Write down a mathematical sentence to describe this.	• 0 + 3 = 3	» Ask a wide range of questions to ensure students are well practised in using the number line.	
» Starting at +4 on your number line, go 3 places in the negative direction. What number are you now at?	• 1		
» Write down a mathematical sentence to describe this.	• + 4 - 3 = 1		
» Starting at -5 on your number line go 4 places in the negative direction. What number are you now at?	• -9	» Distribute **Section B: Student Activity 2**.	
» Write down a mathematical sentence to describe this.	• - 5 - 4 = -9	» Circulate and listen to what students are saying.	» Are students comfortably using the correct directions on the number lines?
» Complete **Section B: Student Activity 2**.			

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
Section D: Multiplication of positive and negative integers			
» We are going to have a look at what happens when positive and negative numbers are multiplied together. » Can anyone give me an example of a sentence where 'two negatives make a positive'?	• "I can't not go" means you have to go. • "I haven't got no money" means you have money. • "I ain't having none of it" means you're having some of it.	» Remind students about what happens in the English language when you have two negatives. » Facilitate a discussion on how 'two negatives make a positive' in language.	
» If we think about money, owing money is negative. What's another way of saying 'owing money'. » Is being 'in debt' positive or negative? » Is 'taking away' positive or negative? » So if we take away a debt, are we using positives or negatives? » If we compare this to the language example, what type of answer will we get? » Which direction would 'taking away a debt' go on a numberline? » Working in pairs, complete **Section D: Student Activity 3**.	• Being in debt • Negative • Negative • Two negatives • If it's two negatives, it makes a positive? • To the right	» Draw a number line on the board and ask students to show examples of taking away a debt. » Distribute **Section D: Student Activity 3**. » Allow students to present their work and facilitate discussion and questions from other groups.	» Are students comfortable with 'two negatives making a positive'?

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
» What about - (-6)		» Use the Powerpoint show available at www.projectmaths.ie OR Use the method outlined in **Appendix 1, Class Demonstration**. » Have a number line on the wall, projected or as a poster, and relate this exercise to the number line. » Guide the students language towards using zero rather than nothing.	» Can students relate the demonstration to the operations on the number line? » Are students using zero instead of nothing by the end of the lesson?
		» For practise, do the first exercise in **Appendix 2, Mental Maths**, with the students.	
» Working in pairs, complete **Section D: Student Activity 4**.		» Distribute **Section D: Student Activity 4**. » As you circulate, ask the students to explain their solutions i.e. verbalise their reasoning. » Ask individual students to write the solutions on the board and explain what they are doing.	» Are students becoming familiar with the operations and relevance of the signs outside the brackets? » Are students able to answer the questions without too much hesitancy?

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning

Section E: Multiplication of positive and negative integers

» Complete Tables 1 and 2, from **Section E: Student Activity 5**.

3 Times	Result	
3 × 4	**= 12**	**+ Ans**
3 × 3	**= 9**	**+ Ans**
3 × 2	**= 6**	**+ Ans**
3 × 1	**= 3**	**+ Ans**
3 × 0	**= 0**	**0**
3 × (-1)	**= -3**	**- Ans**
3 × (-2)	**= -6**	**- Ans**
3 × (-3)	**= -9**	**- Ans**
3 × (-4)	**= -12**	**- Ans**
3 × (-5)	**= -15**	**- Ans**

5 Times	Result	
5 × 4	**= 20**	**+ Ans**
5 × 3	**= 15**	**+ Ans**
5 × 2	**= 10**	**+ Ans**
5 × 1	**= 5**	**+ Ans**
5 × 0	**= 0**	**0**
5 × (-1)	**= -5**	**- Ans**
5 × (-2)	**= -10**	**- Ans**
5 × (-3)	**= -15**	**- Ans**
5 × (-4)	**= -20**	**- Ans**
5 × (-5)	**= -25**	**- Ans**

» Distribute **Section E, Student Activity 5.**

» Circulate to monitor progress and guide or prompt where necessary.

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
	• Completed sentence (a) A positive number multiplied by a positive number gives a positive number (b) A positive number multiplied by a negative number gives a negative number.	» Ask a student to write completed sentences on the board and explain their reasoning. » Write the following on the board: • 2 x -3 = • -3 x 2 = • 5 x -3 = • -3 x 5 = • 1 x -10 = • -10 x 1 =	» Can the students complete the sentences correctly?
» What do you notice about 5 x (-3) and -3 x (5)? » This is called the Commutative Law. » Write down examples of the Commutative Law.	• The order of the numbers doesn't matter for multiplication. • 2 x -3 = -3 x 2 • 10 x -2 = -2 x 10	» Write Commutative Law on the board or add it to the Word Bank.	

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
» Complete Tables 3 and 4 from **Section E: Student Activity 5**.	(tables below)	» Circulate as students work. » Ask students to check for the Commutative Law for Multiplication. » If students are having difficulties allow them to talk through them so that they can identify their misconceptions for themselves. » Emphasise that we can only multiply integers in pairs (or two at a time).	

-2 Times	Result	
-2 × 4	= -8	- Ans
-2 × 3	= -6	- Ans
-2 × 2	= -4	- Ans
-2 × 1	= -2	- Ans
-2 × 0	= 0	0
-2 × (-1)	= 2	+ Ans
-2 × (-2)	= 4	+ Ans
-2 × (-3)	= 6	+ Ans
-2 × (-4)	= 8	+ Ans
-2 × (-5)	= 10	+ Ans

-4 Times	Result	
-4 × 4	= -16	- Ans
-4 × 3	= -12	- Ans
-4 × 2	= -8	- Ans
-4 × 1	= -4	- Ans
-4 × 0	= 0	0
-4 × (-1)	= 4	+ Ans
-4 × (-2)	= 8	+ Ans
-4 × (-3)	= 12	+ Ans
-4 × (-4)	= 16	+ Ans
-4 × (-5)	= 20	+ Ans

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Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
» Summarise what you know about multiplying signs.	• A positive number multiplied by a positive number gives a positive number. • A positive number multiplied by a negative number gives a negative number. • A negative number multiplied by a positive number gives a negative number. • A negative number multiplied by a negative number gives a positive number. **NOTE:** Some discussion as to what should be included here. Three separate bank accounts all of which have an overdraft of €100 gives a total debt of 3(€100.00), for example. Or three jumps to the left of 5 units on the number line moves us how many units in the negative direction? etc.		» Are students correctly completing the sentences?

Teacher Reflections

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
Section F: Order of Operations (BIMDAS)			
» Evaluate 2 + 4 x 7	» Students offer their solutions and then explain how they arrived at them.	» Write 2 + 4 x 7 on the board.	
» How did you get your answers?	• Multiply 4 by 7 and add 2 • Add 2 and 4 together and multiply by 7	» Write the answers 30 and 42 on the board and allow students to talk through their work so that they can identify misconceptions.	
» Which of the answers is correct?	• Both? • We're not sure	» If students have difficulty coming up with ideas, prompt with an example like "Does it matter in what order you go through the classes in primary school?"	
» Think of examples in real life where order matters.	• Following a recipe • Doing a science experiment • Getting dressed		
» Let's look at another situation and see if we can decide. You charge €7 per hour for babysitting. If you do 4 hours babysitting and get a €2 tip, how much will you have?	• €30		

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
» Write the mathematical sentence, in as many ways as possible, to represent this.	• 2 + 4 x 7 • 4 x 7 + 2 • (4 x 7) + 2 • 2 + (4 x 7)	» Get students to write their answers on the board and explain how they got them. **Note:** Allow students time to adopt an investigative approach here. Delay giving the procedure. Engage students in talking about which of €30 or €42 is correct.	» Are students recognising the need for order?
» Just looking at the mathematical sentences, are there any other possible answers?	• €42		

» Where does the €42 come from?	• 2 + 4 = 6 and 6 x 7 = 42	» On the board, write up the term BIMDAS as
		
		      B____I
		   M_____D
		 A_____S
		
		and explain the order of operations. Ask students to come up with their own mnemonics to remember the acronym. (This format will overcome any anomalies in expressions of the form 8 ÷ 2 x 4)	

» Can we have two correct answers or do we need to decide on one method or the other?			
» There is an agreed set of rules, which if followed, means there is only one correct answer for each expression.			
» Can you put in words what we have done.	• We multiplied 6 x 7 and then added 4.		
» In pairs, develop problems that represent BIMDAS.	» Students should compare their questions and answers.	» Check the examples that students are devising.	» Can students develop problems that represent BIMDAS?
» Working in pairs, do the questions from **Section F: Student Activity 6**.		» Distribute **Section F: Student Activity 6**. » Ask students to present and explain their answers. Allow for discussion.	

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
Section G: Multiplying a number of integers by each other			
» Working in pairs,answer questions 1-3 from **Section G: Student Activity 7**. » Now complete **Section G: Student Activity 7**. » What patterns did you notice?	• Anything multiplied by 5 ends in 0 or 5 • Anything multiplied by an even number is an even number • Anything multiplied by 0 = 0 • Minus x minus = plus • Minus x plus = minus • Plus x plus = plus • Plus x minus = minus • Like signs multiplied together have a plus answer. » Students write into their copybooks what they have learned.	» Distribute **Section G: Student Activity 7**. » Circulate and check students' work. Engage students in talking about their work. » If students are having difficulties, allow them to talk through them so that they can identify their misconceptions for themselves. » Ask individual students to show their work on the board. » Write students suggestions on the board. » The students may benefit from some practise with Math Walls from **Appendix 2**.	» Are students doing the calculations correctly?

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning

Section H: Division of Integers

Student Learning Tasks: Teacher Input

» Working in pairs, complete **Section H: Student Activity 8**. Fill in the tables using the diagram with the arrows and circles.

» Do you recognise this diagram?

» Do you think order matters for division? In other words, does the Commutative Law hold for division?

» The Commutative Law doesn't hold for division.

Student Activities: Possible Responses

- It looks like the table we filled in for multiplication.

Division	Result	
12 ÷ 4	= 3	+ Ans
25 ÷ 5	= 5	+ Ans
-3 ÷ 1	= -3	- Ans
(-6) ÷ (-2)	= 3	+ Ans
(-10) ÷ 2	= 5	+ Ans
12 ÷ 4	= 3	+ Ans
(-8) ÷ (-2)	= 4	+ Ans
15 ÷ (-3)	= -5	- Ans
(-20) ÷ 5	= -4	- Ans
10 ÷ -2	= -5	- Ans

Division	Result	
(-12) ÷ 3	= -4	- Ans
15 ÷ 5	= 3	+ Ans
20 ÷ (-2)	= 10	+ Ans
8 ÷ (-4)	= -2	- Ans
16 ÷ (-2)	= -8	- Ans
-2 ÷ 1	= -2	- Ans
4 ÷ 4	= 1	+ Ans
6 ÷ 1	= 6	+ Ans
20 ÷ (-4)	= -5	- Ans
(-20) ÷ (-10)	= 2	+ Ans

Teacher's Support and Actions

» It may help to use physical manipulatives to remind students about the meaning of multiplication and division.

» Distribute **Section H: Student Activity 8**.

» Explain the relationship between the numbers in the circles and the numbers on the edges where the arrows point to.

» Circulate, asking questions where necessary and listen to students'conclusions.

» Write completed sentences on the board.

NOTE: Some discussion about the reasons behind these rules should be held here.

» Ask students to put examples on the board to show whether or not it holds.

NOTE: This provides a nice opportunity to discuss with students that when a proposition fails in one instance in Mathematics it fails in all instances (the proof by contradiction).

Assessing the Learning

» Are students using the table correctly to get the answers to the division questions?

» Can students verbalise to the class what they know about:
1. Ordering
2. Addition and subtraction of integers
3. Multiplication of signs outside brackets
4. Multiplication of positive and negative integers
5. BIMDAS
6. Division of positive and negative integers

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
Section H: Division of Integers			
	• A negative number divided by a positive number gives a negative number.		
	• A negative number divided by a negative number gives a positive number.		
	• A positive number divided by a positive number gives a positive number.		
	• A positive number divided by a negative number gives a negative number.		
	• $6 \div 2 = 3$		
	• $2 \div 6 = \frac{1}{3}$		
Reflection: Summarise what you know about integers.	• Students write into their copybooks what they have learned.		

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Teacher Reflections

Section B: Student Activity 1

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Ordering

1. **Mark in all the integers from -12 to 12 on the number line shown below.**

    0

2. **Use the number line above to say which is greater**

    (a) 0 or 7 _____ (b) -1 or 2 _____ (c) -6 or 7 _____
    (d) -4 or 2 _____ (e) 7 or -5 _____ (f) -10 or -2 _____
    (g) -10 or -12 _____ (h) -5 or -6 _____

3. **Use the number line above to say which is smaller**

    (a) 1 or 4 _____ (b) 7 or 5 _____ (c) -6 or 8 _____
    (d) -4 or -2 _____ (e) -7 or 6 _____ (f) -8 or -2 _____
    (g) -10 or -3 _____ (h) -8 or -12 _____

4. **Complete the statement: If one number is larger than another it lies**
    _____on the number line.

5. **From the following groups, list the numbers from the highest to the lowest (i.e. in decreasing order)**

    (i) 3, -5, 7, -3, 9. _____

    (ii) 8, -6, -2, -3, 5. _____

    (iii) -9, -7, 6, -3, -2. _____

    (iv) 4, -7, 0, -4, -5. _____

5. **Which of the following temperatures are colder?**

    (i) 8°C or 5°C _____

    (ii) -7°C or 2°C _____

    (iii) 0°C or 5°C _____

    (iv) -8°C or -5°C _____

6. **Insert the correct symbol > or < between the following numbers.**
    Example: 10 > 5 because 10 is bigger than 5

    i) 14   5   ii) -2   5   iii) 7   -3
    iv) -10   -8   v) -100   -102   vi) 76   -76.

Section C: Student Activity 2

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Addition and Subtraction

1. The number line shows -4 + 6 which gives 2

| Start | Forward Steps | Answer |

-13 -12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12 13

Use the number lines below to show the answers to the following questions. Show your work on the number lines.

i) 2 + 4

-13 -12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12 13

Answer _____

ii) -8 + 10

-13 -12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12 13

Answer _____

iii) -6 + 5

-13 -12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12 13

Answer _____

iv) -12 + 8

-13 -12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12 13

Answer _____

v) -11 + 4

-13 -12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12 13

Answer _____

2. Find the values of each of the following

i) 2 + 7 = _____   ii) -5 + 7 = _____   iii) -7 + 2 = _____
iv) -3 + 1 = _____   v) -5 + 10 = _____   vi) -9 + 2 = _____
ii) -6 + 12 = _____   iii) -7 + 13 = _____   iv) -2 + 0 = _____
(v) -11 + 7 = _____

3. Explain how you got the answers to any one of the questions in question 2 above _____

_____

_____

Section C: Student Activity 2 (continued)

4. The number line shows 2 - 7 which gives -5

| Start | Backward Steps | Answer |

-13 -12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12 13

Use the number lines below to show the answers to the following questions. Show your work on the appropriate number line in each case.

i) 5 - 4

-13 -12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12 13

Answer _____

ii) 12 - 10

-13 -12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12 13

Answer _____

iii) -5 - 4

-13 -12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12 13

Answer _____

iv) -2 - 8

-13 -12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12 13

Answer _____

v) -9 - 4

-13 -12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12 13

Answer _____

Section C: Student Activity 2 (continued)

5. **The table below shows the temperatures in a number of cities. If the temperature in Galway is 15°C, fill in the table below to find the temperature in the various cities.**

CITY	Warmer/Colder	Temperature
Galway		15°C
Dublin	2° colder than Galway	
Paris	10° warmer than Galway	
Moscow	20° colder than Galway	
New-York	4° warmer than Moscow	
Sydney	25° colder than Galway	
Cairo	10° warmer than Paris	
Oslo	3° colder than Moscow	

6. **Find the values of each of the following.**

   a) **2 + (-4) + 5 Answer** _____

   b) **7 + 6 + (-10) Answer** _____

   c) **8 + (- 7) + (-6) Answer** _____

   d) **-2 + (-3) + (-3) Answer** _____

7. **Find the value of each of the following**

   a) **11 + (-2) + 3 + (-1) + 6 + (-4) + (-3) Answer** _____

   b) **-8 + 12 + (-2) + (-12) + 7 + (-7) Answer** _____

   c) **10 + (-2) + 14 +(-11) + 3 + (-8) +12 Answer** _____

Section C: Student Activity 2 (continued)

8. The scores, compared to par, for ten players in a golf tournament are listed in the following table. (See below for definition of par.)

Name	Round 1	Round 2	Round 3	Round 4	Total score compared to par
O'Brien	-2	-3	+3	Level (0)	
McCarthy	-1	+2	+2	+1	
Collins	Level (0)	Level (0)	-1	Level (0)	
O'Connor	+1	+2	-3	Level (0)	
Gallagher	-4	-1	+1	-2	
Ryan	+2	-2	Level (0)	+3	
Conneely	-1	Level (0)	-2	+4	
Cleary	-4	-3	-1	Level (0)	
Scanlon	+1	-3	Level (0)	-1	
Lyons	-3	+3	Level (0)	-1	

Use this table to answer the following questions

i)   Fill the total score, compared to par, for each player into the table above.

ii) Which player had the lowest score? _____

iii) Which player had the highest score? _____

iv) Who is the best golfer, according to the above table?_____

Definition: Par is the number of strokes an expert golfer is expected to need to complete each individual hole, or all the holes on a golf course.

9. Mr. McKeon has €500 in his bank account. During the day he withdraws €275 from an ATM and a cheque for €370 is also debited from his account. What is the account balance at the end of the day?

   _____

Section D: Student Activity 3

Two Negatives

Example: Mary borrowed €20 from her brother and after doing some babysitting was able to pay back €12. How much did she owe then? €8

Write out a Mathematical Sentence to show this and illustrate the process on a number line.
-20 + 12 = -8 (Owing €20 and subsequently taking away €8 of the debt)

I took away €12 of the debt→

←I borrowed €20 from my brother

```
-26  -24  -22  -20  -18  -16  -14  -12  -10  -8   -6   -4   -2   0    2    4
```
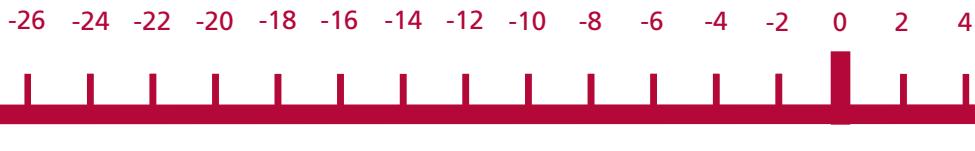
Use the number lines given to answer each of the following questions. In each case show your work on the number line. Write a Mathematical Sentence to illustrate your answers.

1. **My sister had to borrow €100 from my Dad for her school tour. She paid him back €20 per week. How much did she owe at the end of the first week? _____**

```
-160  -140  -120  -100  -80  -60  -40  -20   0    20   40   60   80   100  120  140
```

Mathematical Sentence_____

How many weeks did it take to pay back the full amount?_____

2. **Joan borrowed €25 from her sister and promised to pay €30 in return. Her sister had €50 to begin with. How much did she have at the end?_____**

```
-32 -30 -28 -26 -24 -22 -20 -18 -16 -14 -12 -10 -8 -6 -4 -2  0  2  4  6  8  10 12 14 16 18 20 22 24 26 28 30
```

Mathematical Sentence_____

Section D: Student Activity 3 (continued)

3. i) I needed €42 and had to borrow €7 from each of my 6 friends. Illustrate this using a mathematical sentence.

    ii) I borrowed an additional €3 from one of them. Show in steps, on the number line how I paid him back in full.

```
-46 -44 -42 -40 -38 -36 -34 -32 -30 -28 -26 -24 -22 -20 -18 -16 -14 -12 -10 -8 -6 -4 -2  0  2  4  6  8  10  12  14  16  18
```

Maths sentence (i)_____ Maths sentence (ii)_____

4. Look at the number line below. Write a story using two negatives. Indicate the direction on the number line and add to the diagram if necessary.

    _____

    _____

    _____

    _____

    _____

    _____

```
    -5      -4      -3      -2      -1      0      1      2      3
```

5. Using the same number line, write a story for a negative and a positive. Indicate the direction on the number line and add to the diagram if necessary.

    _____

    _____

    _____

    _____

    _____

    _____

Section D: Student Activity 4

Multiplication of Signs Outside Brackets

1. **Simplify by removing the brackets.**

   a) - (+ 1) = _____   b) + (-2) = _____   c) - (-4) = _____

   d) + (-8) = _____   e) - (-12) = _____   f) - (+ 5) = _____

   From the above, complete the following sentences.
   A minus outside a bracket changes the _____ of the number inside the bracket.
   A plus outside a bracket has no effect on the _____ of the number inside the bracket.

2. **Simplify each of the following by removing the brackets.**
   Example: 5 + (-1) = 5 - 1 = 4

   a) 4 + (-1) = _____   b) 6 + (-2) = _____   c) 5 - (-3) = _____

   d) 7 + (-8) = _____   e) 12 - (-12) = _____   f) 0 - (+ 5) = _____

3. **Simplify each of the following.**

   a) -3 + (-4) + 6 =_____   b) -5 + 8 + (-3) =_____   c) 2 + (-6) + (-8) =_____

   d) -6 + (-4) + (-1) =__   e) 9 - (-2) + 7 =_____   f) 11 + (-3) + 9 =_____

   f) 13 + (-6) + 9 =_____   g) -5 + (-5) + 3 =_____   h) 15 + (-12) + (-5) =__

4. **Simplify each of the following**

   a) 4 - (-7) + 4 - (-5) = _____

   b) - 9 - 2 (-5) - 4 (-3) = _____

   c) - 7 - 4 (-4) - (-1) = _____

   d) - 8 + 2 (-3) - (-7) + 2 = _____

5. **Simplify each of the following**

   a) 7 + (-1 + 2) - (6 - 4) = _____

   b) - 8 - (-2+ 5) - (-1 - 2) = _____

   c) 7 - (-2) - (2 - 1) = _____

   d) - 13 + (5 - 5) - (- 8 - 8) + 3 = _____

Section E: Student Activity 5

Project
Maths
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Multiplication of Positive and Negative Integers

Use number patterns to complete the following tables.

Table 1			Table 2		
3 Times		**Result**	**5 Times**		**Result**
3 × 4	= 12	**Positive Answer**	**5 × 4**	= 20	**Positive Answer**
3 × 3	= 9	**Positive Answer**	**5 × 3**	= 15	**Positive Answer**
3 × 2	=		**5 × 2**	=	
3 × 1	=		**5 × 1**	=	
3 × 0	= 0	0	**5 × 0**	= 0	0
3 × (-1)	=		**5 × (-1)**	=	
3 × (-2)	=		**5 × (-2)**	=	
3 × (-3)	=		**5 × (-3)**	=	
3 × (-4)	=		**5 × (-4)**	=	
3 × (-5)	=		**5 × (-5)**	=	

Use the completed tables to fill in the appropriate term in the following sentences.

a) **A positive number multiplied by a positive number gives a _____ number.**

b) **A positive number multiplied by a negative number gives a _____ number.**

Section E: Student Activity 5 (continued)

Use number patterns to complete the following tables.

Table 3				Table 4			
-2 Times		**Result**		**-4 Times**		**Result**	
-2 × 4	= -8	Negative Answer		-4 × 4	= -16	Negative Answer	
-2 × 3	= -6	Negative Answer		-4 × 3	= -12	Negative Answer	
-2 × 2	= -4			-4 × 2	= -8		
-2 × 1	=			-4 × 1	=		
-2 × 0	= 0	0		-4 × 0	= 0	0	
-2 × (-1)	=			-4 × (-1)	=		
-2 × (-2)	=			-4 × (-2)	=		
-2 × (-3)	=			-4 × (-3)	=		
-2 × (-4)	=			-4 × (-4)	=		
-2 × (-5)	=			-4 × (-5)	=		

Use the completed tables to fill in the appropriate term in the following sentences.

a) A negative number multiplied by a positive number gives a _____ number.

b) A negative number multiplied by a negative number gives a _____ number.

Section F: Student Activity 6

Order of Operations - BIMDAS

1. **A mechanic charged a customer €45 for parts. He charged €15 per hour and it took 4 hours to fix the car. How much was the bill? _____**

2. **4 x 6 + 2 + 3 x 6 = 50. Add brackets, where appropriate, to make this statement correct.**

3. **Mike cuts 6 lawns every Saturday. He charges €8 per lawn. Two of the houses always give him a €2 tip. How much will he earn after four Saturdays? _____**
   Write out a mathematical sentence to represent this.

   _____

4. **In a factory, the standard rate per hour is €9.50. The rate for working on Saturday is €14 per hour. Circle the following statements which correctly represent working 36 hours at the standard rate and 6 hours of Saturday work? There may be more than 1 correct answer.**

   a) **9.50 x 36 + 14 x 6**   b) **9.50 x (36 + 14) x 6**

   c) **9.50 + 14 x 6 + 36**   d) **(14 x 6) + (9.50 x 36)** e) **23.5 x 42**

5.

   Dry Cleaners Price list

   Jacket   €12.50
   Trousers   € 8.00
   Dress   €7.50
   Coat   €15.00

   A dry cleaners is trying to drum up business.

   The following mathematical sentence represents Mary's bill and the special offer. Write out what Mary got cleaned and the special offer.

   3 x 12.50 + 8 + 2 x 7.50 + 3 x 15 - (2 x 7.50)

6. **Use brackets, where necessary, to make to following correct.**

   i) **23 + 2 x 7 - 5 x 4 = 17** _____

   ii) **23 + 2 x 7 – 5 x 4 = 680** _____

   iii) **23 + 2 x 7 – 5 x 4 = 128** _____

   iv) **23 + 2 x 7 – 5 x 4 = 200** _____

Section G: Student Activity 7

Multiplying a Number of Integers

Example 1: Find the value of -7 × (-4) × 2

-7 x (-4) x 2

+28   x 2

+56

Example 2: Find the value of -1 × (-3) × (-2) × (2)

-1 x (-3) x (-2) x (2)

+3   x   -4

-12

Complete the following questions on multiplication of integers.

1. **Multiply each of the following.**

    a) $7 \times (-3) =$ _____   **b)** $-8 \times (-1) =$ _____   **c)** $3 \times (-4) =$ _____

    d) $-4 \times (-2) =$ _____   **e)** $-9 \times (-4) =$ _____   **f)** $5 \times (-3) =$ _____

    g) $(-2) \times (-3) =$ _____   **h)** $5 \times (-5) =$ _____   **i)** $-5 \times (-2) =$ _____

    j) $-6 \times (-3) =$ _____   **k)** $5 \times (-8) =$ _____   **l)** $-5 \times (-6) =$ _____

2. **Simplify the following.**

    a) $5 \times (-1) \times (-3) =$ _____

    b) $-3 \times (-1) \times 4 =$ _____

    c) $(-3) \times (-4) \times (-4) =$ _____

    d) $-3 \times (-1) \times (-5) =$ _____

3. **Simplify the following.**

    a) $6 \times (-3) \times (-2) \times (-4) =$ _____

    b) $-2 \times (-1) \times 4 \times (-6) =$ _____

    c) $-7 \times (-4) \times (-1) \times (-2) =$ _____

    d) $5 \times (-8) \times 4 \times (-6) =$ _____

Section G: Student Activity 7(continued)

Complete the multiplication table below and then answer the questions that follow.

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
-5											
-4											
-3											
-2											
-1											
0											
1											
2											
3											
4											
5											

List three patterns you see on the completed table.

1. _____

_____

2. _____

_____

3. _____

_____

Section H: Student Activity 8

Division of Integers

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
-5	25	20	15	10	5	0	-5	-10	-15	-20	-25
-4	20	16	12	8	4	0	-4	-8	-12	-16	-20
-3	15	12	9	6	3	0	-3	-6	-9	-12	-15
-2	10	8	6	4	2	0	-2	-4	-6	-8	-10
-1	5	4	3	2	1	0	-1	-2	-3	-4	-5
0	0	0	0	0	0	0	0	0	0	0	0
1	-5	-4	-3	-2	-1	0	1	2	3	4	5
2	-10	-8	-6	-4	-2	0	2	4	6	8	10
3	-15	-12	-9	-6	-3	0	3	6	9	12	15
4	-20	-16	-12	-8	-4	0	4	8	12	16	20
5	-25	-20	-15	-10	-5	0	5	10	15	20	25

Above is the completed table from Student Activity 7.

The circle and the arrows represent division, which is read as

12 ÷ 4 = 3 or 12 ÷ 3 = 4

25 ÷ 5 = 5

(-3) ÷ (-3) = 1 or (-3) ÷ 1 = (-3)

Use the table above to complete the tables on the next page.

Section H: Student Activity 8 (continued)

Table 1		
Division	**Result**	
12 ÷ 4	= 3	**Positive Answer**
25 ÷ 5	= 5	**Positive Answer**
-3 ÷ 1	= -3	
(-6) ÷ (-2)	=	
(-10) ÷ 2	=	
12 ÷ (-4)	=	
(-8) ÷ (-2)	=	
15 x (-3)	=	
(-20) ÷ 5	=	
10 ÷ (-2)	=	

Table 2		
Division	**Result**	
(-12) ÷ 3	= -4	**Negative Answer**
15 ÷ 5	=	**Positive Answer**
20 ÷ (-2)	=	
8 ÷ (-4)	=	
16 ÷ (-2)	= 0	
(-2) ÷ 1	=	
4 ÷ 4	=	
6 ÷ 1	=	
20 ÷ (-4)	=	
(-20) ÷ (-10)	=	

Use the completed tables to fill in the following spaces.

a) A positive number divided by a positive number gives a _____ number.

b) A negative number divided by a negative number gives a _____ number.

c) A positive number divided by a negative number gives a _____ number.

d) A negative number divided by a positive number gives a _____ number

Simplify each of the following.

a) 12 ÷ 3 = _____

b) -12 ÷ 2 = _____

c) -14 ÷ 7 = _____

d) (-12) ÷ (-4) = _____

e) 6 ÷ -3 = _____

f) (-24) ÷ (-6) = _____

g) -(12 ÷ 11) = _____

e) -(-32 ÷ 4) = _____

f) (-26) ÷ 9 = _____

Appendix 1

Project **Maths**
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Class Demonstration

- **Place an empty box or the outline of a square on the table.
  Ask the students what quantity is in the box/square.**
  Answer: *Nothing/Zero*

- **Place a card with +6 on it in the box.
  Again, ask the students what quantity is in the box.**
  Answer: *6 or plus 6 or positive 6*

  +6

- **Remove this card and put in a different one with -6 on it.
  Ask the students what quantity is in the box now.**
 **Answer:** *Minus 6 or take away 6 or negative 6*

  -6

- **Place the +6 in the box with the -6.
  Ask the students what quantity is in the box at this stage.**
  Answer: *Nothing/zero*

 -6 +6
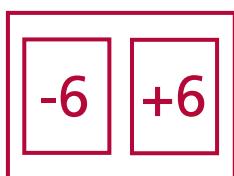
Appendix 1 (continued)

- **Repeat this exercise with different integers to reinforce the idea that a '+' and '-' of the same number gives zero.**

- **Now, write -(-6) on the board and ask students to tell you what it means.**
  Answer: *Take away minus 6 or minus minus 6 or take away negative 6.*

- **We have zero in the box and we are going to take away -6.**
  What quantity is left in the box now?
  Answer: *6 or +6 or positive 6*
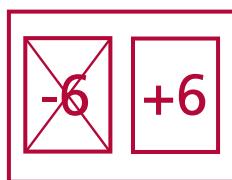
- **So if we have 0 -(-6), what is left in the box?**
  Answer: *+6*   **So -(-6) is +6**
  Repeat this exercise with different integers to reinforce what is happening.

- **Let's try this exercise with a '+' outside the bracket.**
  What quantity is in the box?
  Answer: *Zero/nothing*
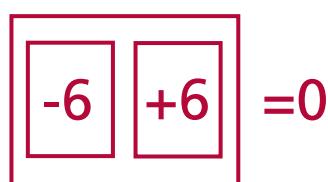
- **Explain what +(-6) means.**
  Answer: *Add minus 6/plus negative six/add negative 6*

- **So if I add (-6) to the box, what quantity is in there now?**
  Answer: *Negative 6/minus 6*
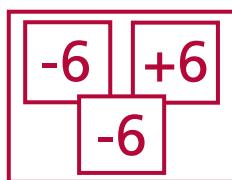
- **So if we have 0 + (-6), what quantity is in the box?**
 **Answer:** *-6*   **So +(-6) is -6**

APPENDIX 2

Mental Maths

1. Small white boards

These are small, white, laminated boards approximately 30cm x 20cm. Each student in the class is given one of these boards with a small white board marker and a cloth. The teacher calls out a question such as -4 multiplied by -2. Each student writes down his or her answer and the teacher says "show me". Each student then holds up the board with his or her answer so that the teacher can scan around the room quickly and check to see who is getting the mental maths right or wrong. The teacher then says "wipe" and each student wipes his or her board clean. The teacher proceeds to call out more questions. This enables the teacher to identify students who are having difficulty with a particular aspect of the topic.

2. Maths walls

These are useful from the point of view that they hold the students attention over a period of time. Below are two examples of five block walls, one for addition and one for multiplication. You can also have 7, 9,… block walls but these sometimes take too long to complete and therefore do not fall into the "mental maths" category.

5 Block Addition Wall	5 Block Multiplication Wall
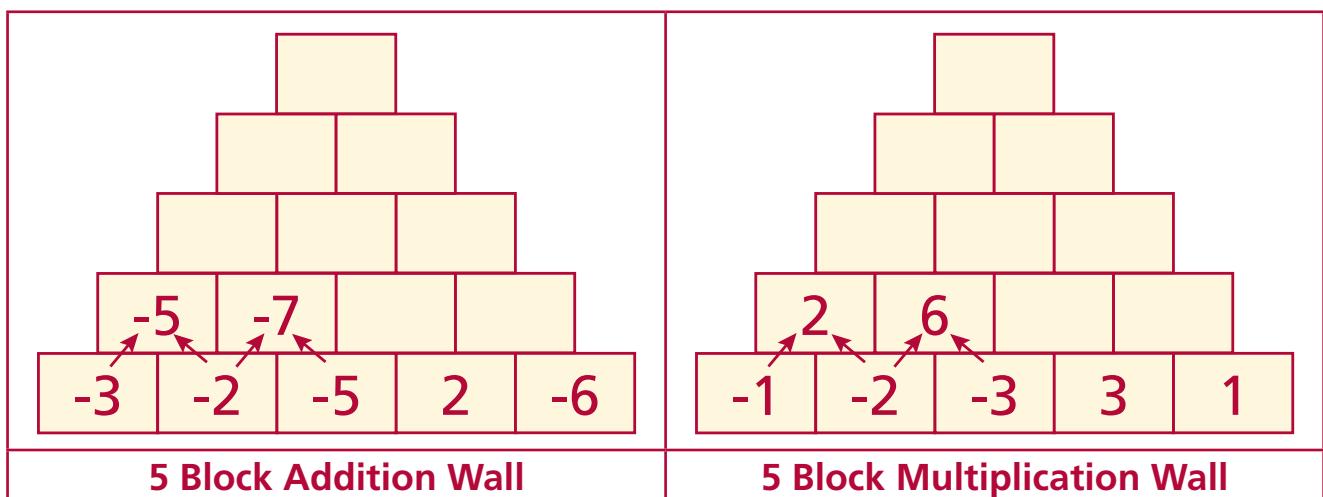
Note: Keep the integers for the multiplication wall small as wall increases in difficulty with larger integers.