Teaching & Learning Plans

Integral Calculus

Leaving Certificate Syllabus

Project Maths
Tionscadal Mata
Development Team

The Teaching & Learning Plans are structured as follows:

Project Maths
Tionscadal Mata
Development Team

Aims outline what the lesson, or series of lessons, hopes to achieve.

Prior Knowledge points to relevant knowledge students may already have and also to knowledge which may be necessary in order to support them in accessing this new topic.

Learning Outcomes outline what a student will be able to do, know and understand having completed the topic.

Relationship to Syllabus refers to the relevant section of either the Junior and/or Leaving Certificate Syllabus.

Resources Required lists the resources which will be needed in the teaching and learning of a particular topic.

Introducing the topic (in some plans only) outlines an approach to introducing the topic.

Lesson Interaction is set out under four sub-headings:

i. **Student Learning Tasks – Teacher Input:** This section focuses on possible lines of inquiry and gives details of the key student tasks and teacher questions which move the lesson forward.

ii. **Student Activities – Possible Responses:** Gives details of possible student reactions and responses and possible misconceptions students may have.

iii. **Teacher's Support and Actions:** Gives details of teacher actions designed to support and scaffold student learning.

iv. **Assessing the Learning:** Suggests questions a teacher might ask to evaluate whether the goals/learning outcomes are being/have been achieved. This evaluation will inform and direct the teaching and learning activities of the next class(es).

Student Activities linked to the lesson(s) are provided at the end of each plan.

Teaching & Learning Plans: Integral Calculus

Project Maths
Tionscadal Mata
Development Team

Aims[1]

The aim of this series of lessons is to enable students to:

- To understand the process of anti-differentiation.
- To recognise the problem of calculating areas bounded by non-linear functions.
- To understand how the limit of the sum of rectangles may be used to calculate the area bounded by a function.
- To understand that the area bounded by a function may itself be described by a function.
- To understand that an anti-derivative of a function may be used to calculate the area under the function.
- To understand that the area under a function is equal to the sum of the function.
- To understand the idea of the average value of a function and to calculate it.
- To apply knowledge and skills relating to anti-differentiation to solve problems.
- To use mathematical language, both written and spoken, to communicate understanding effectively.

Prior Knowledge

Students have prior knowledge of:

- Functions
- Differential Calculus
- Pattern analysis
- Area
- Distance, speed and time
- Indices
- Limits

1 This Teaching & Learning Plan illustrates a number of strategies to support the implementation of *Literacy and Numeracy for Learning and Life: the National Strategy to Improve Literacy and Numeracy among Children and Young People 2011-2020* (Department of Education & Skills 2011). Attention to the recommended strategies will be noted at intervals within the Lesson Interaction Section of this Teaching and Learning Plan.

Learning Outcomes

As a result of studying this topic, students will be able to:

- Find the indefinite form of the anti-derivative of a function.
- Find a distinct anti-derivative of a function.
- Use anti-differentiation to solve real world problems in which rate-of-change information is given.
- Understand that the area bounded by a function may be calculated using
  $\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$
- Understand the meaning of $\int_a^b f(x)dx$
- Use anti-differentiation to derive an area function.
- Calculate the area under a function between two extremes.
- Use anti-differentiation to calculate the average value of a function.
- Understand how anti-differentiation and the average value of a function might be used in a range of every-day applications.

Catering for Learner Diversity

In class, the needs of all students, whatever their level of ability level, are equally important. In daily classroom teaching, teachers can cater for different abilities by providing students with different activities and assignments graded according to levels of difficulty so that students can work on exercises that match their progress in learning. Less able students, may engage with the activities in a relatively straightforward way while the more able students should engage in more open-ended and challenging activities.

In interacting with the whole class, teachers can make adjustments to meet the needs of all of the students.

Apart from whole-class teaching, teachers can utilise pair and group work to encourage peer interaction and to facilitate discussion. The use of different grouping arrangements in these lessons should help ensure that the needs of all students are met and that students are encouraged to articulate their mathematics openly and to share their learning.

Project Maths Tionscadal Mata Development Team

Relationship to Leaving Certificate Syllabus

Topic	**Learning outcomes** *In addition students working at HL should be able to*

5.2 Calculus	• recognise integration as the reverse process of differentiation
	• use integration to find the average value of a function over an interval
	• integrate sums, differences and constant multiples of functions of the form:
	  • x^a, where $a \in Q$
	  • a^x, where $a \in R$
	  • $Sin\ ax$, where $a \in R$
	  • $Cos\ ax$, where $a \in R$
	• determine areas of plane regions bounded by polynomial and exponential curves

Resources Required

Whiteboard, rulers, *GeoGebra* and calculators.

Project Maths Tionscadal Mata Development Team

Teacher Reflections

Lesson Interaction

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
Section A - The Anti-Derivative			
» We will now revise the process of differentiation. In **Section A: Student Activity 1** there are 10 functions together with their derivatives. Working in pairs, I want you to match the correct derivative to each function.		» Distribute **Section A: Student Activity 1**. » Circulate to observe students' work. » Assist students as required. » Remind students that they may refer to page 25 of the *Formulae and Tables Booklet* for help with the differentiation process.	
» When you complete the task I want you to examine the function-derivative pairs. Is there anything interesting/ noteworthy about your results? » We now see that different functions may have the same derivative.	• There seem to be functions which have the same derivative. • We have several derivatives which are the same. • Different functions have the same derivative.	» Write the list of functions (unsorted) on the board and ask students in the class to come to the board and fill in the correct derivative beside each one. » Circle identical derivatives on the board. Highlight the fact that they come from different functions. » Sort the function-derivative pairs so that functions having the same derivatives are placed together. » Highlight the fact that $f(x) = 5x$, $g(x) = 5x + 2$ and $h(x) = 5x - 10$, all have the same derivative; $\frac{dy}{dx} = 5$	» Are students able to differentiate each function to find its matching derivative? » Do students recognise that different functions may have the same derivative? » Can students express themselves clearly and develop their arguments logically?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» Can you explain why different functions may have the same derivative? Discuss your reasoning with your partner. » Do you understand what the word derivative means in this context? How might you use this word in another context?	• The functions which have the same derivative are all similar to each other. • Derivative means slope and $f(x) = 5x$, $g(x) = 5x + 2$ and $h\ (x) = 5x$ -10, all have the same slope of 5. • $g(x) = 5x + 2$ and $h\ (x) = 5x$ -10 and are just vertically shifted versions of $f(x) = 5x$, and vertically shifting a functions doesn't affect its slope (derivative). • When you differentiate a constant you get zero. • $g(x) = 5x + 2$ is a function made up of two functions added together: $g_1\ (x) = 5x$ and $g_2\ (x) = 2$. The slope of $g_1\ (x)$ is 5 and the slope of $g_2\ (x)$ is 0 (because it's a horizontal line) so adding $g_2\ (x) = 2$ to $g_1\ (x) = 5x$ has no effect on its slope (derivative).	» Support student discussion with appropriate questions: - What does the derivative of a function mean? - Could you use a sketch to explain what the derivative of a function represents? - Could you use a sketch to explain why different functions may have the same derivative? » Sketch a cubic function on the board and ask students to sketch its derivative. Sketch a vertically-shifted version of the cubic function on the board and ask students to sketch its derivative. » Observe students' work and support as necessary. » Use the *GeoGebra* file "Derivative of a Shifted Function" to demonstrate that vertically shifting a function has no effect on its derivative: - Open *GeoGebra* file. - Click "Show Tangents" to show the tangent to the function. - Drag the point around function to trace out its derivative in the second graphics window. - Click "Show Tangents" to hide the tangent to the function. - Click "Show Shifted Function" to show the shifted function. - Drag the slider "Shift" to move the shifted function up or down. - Click "Show Tangents" to show the tangents to both functions. - Drag the point around the original function to show that both functions have equal slopes. - Highlight the fact that the slope of the shifted function traces out the same derivative as the original function in the second graphics window. - Click "Show Algebra" to demonstrate that the two functions only differ by a constant value.	» Do students understand what the derivative of a function means? » Can students sketch the derivative of a cubic function? » Can students relate the slope of a tangent to a function to its derivative? » Do students recognise that when several functions have the same derivative, each function is just a vertically-shifted version of the others? » Can students use mathematical terms when communicating the information?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» Often in mathematics when we carry out some operation we like to be able to undo the operation or reverse it. For example when we square a number, to recover the number we started off with we use a square-root. When we multiply, to recover the number we started with, we divide and so on.			
» Can we do the same with a derivative function? That is – if we took a function and found its derivative, could we, starting with the derivative, recover the function we started off with? In other words can we reverse the differentiation process?	• Yes we can. • No we cannot. • We can determine the general form of the function but we cannot recover the exact function.	» Write the function $g(x) = 5x + 2$ and its derivative $g'(x) = 5$ on the board linking with an arrow. Rub out $g(x) = 5x + 2$, reverse the direction of the arrow and ask students if it's possible to recover the function $g(x) = 5x + 2$?	» Do students understand that there is an ambiguity which arises when we want to reverse the differentiation process?
» What is the difficulty in reversing the differentiation process?	• We don't know if the function contains a constant term or not. • We can't find the vertical shift of the function since the derivative only contains information about the slope of the original function. • We saw in **Section A: Student Activity 1** that different functions may have the same derivative so if we have the derivative we have no way of knowing the exact function which produced it. • We cannot recover the exact function.	» Refer to the diagram from **Section A: Student Activity 1** which shows the functions $f(x) = 5x$, $g(x) = 5x + 2$ and $h(x) = 5x - 10$, matched to the same derivative $\frac{dy}{dx} = 5$	» Can students communicate what the difficulty with reversing the differentiation process is? » Do students understand that several functions may have the same derivative such that when you reverse the differentiation there is an ambiguity about the identity of the original function?

Project Maths Tionscadal Mata Development Team

Teacher Reflections

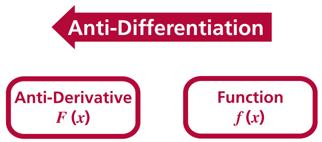
Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» How many functions have a derivative $\frac{dy}{dx} = 5$?	• Several. • An infinite number of functions.	» Write up some student examples of functions whose derivative is 5.	» Do students understand that there are an infinite number of functions whose derivative is 5?
» We cannot identify a single function whose derivative is 5 but can we identify a general pattern in all functions whose derivative is 5?	• $5x$ + *some number*. • All the functions have a slope of 5.	» Write up some examples of functions whose derivative is not 5 e.g. $f(x) = 3x + 1$, $g(x) = x^3 + 4x + 3$ and ask students why these cannot be the functions we're looking for. » Circle all the $5x$ terms in the list of functions whose derivative is 5. Circle all the different constant terms in the same list.	» Can students identify a common form to the functions whose derivative is 5? » Do students understand that all functions whose derivative is 5 will be of the form $5x + C$?
» Can we represent this pattern algebraically? » There are an infinite number of functions whose derivative is 5. Each function differs from the others by a constant value.	• Yes • $5x$ + *some number*. • $5x + C$.	» Demonstrate the idea of $5x + C$ graphically using the *GeoGebra* file "Derivative of a Shifted Function": - Click "Show Shifted Function" to show the shifted function. - Click "Show Tangents" to show tangents to both functions. - Click "Trace" to turn the trace on the shifted function. - Drag the point along the function to trace out the derivative in the second graphics view. - Drag the slider "Shift" to create several shifted functions. - Highlight that each of these functions has the same derivative such that if you started with the derivative there are an infinite number of functions which could have produced it.	» Do students understand that C may be any real number?

Project Maths Tionscadal Mata Development Team

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» We now know that it is possible to reverse the differentiation process (at least in part). We now need to come up with suitable language to describe this?			
» When we take a function and find its rate of change – what do we call this process?	• Differentiation.	» On the board, complete the following flow diagram as students come up with the relevant terms. Differentiation → Function $f(x)$ → Derivative $f'(x)$	» Do students understand function and derivative notation?
» What do we call the new function we derive – the function which describes the rate of change?	• The Derivative Function. • The Derivative. • The Slope Function.		» Can students use notation accurately to describe functions and their derivatives?
» What notation do we use to identify the function we start with and its derivative?	• If the function we start with is $f(x)$ then the derivative with respect to x is $f'(x)$. • If the function y depends on x, the derivative is denoted by $\frac{dy}{dx}$.	» Include some examples of functions where the independent variable is something other than x and the dependent variable is something other than y e.g. For the function $A(r)$ the derivative with respect to r is denoted by $A'(r)$ or $\frac{dA}{dr}$.	» Do students understand the language used to describe the process of reversing differentiation?
» If we start with the derivative and reverse the differentiation process what language could we use to describe this process?	• I'm not sure.		» Do students understand the language used to describe reversing the differentiation process?
» When we get a virus on our computers, what do we use to reverse the effects of the virus?	• Anti-Virus Software.		» Do students understand the rationale for using this language?

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» Can you suggest language to describe reversing the differentiation process?	• Anti-Differentiation.	» As students respond to questioning change the previous diagram so that it now looks as follows: 	
» If we start with some function and anti-differentiate it what could we call the new function we produce?	• The Anti-Derivative.		
» If we have a function $f(x)$ and we anti-differentiate it we denote the new function, the anti-derivative, by $F(x)$.			
» Given this, if I had a function $p(h)$, how would you label its anti-derivative?	• $P(h)$	» Add $p(h)$ and $P(h)$ to the flow diagram.	» Can students use the correct notation for the anti-derivative?

Project Maths
Tionscadal Mata
Development Team

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» We now know that it is possible to reverse the differentiation process and we have the language to describe the process. We will now look at taking a function and finding its anti-derivative using nothing but our knowledge of differentiation and one of our problem-solving strategies: that of trial and improvement.			
» Suppose you were asked to find the anti-derivative of the function $d(x) = x^2$, could you do it?	• Yes. • No. • I don't know.	» Write the function $d(x) = x^2$ on the board.	
» Do you understand what the question is asking? Could you re-write the question in your own words? What does the anti-derivative mean?	• The anti-derivative of $d(x) = x^2$ is the function whose derivative is x^2. • I am being asked to find a function which, when I differentiate it, will give me x^2.	» Encourage students to discuss what an anti-derivative is.	» Can students explain to each other what the anti-derivative means? » Can students themselves create meaningful and insightful questions relating to the anti-derivative?
» Suppose I just guessed an anti-derivative e.g. $2x$. Is there any way I could check my guess?	• Yes, I can check it by differentiating it.	» Write the proposed anti-derivative function $D(x) = 2x$ on the board.	» Do students use relevant mathematical language when in group and class discussions relating to the anti-derivative'.
» Is $2x$ the anti-derivative of x^2? » This approach of guessing could take a long time to yield the correct anti-derivative so we need to inform our guesses using some prior knowledge.	• No. • No, because when I differentiate $2x$, I get 2 and not x^2. • No, $2x$ is the derivative of x^2.	» Differentiate $2x$ and write up the solution of 2.	» Do students understand how to check their anti-derivative to determine if it is correct? » Do students recognise that $2x$ is the derivative of x^2 and not the anti-derivative?
» If we take a polynomial function e.g. x^5 and differentiate it what would my answer be?	• $5x^4$	» Write up the function x^5 and its derivative $5x^4$.	Can students describe the process of finding the derivative of a polynomial function?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» Can you describe how you formed the derivative?	• Multiply by the exponent and reduce the exponent by 1.		
» Given this, if we have a function x^2 and want to find its anti-derivative what can we say about the exponent of the anti-derivative?	• It has to be greater than 2. • The exponent must be 3. • The exponent must be 3 so that when I differentiate it I'll get an exponent of 2.	» Encourage students to explain their reasoning.	» Do students understand that the anti-derivative of a polynomial function must have a larger exponent than the function?
» Is x^3 the anti-derivative of x^2?	• No. • No, because when I differentiate it, I get $3x^2$ and not x^2. • No, but it is close.	» Write up the function x^3 and its derivative $3x^2$.	» Can students check if x^3 is the anti-derivative of x^2?
» What is wrong with x^3 as the anti-derivative of x^2?	• It's too big. • It's three times too big.		» Can students explain that the derivative $3x^2$ is three times too big?
» Could you make an improved guess for the anti-derivative of x^2? Can you check your latest guess?	• $\frac{1}{3}x^3$ • Three times smaller than x^3 • One third of x^3.	» Write up the function $\frac{1}{3}x^3$ and its derivative x^2. » Encourage students to check their latest suggestion.	» Can students identify $\frac{1}{3}x^3$ as the anti-derivative of x^2?
» Is $\frac{1}{3}x^3$ the only anti-derivative of x^2?	• No, $\frac{1}{3}x^3 + 1$, $\frac{1}{3}x^3 + 5$ etc. are also possible anti-derivatives of x^2.	» Ask students to suggest additional anti-derivatives of x^2. » Write up some possible anti-derivatives of x^2 and highlight the fact that when you differentiate each one you get x^2.	» Do students recall that each function has an infinite set of anti-derivatives?

Project Maths
Tionscadal Mata
Development Team

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» How can we write down the entire set of anti-derivatives of x^2?	• $\frac{1}{3}x^3 + C$	» Reinforce the common pattern followed by all the anti-derivatives and the fact that it may be represented generally by $\frac{1}{3}x^3 + C$.	» Can students write down the indefinite form of the anti-derivative of x^2? » Do students understand that $\frac{1}{3}x^3 + C$ represents an infinite set of functions?
» We call the set of all possible anti-derivatives of a function the indefinite form of the anti-derivative (since we don't know the value of C).		» Encourage students to write an explanation of the term indefinite form of the anti-derivative into their journals.	» Do students understand what the indefinite form of the anti-derivative means?
» We started with the function $d(x) = x^2$ and found its anti-derivative. How would you correctly name/label the anti-derivative?	• $D(x) = \frac{1}{3}x^3 + C$	» Write up the function $d(x) = x^2$ and its anti-derivative $D(x) = \frac{1}{3}x^3 + C$.	» Can students label the anti-derivative correctly?
» Let's recap on what we've learned. » Can you identify the key findings of the lesson?	• Many functions may have the same derivative. Such functions differ only by a constant. • Anti-differentiation is the process of reversing differentiation. • The anti-derivative of a function is such that when you differentiate it you get the function you started with. • A function has an infinite number of anti-derivatives. These anti-derivatives differ only by a constant. • We can check an anti-derivative to see if it is correct by differentiating it. • When we anti-differentiate a function we generate a new function.	» Encourage students to discuss with each other what they consider to be the most important findings of the lesson. » Limit students to a small number of key findings. » Encourage students to write down their key findings in their journal. » Ask different groups of students for one key finding and write on the board. Ask each group for a finding that has not already been written up on the board. Encourage students to write any findings they did not identify into their journal.	» Can students identify the key ideas of anti-differentiation? » Can students describe/explain what they have learned? » Do students have a good overall understanding of the topic so far?

Project Maths Tionscadal Mata Development Team

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» In all areas of life, when a new skill is introduced it is important to practice it so that you improve at it. Anti-differentiation is no different. We will now practice the process of anti-differentiation.			
» It is important to ensure that students develop skills that enable them to carry out procedures flexibly and accurately. » Working in pairs, complete **Section A: Student Activity 2**. Each time you form an anti-derivative, check your solution. The first question is completed for you.	» In pairs, students complete **Section A: Student Activity 2**.	» Distribute **Section A: Student Activity 2** to students. » Observe students as they complete the task – helping out as required. » Ask students to use the correct notation when writing down the anti-derivative. » Encourage students to explain their approach to finding the anti-derivatives. » Observe if students can find the correct anti-derivative more quickly and, if they can, ask them how they are doing so.	» Are students able to use their knowledge of differentiation to work backwards to find anti-derivatives? » Can students use the correct anti-derivative notation? » Do students have a strategy for finding the anti-derivative? » Can students check their answer? » Can students identify a common process for finding the anti-derivative of a polynomial function?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» Let's see how much better you are at anti-differentiating. I am going to give you a polynomial function and, in your groups, I want you to write its anti-derivative on a whiteboard. Hold up your answer when you're done.			
» The function is $q(x) = x^{17}$. Write down its anti-derivative.	• x^{18} (incorrect) • $\frac{x^{18}}{18}$ (partially correct) • $\frac{x^{18}}{18}+C$ • $Q(x)=\frac{x^{18}}{18}+C$ • $Q(x)=\frac{1}{18}x^{18}+C$	» Observe how quickly and accurately each group writes down the anti-derivative. » Observe if any groups are having problems and assist as required. » Write up each group's answer on the board. » Ask students to identify which answers are correct and where a mistake is made to identify the mistake made. » Use the examples to reinforce the indefinite form of the anti-derivative and correct notation.	» Can all groups anti-differentiate a polynomial function? » Have all groups spotted the common process for finding the anti-derivative of a polynomial function? » Are students writing down the indefinite form of the anti-derivative? » Are students using the correct anti-derivative notation? This is important to ensure they are developing their ability to communicate their ideas effectively.
» You wrote down the anti-derivative quite quickly – how were you able to do so?	• Increase the exponent by one and divide by the new exponent. Then add on C. • There is a common relationship between a polynomial function and its anti-derivative.	» Write up some other examples of functions and ask students to apply this rule to find their anti-derivatives. Confirm answers by differentiation.	» Can students describe the common rule for finding the anti-derivative of a polynomial function?

Project Maths Tionscadal Mata Development Team

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» Given the general polynomial function $f(x) = x^n$, could you write down its anti-derivative? » We have discovered a quick rule for finding the anti-derivative of a polynomial function. If we were to anti-differentiate other function types (e.g. trigonometric functions, exponential functions) using the "reverse process" idea we would find that similar rules exist for these functions also. It is time-consuming to do so. For this reason the anti-derivatives of a number of commonly-used functions are presented on page 26 of the *Formulae and Tables Booklet.* » We will now look at using the *Formulae and Tables Booklet* to find the anti-derivative of various types of function. Working in pairs, I want you to complete **Section A: Student Activity 3**.	• $F(x)=\dfrac{x^{n+1}}{n+1}+C$	» Encourage students to describe the rule they've come up with using mathematical notation. » Encourage students to describe the rule they've come up with using mathematical notation, orally and in writing. » Ask students to open their *Formulae and Tables Booklet* on page 26. » Highlight similarities between page 25 and page 26 of the *Formulae and Tables Booklet* to reinforce the idea of anti-differentiation as the reverse process of differentiation. » Distribute **Section A: Student Activity 3** to students. » Circulate to check that students understand what they are being asked to do. » Observe students' work and assist as required.	» Can students express this rule using mathematical notation? » Do students recognise similarities between the pairs of functions on page 25 and page 26 of the *Formulae and Tables Booklet*? » Do students recognise differences between the pairs of functions on page 25 and page 26 of the *Formulae and Tables Booklet?* » Can students use the *Formulae and Tables Booklet* to find anti-derivatives of functions? » Do students understand that the *Formulae and Tables Booklet* contains a limited number of anti-derivatives? » Do students understand that the anti-derivative of the sum of two functions is the sum of the anti-derivatives of each function?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» Up to now when we were asked to find the anti-derivative of a function, we wrote down the indefinite form of the anti-derivative i.e. the set of all possible anti-derivatives of the function. We are now going to do something a little different.		» Distribute **Section A: Student Activity 4** to students.	
» Take a couple of minutes to read through **Section A: Student Activity 4**. » Can you explain to each other what you are asked to do?	• We want to find the anti-derivative of $f(x) = 3$. • We want to find one of the anti-derivatives of $f(x) = 3$. • We want to find a particular anti-derivative of $f(x) = 3$, the one which passes through the point (1, 5).		» Can students explain what they are being asked to do? » Do students recognise this as a different question to the ones they previously encountered?
» How is this task different to the other anti-derivative tasks we've encountered?	• In previous questions we found the indefinite form of the anti-derivative. • Here we're asked to find a particular anti-derivative as opposed to all possible anti-derivatives. • We're going to have to find the indefinite form of the anti-derivative and then find out the value of C.		» Can students explain the differences between this question and previous anti-derivative questions?
» In pairs, work through **Section A: Student Activity 4** answering all questions.		» Circulate to check that students are completing the task successfully. » Question students to see if they have a clear understanding of the task and their approach.	» Can students represent the indefinite form of the anti-derivative graphically? » Can students identify the anti-derivative that they need from the graph?

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» What is the anti-derivative of $f(x) = 3$ which passes through the point (1, 5)?	• $F(x) = 3x + 2$	» Write the solution to the task on the board.	» Are students using the correct notation? » Can students explain how they got the single anti-derivative?
» How did you identify the anti-derivative you were looking for?	• I plotted several anti-derivatives and picked out the one which passed through the point (1, 5).	» Demonstrate to students how to identify the correct anti-derivative using a graph (You may use the *GeoGebra* File "Indefinite to Distinct Anti-Derivative") to do so: - Move the Slider "C" to demonstrate a number of the possible anti-derivatives. - Click the "Trace" button to turn on trace. Move the slider again, to trace out the various anti-derivatives. - Click the "Show Point" button to show the point through which the anti-derivative we're looking for must pass. - Move the slider until the correct anti-derivative is selected. - Read the algebra of the anti-derivative from the graph.	
» What does it mean for a function when its graph passes through the point (1, 5)?	• When the input to the function is 1, the output is 5. • When the *x value* is 1, the *y value* is 5.		» Can students explain what the coordinates of the graph of a function mean in terms of the function?
» Can we use this fact to find the anti-derivative we want in another way?	• Yes • I know the indefinite form of the anti-derivative $F(x) = 3x + C$. I also know that when $x = 1$, $y = 5$ or $F(x) = 5$. If I solve the linear equation $5 = 3\ (1) + C$ I can find the correct value of $C = 2$ and so have the single anti-derivative I am looking for.	» Encourage students to write down the useful information they have. Write this on the board: $F(x) = 3x + C$	» Can students take the information at hand and use algebra to find the anti-derivative they are looking for?

Project Maths © Tionscadal Mata
Development Team

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
How many ways do we have for finding a particular anti-derivative from the indefinite form of the anti-derivative?	• Two. Using a graph and using algebra.	» Demonstrate how these two pieces of information may be linked to find out the value of C. » Ask students why they might want two methods for finding a particular anti-derivative from the indefinite form of the anti-derivative?	» Do students understand that they now have two approaches to finding a particular anti-derivative from the indefinite form of the anti-derivative? » Do students understand that both approaches do the same thing but that having more than one approach to completing a task can be useful? » Do students recognise that they needed additional information about the anti-derivative?
» In the current task we were asked to find a particular anti-derivative. What enabled us to do so? In other words what allowed us to turn the indefinite form of the anti-derivative into a particular anti-derivative? » This information is often called the initial condition and allows the indefinite form of the anti-derivative to be changed into a particular anti-derivative.	• The point (1,5). • We were given additional information about the anti-derivative	» Encourage students to re-examine the text of the task. » Encourage students to write a description of this term into their journals. » Encourage students to think of appropriate words for describing one item.	» Do students understand what the term initial condition means?

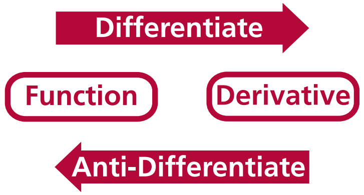
Project Maths
Tionscadal Mata
Development Team

Teacher Reflections

Project Maths Tionscadal Mata Development Team

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» We know that the set of all possible anti-derivatives of a function is called the indefinite form of the anti-derivative. What will we call this particular anti-derivative? Can you suggest any words which are used when describing a single object in a larger group? » The most-commonly used names for a particular anti-derivative are distinct anti-derivative, specific anti-derivative and definite anti-derivative.	• Single • Unique • Specific • Distinct • Definite	» Write up the terms on the board. » Encourage students to write an explanation of each term into their journals	» Do students understand what the terms specific anti-derivative, distinct anti-derivative and definite anti-derivative mean?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» We are now at the stage where we can anti-differentiate with confidence. Why would we want to do so? » Is there any practical reason why we'd want to anti-differentiate? » When we learned to differentiate – what was our motivation for doing so? Is differentiation useful in the real world?	• To recover the function. • If we knew the derivative and wanted to find the function it came from. • Not sure. • Probably. • It allows us to work out the rate of change. • It lets us find rate of change when things are continuously changing. • We can find out the instantaneous rate of change using it. • We can use it to work out slope. • We can use it to find out about tangents.	» Draw the following flow-chart on the board (piece by piece as students answer the associated questions): 	» Can students describe what anti-differentiation does? » Can students describe what differentiation does?

Development Team

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» Can you tell me some cases where rates of change are important in the real world? » Let's consider the case of speed for a minute. To calculate speed, what information do we need? » What does the differentiation process do? » If you were given an expression for speed in terms of time, what would the differentiation process do to it? » Could you now suggest where anti-differentiation might be useful in real-world calculations?	• When working out speed. • When analysing unemployment levels. • When working out acceleration. • Distance and Time. • How distance changes with time. • Distance as a function of time. • It changes the distance-time information into rate-of-change of distance i.e. it changes it into speed information. • Change it into rate-of-change of speed information i.e. into acceleration information. • It changes it into acceleration information. • Where you know speed and want to find out distance. • Where speed is continuously changing and you want to find out the distance travelled. • Where you want to find out speed when you know acceleration. • Anywhere you have a rate-of-change function and want to find the function itself.	» Draw the following flow-charts on the board (piece by piece as students answer the associated questions): $\text{Distance} \underset{\text{Anti-Differentiate}}{\overset{\text{Differentiate}}{\rightleftarrows}} \text{Speed}$ and $\text{Speed} \underset{\text{Anti-Differentiate}}{\overset{\text{Differentiate}}{\rightleftarrows}} \text{Acceleration}$	» Can students recall relevant applications of differentiation? » Do students understand that differentiation allows us to work out speed as a function of time when we know distance as a function of time? » Do students understand that differentiation allows us to work out acceleration as a function of time when we know speed as a function of time? » Do students understand that anti-differentiation allows us to work out distance as a function of time when we know speed as a function of time? » Do students understand that anti-differentiation allows us to work out speed as a function of time when we know acceleration as a function of time?

Project Maths Tionscadal Mata Development Team

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» Are there any real-world situations where you might know rate-of-change information and would like to recover the information from which it came?	• In a car, the speedometer shows speed information but we also want to know how far the car has travelled.	» Show some examples of distance, speed and acceleration: - How far will Felix fall in 30 seconds? http://www.youtube.com/watch?v=7f-K-XnHi9I (last accessed July 9, 2013) - start playing at 2:30.	» Do students recognise that these are calculations which have relevance in the real world?
	• When a projectile is flying through the air – we know its acceleration and might want to find out its speed or how far it travels.		
	• Submarines use anti-differentiation to find out how far they have travelled under the sea.		
		» Students may answer **Section A: Student Activity 5** for an example of the use of anti-differentiation to solve a problem.	» Can students apply their knowledge of anti-differentiation to solve real-world problems?

Project Maths Tionscadal Mata Development Team

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
Section B – Integration			
» We will now turn our attention to a problem at the core of calculus and that is the ability to find the area under a non-linear function. We will see why the ability to do so is so important later in the lesson.			
» We will use a problem grounded in the real-world to make these ideas more intuitive and easier to understand.		» Open the *GeoGebra* file "Area by sum of rectangles" and display on the board.	
» Given the building front can you identify the difficulties in finding its area?	• It's not a regular shape. • Its height is continuously changing. • Its roof is curved.		» Do students recognise the non-linear boundary as the main difficulty in calculating area?
» Is it possible to find the area of the building front?	• Yes. • We can estimate the area but not find it exactly.		
» How could you do so? » We are going to extend our mathematics with the aim of improving the accuracy of our estimation. Ultimately we want to be able to calculate the true area of the building front.	• By counting squares. • By filling it with different geometric shapes and adding up their areas. • Using the trapezoidal rule.	» Drag the slider "Fade" (to 0.65) so that the building front and the coordinated plane are visible. » Emphasis should be placed on the connection between the ideas encountered in exploring the trapezoidal rule and those the students will encounter later in integral calculus.	» Can student use viable area estimation techniques to find a value for area? » Do students recall the trapezoidal rule as a means for estimating area?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» The height of the building is continuously changing. Is there any mathematical tool which could be used to model this continuous change? » While we could try to fit any geometric shape into the building front we will use rectangles as they are simple to deal with.	• A function. • A continuous function.	» Click "Show Function" and highlight that we area naming the function $f(x)$.	» Do students recognise functions as a tool suitable for modelling continuously changing quantities? » Do students appreciate that we will use rectangles instead of trapezoids? » Do students understand that rectangles are simpler shapes than trapezia?
» Given a single rectangle as shown, can you write down an estimate for the area of the building front? » Is this area the true area of the building front? » Can you explain how you know this?	• 99.896 square units • 6.243 (16) • No. • Probably not. • Very unlikely. • The rectangle doesn't fit the building front very well. • There are gaps between the roof of the building and the top of the rectangle.	» Drag the slider "n" so that it has a value of 1. » Encourage students to write down the expression they used to calculate area. » Write the expression for area on the board.	» Can students calculate the given area estimate and write down an expression for calculating this estimate? » Do students recognise this calculation as only an estimate? » Can students explain their reasons for knowing that this calculation is only an estimate of the true area?

Project Maths Tionscadal Mata Development Team

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» Is there any way we could use a rectangle to get a better estimate?	• Fill the area with more rectangles. • Use smaller rectangles. • Use narrower rectangles.	» Drag the slider "n" so that it has a value of 2.	» Can students transfer their knowledge of the trapezoidal rule to identify more rectangles as giving a better estimate of the area of the building front?
» Can you write down a new estimate for the area of the building front?	• 100.757 square units • 6.243(8) + 6.351(8)		
» Could we further improve our estimate?	• Yes. • Yes, by using more rectangles. • Yes by using more rectangles which are narrower.	» Drag the slider "n" so that it has a value of 3.	
» Can you write down your improved estimate for the area of the building front?	• 100.575 square units 6.243(5.33) + 6.646(5.33) + 5.968(5.33)		» Can students write down the expression for the estimate of the area using three rectangles?
» We have seen this approach of increasing the number of shapes to fill an area before. Can you think of where? » We could continue to construct more and more rectangles and calculate their areas numerically. If we want to understand and describe the approach mathematically we are going to have to generalise what we are doing.	• Yes, in the trapezoidal rule.		» Do students recognise this approach as being similar to the trapezoidal rule?

Project Maths Tionscadal Mata Development Team

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» Revisiting the case of a single rectangle, can you explain to me where the two terms in the area expression came from?	• 6.243 is the height of the rectangle. • 16 is the width of the rectangle.	» Drag the slider "n" so that its value is 1.	» Can students explain that their area expression is simply *width* x *height*?
» Can you explain how you calculated the width of the rectangle?	• By subtracting 0 from 16.		
» Could you describe the width in more general terms, using coordinate geometry?	• $x_2 - x_1$ • Δx	» Fill in $x_2 - x_1$ on the diagram. » Fill in Δx on the diagram.	» Do students understand that the width may be thought of as the change in x and called Δx?
» Can you explain how you calculated the height of the rectangle?	• It's the height of the building on the left hand-side. • It's the value of the function on the left-hand side. • It's $f(0)$.		» Do students understand that the height of the rectangle is the output of the function for an input of x_1?
» Could you describe the height in more general terms?	• It's $f(x_1)$.	» Fill in $f(x_1)$ on the diagram.	» Can students write a general expression for the estimate using a single rectangle?
» Can we now write down a more generalised form for the area of the building based on one rectangle?	• Area $\approx f(x_1)\,\Delta x$	» Highlight on the diagram that the area is $f(x_1)\,\Delta x$	
» Could you use two rectangles to get a fresh estimate of the area?	• Area $\approx f(x_1)\,\Delta x + f(x_2)\,\Delta x$	» Drag the slider "n" so that its value is 2. » Fill in $x_2 - x_1$, Δx, $f(x_1)$ and $f(x_2)$ on the diagram.	» Can students apply similar thinking to generalise the area estimates for two and three rectangles? » Do students understand that the width of all the rectangles is the same and that this simplifies our mathematical estimate of area?

Project Maths Tionscadal Mata Development Team

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» In pairs, I want you to complete **Section B: Student Activity 1** by writing down a numerical expression for each estimate and then writing down a generalised expression. We have already completed the first two area estimates.		» Distribute **Section B: Student Activity 1**. » Circulate to ensure students understand the task and address any difficulties as they arise. » Give students time to complete the task. » Encourage students to compare the expressions they arrived at and to identify any differences in approach.	» Can students take each estimate and write out a general expression for it using $f(x)$ and Δx?
» If you fill the building front with 3 rectangles, what is the new estimate?	» Area ≈ 6.243(5.33) + 6.646(5.33) + 5.968(5.33) » Area $\approx f(x_1)\,\Delta x + f(x_2)\,\Delta x + f(x_3)\,\Delta x$	» Write the numerical expression and the general expression on the board.	
» If you fill the building front with 4 rectangles, what is the new estimate?	» Area ≈ 6.243(4) + 6.761(4) + 6.351(4) + 5.752(4) » Area $\approx f(x_1)\,\Delta x + f(x_2)\,\Delta x + f(x_3)\,\Delta x + f(x_4)\,\Delta x$	» Write the numerical expression and the general expression on the board.	
» If you fill the building front with 5 rectangles, what is the new estimate?	» Area ≈ 6.243(3.2) + 6.806(3.2) + 6.538(3.2) + 6.132(3.2) + 5.619(3.2) » Area $\approx f(x_1)\,\Delta x + f(x_2)\,\Delta x + f(x_3)\,\Delta x + f(x_4)\,\Delta x + f(x_5)\,\Delta x$	» Write the numerical expression and the general expression on the board.	

Project Maths
Tionscadal Mata
Development Team

Teacher Reflections

Project Maths Tionscadal Mata Development Team

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» The expressions are getting a bit unwieldy to write out. Is there a more efficient way to write out sum of the five terms in the last area expression?	• Using sum notation. • Using Sigma notation. • Area $\approx \sum_{i=1}^{5} f(x_i)\Delta x$	» Write Area $\approx \sum_{i=1}^{5} f(x_i)\Delta x$ on the board.	» Can students rewrite the latest area expression using Sigma notation? » Can students correctly interpret Sigma notation and verbalise the meaning of the expression for area?
» Why did we change our area estimate from a single rectangle to two, three, four and five rectangles? » How do we know that doing so improves our estimate? » We can confirm this by comparing our estimate to the true area of the building front.	• To improve our area estimate. • For greater accuracy. • There is less space between the tops of the rectangles and the top of the building.	» Drag the slider "n" from 1 → 5 and emphasise how the space between the rectangles and the top edge of the building decreases. » Click "Show Area Info" and drag the slider "n" from 1 → 5 to highlight the decrease in percentage error.	» Do students understand the need for a more efficient way for writing the area expression? » Do students understand why they would want to increase the number of rectangles?
» Why does increasing the number of rectangles improve our estimate?	• The rectangles fit the area much better. • The rectangles are narrower and so fit into the irregular area better.	» Drag the slider "n" from 1 → 5 and emphasise how rectangles are getting narrower. » Drag the slider "n" so that its value is 10.	» Can students explain how they know that more rectangles mean a better estimate?
» Could we improve our estimate even further?	• Yes. • Yes, by using more rectangles. • Yes, by using more rectangles which are even narrower.	» Highlight the decrease in percentage error. » Highlight the decrease in the width of the rectangles.	» Can students explain why more rectangles mean a better estimate?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» Can you write down a general expression to describe our latest estimate? » Could we improve our estimate even further? » Can you write down a general expression to describe our latest area estimate?	• Area $\approx \sum_{i=1}^{10} f(x_i)\Delta x$ • Yes – by filling our area with even more rectangles. • Area $\approx \sum_{i=1}^{100} f(x_i)\Delta x$	» Drag the slider "n" so that its value is 100. » Highlight the decrease in percentage error. » Highlight the decrease in the width of the rectangles.	» Do students understand that they can improve their estimate further by using even more rectangles? » Can students write down the sum of the area of 10 rectangles using Sigma notation?
» How many rectangles do you think we'd need to reduce the percentage error to zero i.e. to calculate the true area of the building front?	• 500 • 1,000 • 1,000,000 • An infinite number of rectangles.	» Drag the slider "n" to increase its value.	» Can students write down the sum of the area of 100 rectangles using Sigma notation?
» How many rectangles do you think we'd need to calculate the true area? » Can you write down an expression to represent this area estimate?	• An infinite number of rectangles. • Area; $\sum_{i=1}^{\infty} f(x_i)\Delta x$	» Highlight the fact that while the percentage error is decreasing, it is still not zero.	» Do students recognise that, even though more rectangles means a better area estimate, we still only have an estimate as opposed to the true area?

Project Maths Tionscadal Mata Development Team

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» What happens when you use an infinite number of rectangles to estimate the area? » In pairs I want you do discuss and explain why this happens.	• It doesn't work. • We get 100% error. • Our sum breaks down.	» Click on the button "$n = \infty$". » Highlight the fact that the area estimate is now zero and the percentage error is 100%. » Encourage students to discuss the problem with each other.	» Do students recognise that they need an infinite number of rectangles to fill the area if they are to find the true area? » Do students recognise that the sum of the areas of an infinite number of rectangles gives an area of zero?

Project Maths Tionscadal Mata Development Team

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» Why doesn't our infinite sum give the correct area?	• When we have an infinite number of rectangles each rectangle has no width. • When $n = \infty$, $\Delta x = 0$. • $\Delta x = 0$ so $f(x_i)\,\Delta x = 0$.	» Assist students using suitable questions such as: - How could our sum produce an answer of zero? - Can you describe what happens to the rectangles the more of them we have? » Drag the slider "n" from 1 → 500, highlighting the fact that Δx is getting smaller as we do so.	» Can students explain why the sum of areas of an infinite number of rectangles gives an area of zero?
» So we have a problem. If we want to calculate the true area, we need an infinite number of rectangles but when we use an infinite number of rectangles our sum returns an area of zero. » Somehow we need to let our number of rectangles get closer and closer to infinity without ever being equal to infinity. Do we have any mathematical tools which allow us to do so? » Could you now write down an expression for the true area of the building front? » This is an amazing discovery. Using a limit together with rectangles it is possible to calculate the area under any boundary. » This process of finding area is known as Integration or Integral Calculus.	• Yes we do. • Limits • Area = $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n} f(x_i)\Delta x$ • Area = $\lim\limits_{\Delta x\to 0}\sum\limits_{i=1}^{n} f(x_i)\Delta x$	» Click the button "$n \to \infty$" to reveal the true area and zero percentage error.	» Can students identify a limit as a tool for finding the true area? » Can students write down an expression for area using limits?

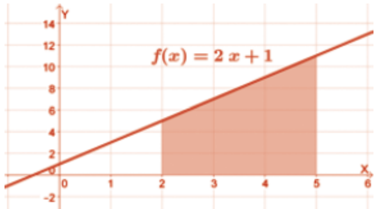
Project Maths Tionscadal Mata Development Team

Teacher Reflections

Project Maths
Tionscadal Mata
Development Team

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» The notation for describing the process is itself a bit unwieldy. Gotfreid Wilhelm Liebniz developed a shorthand version of the notation. Here is what Leibniz's notation looks like.		» Write Area = $\lim\limits_{\Delta x \to 0}\sum\limits_{i=1}^{n} f(x_i)\Delta x$ on the board. Write up the Leibniz form of this expression: Area = $\int_a^b f(x)\,dx$.	» Do students understand that Integration of Integral Calculus is the name given to finding an area using the limit of the sum of rectangles?
» I would like you to discuss what each part of the notation means. To help you out I will write up the notation for the building example we've just looked at.		» Encourage students to compare the two expressions and to convince themselves that they are identical. » Write up Area = $\int_0^{16} f(x)\,dx$ on the board.	» Do students understand what the notation $\int_a^b f(x)\,dx$ means and how it's arrived at?
» What do you think the $f(x)\,dx$ part of the expression means?	• The area of a really narrow rectangle.	» Highlight the fact that dx means an infinitesimal width (just as in differential calculus).	
» What do you think the $\int$ symbol means?	• It must mean the limit of the sum of all the areas as the number of rectangles tends to infinity.	» Highlight the fact that $\int$ means the limit of an infinite sum.	
» What do you think a and b mean?	• They mean the distance along the x - axis. • They mean what portion of the function you're finding the area of.	» Identify a and b on the diagram.	

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» What does the entire expression Area = $\int_a^b f(x)dx$ mean?	• The area under a function between the vertical lines $x = a$ and $x = b$.	» Encourage students to write this notation into their journals with an explanation of what it means.	» Can students apply their understanding of integral notation to complete **Section B: Student Activity 2**?
» Working in pairs, I want you to complete **Section B: Student Activity 2**.		» Distribute **Section B: Student Activity 2** to students.	
» What is $\int_2^5 (2x+1)dx$?	• 24 square units.	» Circulate to check if students understand the integral notation.	» Do students understand that this activity may be answered using their knowledge of geometry?
» What does the question mean?	• Find the area under the graph of the function $f(x) = 2x + 1$ from $x = 2$ to $x = 5$.	» Help students having difficulties through suitable questioning strategies	» Do students sketch the function to get a picture of what they are being asked to do?
» How did you do it?	• I sketched my function to see what it looked like. • The area I want is just a trapezium so I used the area of a trapezium formula.	» Sketch the function and required area on the board: 	
» Did you need to split the area up into lots of different rectangles?	• I used a combination of a rectangle and triangle to calculate the area in question. • No. • No because the area is a regular shape so I can just use geometry to work it out.	» Demonstrate various ways to find the required area using geometry.	

Project Maths Tionscadal Mata Development Team

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» We've discovered a lot of new mathematics throughout the last two activities. Let's recap on what we've learned.		» Encourage students to summarise their key findings of the lesson in their journals.	» Can students summarise the key points of this part of the lesson? » Do students use a format (genre) appropriate to mathematics when writing?
» If you're presented with a non-linear function and asked to find the area under a part of it, how would you do it?	• By adding up the area of lots of narrow rectangles. • By using the limit of an infinite sum.		» Can students describe the integral-calculus approach to calculating area?
» Do you understand what the following notation means? $\int_a^b f(x)\,dx$	• It means area. • It means the area under the function $f(x)$. • It means the area under the function $f(x)$ from a to b. • It means the limit of the sum of the areas of an infinite number of infinitesimally-wide rectangles.		» Can students explain what integration notation means?

Project Maths Tionscadal Mata Development Team

Teacher Reflections

Project Maths Tionscadal Mata Development Team

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
Section C - The Area Function			
» We now have a method for calculating the area under a continuous function. While this method works, it is not without its limitations. Using $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n} f(x_i)\Delta x$ to calculate area is time consuming. The limit has to be re-calculated every time one wishes to calculate the area under the function between two different extremes. » We encountered a somewhat similar problem when we started our investigations in differential calculus.			» Do students recall the need for differential calculus? » Do students recall the central ideas underpinning differential calculus?
» What problem were we trying to solve when we introduced differential calculus?	• Find slope. • Find slope at a point. • Find the slope at a point along a continuously-changing function. • Find instantaneous rate of change.	» Sketch the graph of a non-linear function on the board and refer to as needed.	
» How did we solve the problem?	• Using a limit. • By taking the limit of our slope formula. • By using $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ • By using $\lim\limits_{\Delta x\to\infty}\dfrac{\Delta y}{\Delta x}$ • By finding the slope of a tangent. • Using differentiation from first principles.	» Write $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ and $\lim\limits_{\Delta x\to\infty}\dfrac{\Delta y}{\Delta x}$ on the board. Highlight that the two expressions are equivalent.	

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» Were there any issues with this approach?	• It was time consuming. • It was complicated. • The more complex the functions are, the more difficult it is to calculate this limit.	» Encourage students to recall their initial approach to differentiation (First Principles) and the different approach (Differentiation by rule) that they use in real examples.	» Do students recall moving on from differentiation by first principles to differentiation by rule and the reasons for doing so?
» How did we overcome these issues?	• We used pattern analysis to derive slope functions for a number of functions. • We discovered some simple rules for finding functions which describe the slope everywhere along other functions. • We discovered that there is a pattern to the slopes of many functions. We call this the derivative. • We used the rules of differentiation from Page 25 of the *Formulae and Tables Booklet*.	» Remind students that the pairs of functions listed on Page 25 of the *Tables and Formulae Booklet* are functions and their slope functions.	

Project Maths Tionscadal Mata ©
Development Team

Teacher Reflections

Project Maths Tionscadal Mata Development Team

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» Perhaps a similar approach will work for calculating area. Is it possible to find a function which describes the area at all points under another function?	• I don't know. • Maybe. • Probably.		
» In groups I would like you to complete **Section C: Student Activity 1**. For our initial investigation we will start with simple linear functions – since we already know how to calculate areas under such functions using geometry. » The task assigned in a structured cooperative group work approach should facilitate student discussion. Along with the content to be taught and learned, there will also be a focus on the development of students' skills in the use of mathematical language.		» Distribute **Section C: Student Activity 1** to students. There are five different parts to the activity. It is intended that each group of students would complete two tasks only. The five parts of the activity are graded in terms of difficulty level (Group A is easiest, Group E is most difficult) and may be assigned to different groups of students accordingly. Alternatively the activity may be given as a homework exercise, with discussion of the key learning outcomes to take place during the following lesson. The function designated to Group B is used to discuss the key learning outcomes so it is advised that all students complete this part of the activity.	» Are students clear as to the purpose of the task? » Do students understand what to do? » Can students use prior knowledge of pattern analysis to work their way through the activity? » Can students sketch the area function? » Can students write down the algebraic form of the area function? » Do students understand what the area function represents?

Teacher Reflections

<table>
<tr><th>Student Learning Tasks: Teacher Input</th><th>Student Activities: Possible and Expected Responses</th><th>Teacher's Supports and Actions</th><th>Assessing the Learning</th></tr>
<tr><td>» Does the area under your function follow a predictable pattern?</td><td>• Yes</td><td>» Observe students' work to see if they understand the task.

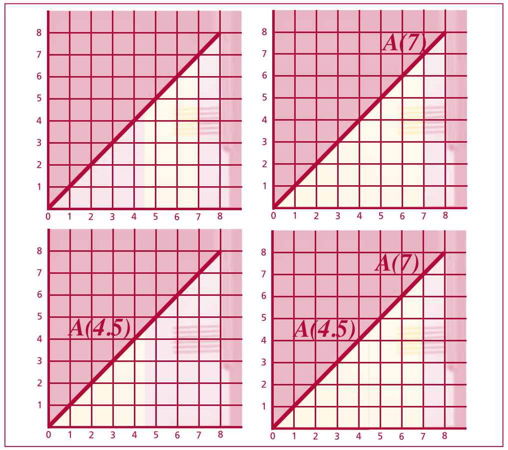
» Assist students as required using appropriate questioning techniques to help their understanding.</td><td>» Do students recognise that it is possible to find a function which describes the area under another function?</td></tr>
<tr><td>» Can you describe the pattern algebraically?</td><td>• Yes
• $A\ (x) = 5x$ (Group A)
• $A\ (x) = ½\ x^2$ Group B)
• $A\ (x) = 9x - 27$ (Group C)
• $A\ (x) = x^2$ (Group D)
• $A\ (x) = x^2 - 4x + 4$ (Group E)</td><td>» Write the functions and their corresponding area functions on the board:<table><tr><th>Function</th><th>Area Function</th></tr><tr><td>$h\ (x) = 5$</td><td>$A\ (x) = 5x$</td></tr><tr><td>$h\ (x) = x$</td><td>$A\ (x) = ½\ x^2$</td></tr><tr><td>$h\ (x) = 9$</td><td>$A\ (x) = 9x - 27$</td></tr><tr><td>$h\ (x) = 2x$</td><td>$A\ (x) = x^2$</td></tr><tr><td>$h\ (x) = 2x - 4$</td><td>$A\ (x) = x^2 - 4x + 4$</td></tr></table></td><td>» Have all groups calculated the area functions correctly?

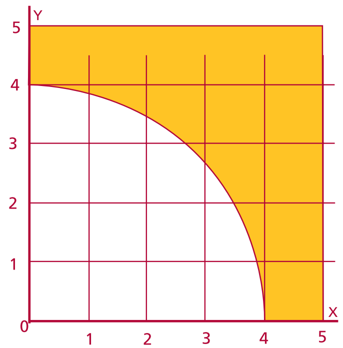
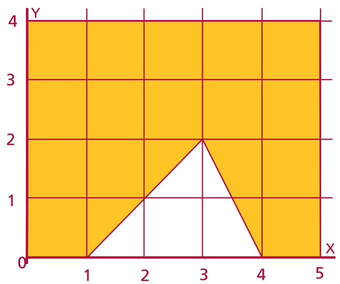
» Can students verbalise what the area function does?</td></tr>
<tr><td>» Given the function $h\ (x) = x$ and its area function, can you tell me what the area function $A\ (x) = ½\ x^2$ means?</td><td>• It tells us what the area under $h\ (x)$ is.

• It lets us find the area under $h\ (x)$ between $x = 0$ and $x =$ *some value.*</td><td>» Display (use a sketch or *GeoGebra*) the graph of $h\ (x) = x$ and the graph of $A\ (x) = ½\ x^2$ on the board starting at $x = 0$.</td><td>» Can students use the area function to calculate a given area?

» Can students write down the S.I. units for area?</td></tr>
</table>

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» Given the function $h(x) = x$ and its area function $A(x) = ½ x^2$, can you tell me what the area under $h(x)$ is between $x = 0$ and $x = 4.5$?	• 10.125 square units	» Encourage students to sketch the functions for themselves and to shade in the required area.	
» Can you explain how you arrived at your answer?	• I calculated the value of $A(x)$ when $x = 4.5$. • I used the graph of $A(x)$ to find the value of $A(x)$ when $x = 4.5$.	» Ask a student to write up the calculation of $A(4.5)$ using $A(x)$ on the board. » Ask a student to demonstrate how the graph of may be used to find $A(4.5)$.	» Can students explain how they calculated $A(4.5)$? » Can students identify different ways of calculating $A(4.5)$?
» Is there any way you could check your answer?	• The area I'm looking for is the area of a triangle so I could check it this way. • I can calculate the area under $h(x)$ by inspecting the graph of $h(x)$. • I can use geometry to check my answer.	» Ask a student to demonstrate how to use the sketch of $h(x)$ to check the answer for $A(4.5)$.	» Can students suggest a suitable approach for checking their answer?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» Given the function $h(x) = x$ and its area function $A(x) = ½ x^2$, can you tell me what the area under $h(x)$ is between $x = 4.5$ and $x = 7$?	• 14.375 square units	» Encourage students to create a new sketch of the functions for themselves and to shade in the required area.	» Can students work out the correct area value? » Do students have a suitable strategy for working out the area? » Can students shade in the required area on their graph?
» Can you explain how you arrived at your answer?	• I used geometry. • I used the area function to find the area under the function from $x = 0$ to $x = 4.5$ and again to find the area from $x = 0$ and $x = 7$. Then I subtracted my two areas to find the required area. • I used the graph of $A(x)$ to find $A(4.5)$ and also to find $A(7)$. Then I subtracted my smaller answer from my larger one to find the area in between.	» Ask a student to explain how they found the required area? » On the sketch of $h(x)$ demonstrate how the required area may be determined by subtraction of areas calculated using the $A(x)$.  » Emphasise the fact that we want to see how to use the area function to calculate the required area. » Highlight the use of $A(x)$ and its graph to answer the question. » Encourage students to check their answer using geometry.	» Do students understand how to use the area function to calculate the area between two x values? » Can students explain their approach to solving the problem? » Do students understand that the area function calculates the area under a function from some starting point up to some other point? » Do students recognise the required area as a combination of two other areas which themselves may be calculated using the area function?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» Can you re-write the question using integral notation?	• $\int_{4.5}^{7} h(x)\,dx$ • $\int_{4.5}^{7} x\,dx$ • $\int_{4.5}^{7} h(x)\,dx = A(7) - A(4.5)$ where $A(x)$ is the area function of $h(x)$.	» Encourage students to explain the meaning of integral notation. » Encourage students to use mathematical notation to explain their approach to answering the question.	» Do students recall integral notation correctly? » Do students understand the meaning of integral notation? » Can students use integral notation to re-write the question?
» Can you extend the notation to describe how you answered the question?	• Yes, when we're asked to find areas of shapes made up of combinations of different shapes.	» Show some examples of where unknown areas are calculated by subtraction of known areas. 	» Can students use appropriate mathematical notation to describe their approach to solving the problem?
» Have you met this idea of subtraction of areas when solving any other problems?	• Yes, when we're asked to work out probabilities using the standard normal tables.	» Ask students to open their *Formulae and Tables Booklet* on page 36 and note how they got the probabilities from the standard normal distribution.	» Can students identify other times that they have used subtraction of areas to solve a problem? » Do students recognise similarities between this approach and that used in working out probabilities involving the standard normal tables?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» Let's recap on what we've discovered about area functions. » Can you explain what the area function represents?	• It tells us the area under a function between two extremes.	» Encourage students to discuss with each other what an area function is. » Encourage students to write down their description in their journals.	» Can students describe what an area function represents and what it may be used for?
» Suppose you were asked to find the area under $h(x)$ between any two vertical lines $x = a$ and $x = b$. How would you do so?	• By finding $A(a)$ and $A(b)$ and subtracting one from the other. • I would use the graph of $A(x)$ to find the y - value of $A(x)$ when $x = a$ and the y - value of $A(x)$ when $x = b$. I would then subtract the smaller area from the bigger one to find the area inbetween.	» Emphasise the connections between mathematical ideas of different aspects of the syllabus. » Sketch a general function $h\ (x)$and its area function $A\ (x)$on the board and mark in the relevant variables to support student understanding.	» Can students describe how to use an area function to find the area under a function up to a given ordinate? » Can students describe how to use an area function to find the area under a function between any two values?
» Could you describe your approach using integral notation?	• $\int_a^b h(x)\,dx = A(b) - A(a)$	» Write up the area using integral notation as follows $\int_a^b h(x)\,dx = A(b) - A(a)$	» Can students generalise their integral notation to describe mathematically how to find the area under a function between two vertical lines?

Project Maths Tionscadal Mata Development Team

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» We now know that it is possible to describe the area under a function using a new function. » We also know how to use this area function to find areas under a function. » Let's look at calculating the area under another function. » I want you to read through **Section C: Student Activity 2** and discuss how you plan to complete it.		» Distribute **Section C: Student Activity 2**. » Give students time to read the question and discuss how they might answer it. » Listen to students' discussions, using suitable questioning strategies to extend their understanding. » Encourage students to write out a list of tasks they would need to complete to answer the question.	» Can students begin to describe how they would go about solving the problem?
» Can you identify any differences between this activity and the previous activities?	• We are dealing with a non-linear function.	» Display an image of the function on the board.	» Do students recognise that they are dealing with a non-linear function in this case?
» Are there any difficulties with completing this task?	• We have a curved boundary instead of a straight line. • We need the function's area function. • We don't know the area function. • We cannot find the area function as the area is irregular.		» Do students recognise that they are not given the area function?
» How did we find the area function in the previous examples?	• By finding the area of different regions under the graph of the function. • By finding the pattern that the area under the function follows. • By describing the pattern of the areas using algebra.		» Do students understand that in **Section C: Student Activity 1** we discovered the area function using geometry?

Project Maths Tionscadal Mata Development Team

Teacher Reflections

<table>
<tr><th>Student Learning Tasks: Teacher Input</th><th>Student Activities: Possible and Expected Responses</th><th>Teacher's Supports and Actions</th><th>Assessing the Learning</th></tr>
<tr><td>» Can we use the same approach here to find the area function?</td><td>• No because our boundary is not a straight line.
• No because we're not dealing with a regular geometric shape.
• No because the area we are looking at is irregular.</td><td></td><td>» Do students understand that they cannot find the area function using geometry as before?</td></tr>
<tr><td>» We have a problem. We cannot easily find the area function for the given function as we did in the Section C: Student Activity 1. We need a quick and efficient way to find the area bounded by any function but how can we do so?</td><td></td><td></td><td></td></tr>
<tr><td>» Let's re-examine the results from Section C: Student Activity 1.</td><td></td><td></td><td></td></tr>
<tr><td>» In differential calculus we expended a lot of energy finding the slope function of various functions but then we discovered that we could derive the slope function of many functions simply by inspecting the function we started with.</td><td></td><td>» Reproduce the results of Section C: Student Activity 1 on the board.
<table><tr><th>Function</th><th>Area Function</th></tr><tr><td>$h(x) = 5$</td><td>$A(x) = 5x$</td></tr><tr><td>$h(x) = x$</td><td>$A(x) = ½ x^2$</td></tr><tr><td>$h(x) = 9$</td><td>$A(x) = 9x - 27$</td></tr><tr><td>$h(x) = 2x$</td><td>$A(x) = x^2$</td></tr><tr><td>$h(x) = 2x - 4$</td><td>$A(x) = x^2 - 4x + 4$</td></tr></table></td><td>» Do students recall differentiation by rule?

» Can students explain the procedures involved?</td></tr>
</table>

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» Is it possible to do the same here, to derive the area function simply by inspecting the bounding function?	• No. • Not sure. • Maybe. • Yes.	» Draw arrows linking each function to its area function. » Give students time to consider the relationship?	» Do students recognise the area function as an anti-derivative of the boundary function?
» Is there any obvious relationship between the function and its corresponding area function?	• No. • Yes. • The area function is a particular anti-derivative of the boundary function		» Do students recognise the boundary function as the derivative of the area function?
» If you were presented with the area function can you see a way to find the boundary function?	• Yes. • The boundary function is the derivative of the area function. • Yes, you could differentiate it.	» Demonstrate the reverse relationship using different coloured arrows moving in the opposite direction.	
» Given this, can you suggest a relationship between the boundary function and the area function?	• The area function is an anti-derivative of the boundary function.		» Can students explain how to find the area function of any function?
» This is amazing. It seems that a particular anti-derivative of a function gives us that function's area function.	• To find the area function just anti-differentiate the boundary function.		» Can students anti-differentiate correctly?
» I want you to return to **Section C: Student Activity 2** and complete the question.			

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» Are there any remaining difficulties with solving this problem?	• I don't know what C is. • I can only find the indefinite form of the anti-derivative. • I do not know which particular anti-derivative of the function I need.	» Give students time to explore the question and to start answering it. » Observe students to see if they are anti-differentiating correctly. Intervene through appropriate questioning as required.	» Do students recognise that they have a problem of not being able to change the indefinite form of the anti-derivative into the distinct anti-derivative which measures area?
» Can we find the area function of a function by anti-differentiation alone?	• No. • No – we can only find the indefinite form of the area function. • We cannot find the constant term in our area function	» Highlight the presence of different values of C in the area functions on the board and that anti-differentiation alone will not allow us to find the area function.	
» What would we need to find the area function?	• Some additional information. • An initial condition		» Can students recall that they need some additional information to find a distinct anti-derivative?
» For the moment let's just guess a value for C. To give the class a good chance of guessing the correct value, each group must guess a different value of C.	• 0 • 1 • 5 • 11 • Etc. etc.	» Ask each group of students to suggest a value for C making sure that no two groups choose the same value. » Write each group's value for C on the board.	
» Using your value for C, I want you to calculate the area of the building front as originally requested.		» Observe students' work to make sure they are anti-differentiating properly.	» Can students apply their knowledge of anti-differentiation and area functions to calculate the correct area of the building front?

Project Maths
Tionscadal Mata
Development Team

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» What value did you get for the area of the building front?	• 20.26 square units • 20.26 square units • 20.26 square units • Etc. etc.	» Ask a member of each group to come to the board and write their answer beside their value for C.	
» Do you notice anything unusual about the results of each group?	• They're all the same.		
» What is the area of the building front?	• 20.26 square units		» Do students recognise that the value of C has no bearing on the calculation of the given area?
» What is the value of C?	• I don't know. • It doesn't matter (not exactly true). • It can be anything (not exactly true).		
» This is amazing, it seems that the value of C doesn't affect our area calculation.			
» Can you explain why this is i.e. why did everybody get the same (correct) answer for the area?	• I don't know. • When we work out the area between two vertical lines $x = a$ and $x = b$ the C values sum to zero. • When working out the area I calculate the value of $A(6.5)$ - $A(2)$. Even if I don't know the value of C it doesn't matter since: $A(6.5)$ - $A(2)$ $= \frac{-0.53(6.5)^3}{3} + \frac{4.4(6.5)^2}{2} - 3.73(6.5) + C$ $-\left[\frac{-0.53(2)^3}{3} + \frac{4.4(2)^2}{2} - 3.73(2) + C\right]$ and the Cs sum to zero.	» Encourage students to re-examine their calculation of the area. » Encourage students to try an alternative value for C and compare to their first calculation to explain why C does not matter. » Encourage students to explain their reasons to the rest of the class by going through a sample calculation at the board.	» Can students explain why the value of C is not important for this area calculation? » Do students demonstrate the appropriate use of mathematical language in their explanations?

Project Maths Tionscadal Mata Development Team

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» We've discovered a lot of useful mathematics through this activity. Let's recap on what we've learned.			
» If we were given some function $f(x)$ and asked to find the area under the function between the vertical lines $x = a$ and $x = b$, how would you do it?	• I'd do the following: - Find the anti-derivative of the function. - Find the value of the anti-derivative at a. - Find the value of the anti-derivative at b. - Subtract one from the other.	» Encourage students to describe the general steps needed to complete this task. » Encourage students to record their method in their journal.	» Can students describe the general approach to finding the area under a function in words?
» Can you write down your approach using integral notation? » So, in summary the anti-derivative of a function provides an efficient way to calculate the area under the function or in other words anti-differentiation provides a method for integration. For this reason the terms "anti-differentiation" and "integration" are often interchanged. Anti-differentiation allows us to integrate a function.	• $\int_a^b f(x)dx = F(b) - F(a)$ where $F(x)$ is the anti-derivative of $f(x)$.	» Write up the mathematical representation of the approach on the board. » Encourage all students to record this notation and to write their own explanation of what it means.	» Can students adequately describe the general approach to finding the area under a function using integral notation? » Are sentences structured? Is the style appropriate to mathematics?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» Up to now we've focused on linear and quadratic boundary functions when calculating areas. » Let us now look at a different function $k(x) = \sin x$. Working in pairs I would like you to answer Question 1 in **Section C: Student Activity 3**.		» Distribute **Section C: Student Activity 3** to all students. » Circulate to check that students understand the task. » Use appropriate questioning to assess students' understanding.	» Can students use anti-differentiation to correctly calculate: $\int_0^{2\pi} \sin x \, dx$?
» How did you calculate the given area?	• Using anti-differentiation. • I calculated the value of $K(2\pi) - K(0)$. • I found the anti-derivative of $k(x) = \sin x$, substituted $x = 0$ and $x = 2\pi$ into it and subtracted one result from the other.		
» What area value did you get? » Is there any way you could check your answer?	• 0 • We could sketch the function and try to estimate the given area.	» Display the graph of $\sin(x)$ on the board. Alternatively students could sketch the graph of the function themselves.	

Project Maths Tionscadal Mata Development Team

Teacher Reflections

Project Maths Tionscadal Mata Development Team

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» Given the graph of the function $k(x) = \sin x$, can you estimate what the given area is?	• It's approximately 4. • It's more than 3. • It's between 3 and 6.	» Shade in the area we are interested in to highlight that it cannot be zero.	» Can students suggest a practical way to check their answer? » Can students estimate the area from a sketch of the function?
» Does this agree with your calculation in Question 1?	• No.		
» Which value is more correct – the calculation of area using anti-differentiation or the estimate of area?	• The calculation (incorrect). • The estimate (correct).		» Do students recognise that the value they calculated for the area is very different to their estimate of area?
» How do you know that the calculation of area using anti-differentiation is incorrect?	• It's clear from the graph that the area we want cannot be zero.		» Do students recognise that the value they have calculated for area is incorrect?
» We seem to have a problem. Anti-differentiation is not giving us the correct value for the area between the function and the x-axis. Can you suggest why this is so?	• No. • Anti-differentiation treats the area above the x-axis as positive and the area below the x-axis as negative. Since both areas are equal in size, the overall area sums to zero.		

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» We will now investigate why anti-differentiation gave us an area of zero. In pairs I would like you to answer Question 3, Question 4 and Question 5 of **Section C: Student Activity 3**.		» Check that students are completing the task correctly. » Encourage students to consider the key learning outcomes through appropriate questioning.	» Can students use anti-differentiation to correctly calculate $\int_0^{\pi} \sin x \, dx$? and $\int_{\pi}^{2\pi} \sin x \, dx$?
» What value did you get for $\int_0^{\pi} \sin x \, dx$?	• 2. • 2 square units.	» On the board, write "2" on the portion of the area which is located above the x-axis.	
» What value did you get for $\int_{\pi}^{2\pi} \sin x \, dx$?	• -2 • -2 square units.	» On the board, write "-2" on the portion of the area which is located below the x-axis.	» Do students recognise that anti-differentiation gives a positive result for area above the x-axis, and a negative result for area below the x-axis?
» Is it possible to have negative area?	• No. • No, it doesn't make sense.	» Highlight that both pieces of area are the same size but that anti-differentiation assigns a sign to area depending on its location relative to the x-axis.	
» By referring to the graph can you explain what an area of -2 means?	• It means the area is 2 square units in size but is located below the x-axis. • The "-" indicates the area we are talking about is below the x-axis.		

Project Maths Tionscadal Mata Development Team

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» What does an area of +2 mean?	• It means the area is 2 square units in size and is located above the x-axis. • It means the area is above the x-axis.	» On the graph, highlight that the sum of the areas calculated using anti-differentiation is zero.	» Can students use their results to explain why anti-differentiation gave a value of zero for area?
» Given this, can you now explain why anti-differentiation produces an area of zero for $\int_0^{2\pi} \sin x\, dx$?	• The areas above and below the x-axis are equal in size. Anti-differentiation assigns a positive sign to area above the x-axis and a negative sign to area below the x-axis. The areas sum to zero. • The areas above and below the x-axis sum to zero.		
» What is the true size of the area $\int_0^{2\pi} \sin x\, dx$?	• 4. • 4 square units.		
» How did you arrive at this value?	• Both portions of the area are 2 square units in size so I summed them to get a total area of 4 square units. • I added the area above the x-axis to the area below the x-axis.	» On the graph, highlight that both portions of the area are 2 square units in size.	» Can students calculate the true area? » Can students describe how they calculated the true value for area?

Project Maths Tionscadal Mata Development Team

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» If you were now asked to calculate the area between the graph of $k(x) = \sin x$ from $x = 0$ to $x = 2\pi$, could you describe how you would do so?	• Yes I would do the following: - Use anti-differentiation to calculate $\int_0^{\pi} \sin x \, dx$? and $\int_{\pi}^{2\pi} \sin x \, dx$? separately. - Take the modulus of both areas and add them together.	» Encourage students to explain to each other how they would calculate the area using anti-differentiation.	» Can students describe the overall approach to calculating the area of $k(x) = \sin x$?
» Why did you decide to split the area up in this way?	• So that the area above the and the x-axis area below the x-axis are treated separately.	» Write the area question on the board and remove any additional information (such as the graph of the function).	» Can students explain why they divided up their area calculation into two parts?
» How did you know which portion of the area was above the x-axis and which was below the x-axis?	• From our graph. • From the sketch of our function. • I didn't but I figured it out by looking at a graph of the function.	» Highlight the limitations of the information the students are presented with in the question.	» Do students recognise how they knew to divide the area up correctly into that which is above the x-axis and that which is below the x-axis? » Can students correctly calculate the given area?

Project Maths Tionscadal Mata
Development Team

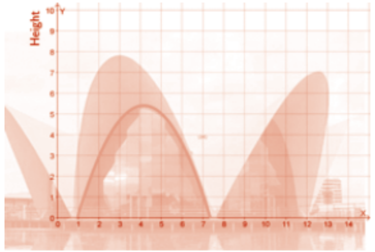
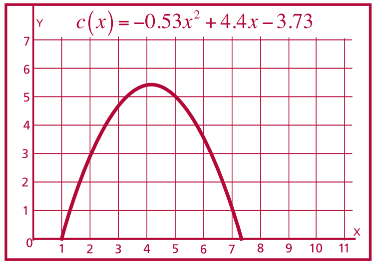
Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» Let's have a look at calculating a different area. In pairs I would like you to answer Question 6 from **Section C: Student Activity 3**.		» Circulate to ensure students understand the task. » Support students in their work through appropriate questioning.	» Can students use a suitable approach to calculate the area?
» What answer did you get for the area?	• $21\frac{1}{12}$ • $-10\frac{5}{12}$ (incorrect)	» Encourage students to compare answers and in the case where answers differ to explore why this is so.	
» Can you describe your approach to calculating the given area?	• I did the following: - I sketched the function to see if it crossed the x-axis in the domain I'm being asked about. - I found where the function crossed the x-axis and used this information to split my area calculation into two parts – one made up of area above the x-axis and one made up of area below the x-axis. - I used anti-differentiation to find the area of each part. - I took the modulus of each part and added them together to find the total area.	» Encourage students to discuss with each other the approach they used to calculating area. » Recognise students' misconceptions as issues to be worked on explicitly and as opportunities to enhance learning through oral discussion.	» Do students recognise that a sketch of the function is an important tool in calculating area using anti-differentiation? » Do students recognise that a sketch of a function may be used to estimate and thereby check their area calculation?

Project Maths Tionscadal Mata Development Team

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» Can you check your answer?	• Yes, by estimating the area using the sketch of my function.	» Encourage students to discuss the key learning outcomes from **Section C: Student Activity 3**.	
» Can you summarise what you've just learned about using anti-differentiation to calculate area? **Note:** In all activities so far we have focused on calculating area between a curve and the x-axis. Students should appreciate that it is possible to calculate areas between a curve and the y-axis. An activity designed to introduce this idea is presented in **Section C: Student Activity 4**. It is envisaged that this activity may be introduced to students as a problem-solving exercise with integration with respect to the y-axis as one possible approach to solving the problem.	• Anti-differentiation assigns a positive sign to areas which lie above the x-axis and a negative sign to areas which lie below the x-axis. • If you want to use anti-differentiation to calculate area you need to find out if the area in question spans the x-axis. If it does you need to calculate each portion of area separately. • It's really important to sketch a function before using anti-differentiation to calculate area. The sketch allows you to split the area up correctly and also provides a way to estimate your answer.	» Ask students to share the learning outcome they consider most important with the rest of the class. » Write the key learning outcomes on the board. » Encourage students to make a note of the key learning outcomes in their journal.	» Can students describe the complete process in finding area with respect to the x-axis using anti-differentiation?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
Section D: Area as sum			
» We have spent a lot of time investigating ways to calculate the area inside a curved boundary. Why is this important?	• Most areas are not regular. • If we want to find areas in the real world.	» Refer to the image used in **Section C: Student Activity 2**.  » Show the graph of a function without a background image.  	» Do students understand that we've been focusing on ways to calculate irregular areas?
» There is another reason. Aside from calculating the area inside a curved boundary, what we've also done is to figure out a way to calculate the area under a function. Why would we want to do this?	• I don't know.		» Do students understand that we've developed a way to find the area under a continuous function?
» When we introduced differential calculus, what problem were we trying to solve?	• Work out slope. • Calculate slope at a point. • Find slope anywhere along a non-linear function.		» Can students recall what the aim of differential calculus was?
» Why is slope so important?	• It tells use the rate of change. • It measures how quickly things change. • Slope also means rate of change and rate of change is important in lots of places.	» Remind students that slope is much more than a measure of steepness but is useful in many situations: - Geometric meaning of slope:  - Slope as rate of change	» Do students understand why slope is so important? » Can students recall that slope means more than the physical $\frac{rise}{run}$ but also measures rate of change?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» In a similar way to slope meaning rate of change, area can also mean something more than the physical space inside a boundary. What else do you think area means?	• I don't know.		» Do students appreciate that just like slope meaning more than the physical $\frac{rise}{run}$ area may mean more than the amount of space within a boundary?
» Let's investigate what else area may be used for.		» Distribute **Section D: Student Activity 1** to students.	» Can students calculate average?
» In groups I want you to read through **Section D: Student Activity 1**.	• In groups, students read and work on **Section D: Student Activity 1**.	» Circulate to check that students understand the task.	» Do students consider that three different averages exist?
» Complete the activity by calculating the average amount of money in Question 1 and the average temperature in Question 2.		» Use appropriate questioning strategies to assess student understanding and to extend their learning.	» Can students describe how they calculate average (mean)?
» As you answer each question I want you to describe how you calculate the average and any similarities or differences between the two questions.		» Ask students what average they intend to calculate (we want the mean). » Encourage students to discuss how they calculate average. » Encourage students to discuss similarities and differences between the two questions.	» Do students recognise similarities between the two questions? » Do students recognise differences between the two questions? » Can students make a reasonable attempt to find the average temperature in Question 2?

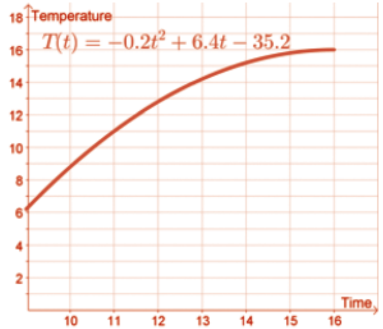
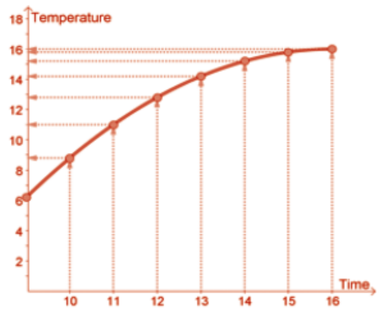
Project Maths Tionscadal Mata Development Team

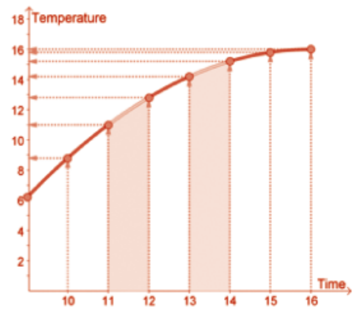
Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» We will now discuss the two questions.			
» What is the average amount of money in Bernie's bank account?	• €14,000.	» Write the calculation and solution on the board.	» Can all students answer Question 1 correctly?
» How did you calculate the average amount of money in Bernie's bank account?	• I added up all the monies and divided by the number of months. • I calculated the sum of all the monies and divided by the time passed to find the average (mean).		» Can students accurately describe how they calculate average?
» Can you write down your approach using mathematical notation?	• $\dfrac{Sum\ of\ monies}{No.\ of\ months}$ • $Average = \dfrac{Sum}{No.\ of\ months}$ • $Average = \dfrac{\sum_{i=1}^{7} Money_i}{7}$	» Write a suitable mathematical expression for average on the board. - e.g. $Average = \dfrac{\sum_{i=1}^{n} x_i}{n}$ where x_i are the amounts of money in Bernie's account each month and n is the number of months.	» Can students write down their approach using suitable mathematical notation? » Can students relate these techniques to areas of their other subjects?
» What is the average air temperature in Cork on the given day?	• I don't know. • I couldn't do it. • 12.5°C [or various numerical answers].		» Are students able to calculate a reasonable answer to Question 2?
» How did you do it?	• I couldn't do it. • I worked out the temperature for every hour using the temperature function. I added up all these temperatures and then divided by the number of hours.		» Can students describe their approach to calculating the average temperature?

Project Maths Tionscadal Mata Development Team

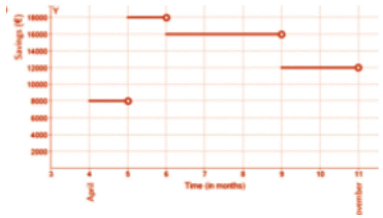
Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» How was this question different to Bernie's bank account?	• We don't have any data given to us. • We had discrete values for Bernie's money but the temperature is continuously changing.	» Show the graph of the temperature function on the board.  	» Do students recognise that Question 2 is different to Question 1 in a number of ways? » Can students identify the key differences between the two questions? » Do students recognise that Question 1 deals with discrete data and that Question 2 deals with continuous data? » Can students use the temperature function to find some temperature values?
» Could I find some temperature values?	• Yes. • Yes, by substituting into the temperature function for different time values.	» Use the graph to show how the function may be used to find discrete temperature values.	» Do students understand that these temperature values may be used to calculate an average temperature?
» Could I then use these values to calculate the average temperature?	• Yes. • Yes but my answer would not be accurate. • Yes but my answer would only be an estimate of the true average temperature?	 	

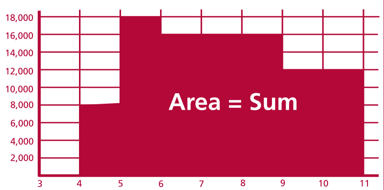
Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
Can anybody identify limitations to this approach? **Note:** the teaching objectives used throughout this problem solving section include: 1. identifying the information necessary to solve the problem 2. representing problems mathematically in a variety of forms 3. breaking problems into smaller steps, choose and use efficient methods and resources 4. presenting and interpreting solutions, explaining and justifying methods and reasoning. 5. enabling the students to hypothesise, generalise and identify exceptional cases or counter examples.	• I don't know what time values to substitute into my function to get temperature values. • Even if I find some temperature values I am still missing out on all the temperature values in between. • The temperature is continuously changing so I can't find all the temperature values and add them up. • Different persons may use the function but choose different temperature values (at different times). This means that different persons might calculate a different average temperature. This is not ideal. • There are an infinite number of temperatures to add up in finding my average and I cannot do this using a traditional sum approach.	» Demonstrate the missing temperature data by means of shading in on the graph (e.g. we miss all the temperature data for $9 < t < 10$, $10 < t < 11$, $11 < t < 12$ etc.)  » Highlight the fact that it is the "Sum" part of $Average = \frac{\sum_{i=1}^{n} x_i}{n}$	» Can students identify and describe limitations to this approach? » Do students appreciate that this approach will only yield an estimate of the average temperature? » Can students explain why this approach will yield an estimate of the average temperature? » Do students recognise the problem of finding the sum of continuously-changing data?
» We have a problem – when dealing with a continuously changing quantity we have no accurate method for calculating average. Why is this?	• We cannot accurately sum all the values. • We can't add up all the values.	which causes us problems when calculating the average value of a continuously-changing function.	

Project Maths Development Team

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» We need an alternative approach to finding a sum.			
» Let's re-examine Bernie's savings account from a different point of view. This time we will use a graphical representation of her savings.		» Display a graph of Bernie's savings on the board. (See page 104 for a larger image.) 	
» Examine the graph of Bernie's savings in **Section D: Student Activity 1**.		» Highlight each term in the expression $Average = \frac{\sum_{i=1}^{n} x_i}{n}$	
» To calculate the average savings in Bernie's account we need two things: - The sum of all the monies. - The number of months.	• Yes it's the width of the domain of the function. • Yes by subtracting 4 from 11. • Yes by subtracting the smallest x-value of the function from the largest x-value.	» Highlight on the graph how to calculate the number of months.	» Can students identify where the number of months may be read from the graph?
» Is the number of months identifiable from the graph?			
» Is the sum of all monies identifiable from the graph?	• Probably. • I can't see it. • Yes.		» Can students identify a way to calculate the total monies using the graph of the function?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» If I asked you to add up the monies for June, July and August using the graph, how would you do it?	• Add €16,000, €16,000 and €16,000. • Multiply €16,000 by 3.	» Highlight the portion of the graph which represents June, July and August. » Write the sum 16,000 x 3 on the board and encourage students to explain where it is visible on the graph. » Link 16,000 to the height of this piece of the function and 3 to the width of this piece of the function.	» Can students describe how to sum the monies for June, July and August? » Can students see that the sum for June, July and August is also the area under the equivalent portion of the graph?
» Can you identify where this sum is represented on the graph?	• It's the area under the piece of the function which covers June, July and August.		
» Why does the area represent the sum?	• Because area is calculated by multiplying height by width. To sum our money we multiply money by no. of months. As our graph is set up the money is the height of the function and the no. of months is the width.		» Can students explain why the area is the same as the sum? » Can students use area to sum the monies for September and October?
» Using the graph could you add up the monies for September and October?	• Yes • Yes, by multiplying €12,000 by 2 months. • Yes, by finding the area under this portion of the function. • Yes, the area under the part of the function representing September and October is €24,000.	» Encourage students to sketch a graphical representation of their calculation on their graphs. » Identify 12,000 and 2 on the graph.	
» Could you now use the graph to add up all the monies from the start of April to the end of October?	• Yes, by finding the area under the function. • Yes, the area under the function is 98,000 so this is the total of all the monies for this time period.	» Link each piece of the sum to an area under the graph of the savings on the diagram on the board.	» Can students use area to find the sum of all monies?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» Can you now describe a graphical approach to finding the average savings in Bernie's account?	• Yes – find the area under the graph of Bernie's savings and then find the width of the graph. Divide area by width to find average. • Find the area under the graph of Bernie's savings and divide it by the width of the graph.	» Encourage students to sketch a graphical representation of how to find the average savings. On the diagram on the board – link the sum to the area under the graph of the function and the number of months to the width of the function. Savings (€)  » Link each term of $Average = \frac{\sum_{i=1}^{n} x_i}{n}$ to the graph.	» Can students describe a graphical approach to finding the average money in Bernie's account? » Do students recognise the sum of monies as the area under the graph? » Do students recognise the no. of months as the width of the graph?

Project Maths Tionscadal Mata Development Team

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» Let's now return to the average temperature problem and look at it from a graphical point of view also.		» Direct students to the graph of temperature in **Section D: Student Activity 1**. » Give students time to attempt solving the average temperature question. » Circulate and observe students' work. » Display the graph of the temperature function on the board.	» Can students apply what they've learned to solve Question 2? » Can students recall the problem with Question 2 which prevented them from finding an accurate average temperature?
» When we first attempted this question, what was our issue with finding the average temperature?	• We couldn't find the sum of all the temperatures.		
» On the graph of the temperature function can you identify where the sum of all the values is represented?	• It's the area under the graph of the function.	» Link the sum of temperatures to the area under the graph of the function.	» Do students appreciate that the area under the graph of the temperature function is the sum of temperatures that they need?
» On the graph of the temperature function can you identify where the number of hours is represented?	• It's the width of the graph of the function.	» Link the number of hours to the width of the graph of the function.	» Can students find the number of hours from the graph?
» Were you able to calculate the average temperature?	• Yes. Yes – by finding the area under the function and dividing it by the width of the function. • Yes – by using $Average = \frac{Total}{n} = \frac{Area}{Width}$	» Ask students to explain how they calculated the average temperature. » Ask a student to go through their calculation of average temperature on the board. » Link the area under the graph of the function to the integral or anti-derivative of the function.	» Are students able to calculate average temperature correctly? » Do students identify integration as the approach needed to calculate the area under the graph of the temperature function i.e. the sum of temperatures?

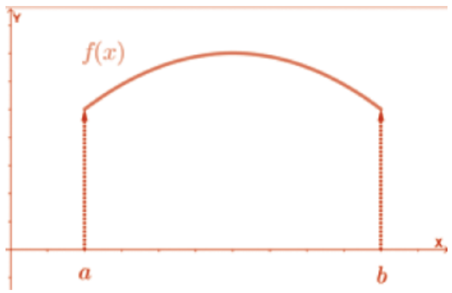
Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» How did you find the area under the function?	• By integration. • Using the anti-derivative. • By finding $F(16)$ - $F(9)$, where $F(t)$ is the anti-derivative of $T(t)$. • Using $\int_9^{16} T(t)\,dt$ • Using $\int_9^{16} [-0.2t^2 + 6.4t - 35.2]\,dt$	» Write up the area under the graph of the function using integral notation: $\int_9^{16} T(t)\,dt$	» Can students explain their approach to finding average temperature? » Can students represent their approach using correct mathematical notation?
» What value did you get for average temperature? » We've discovered a lot of useful mathematics from this activity. Let's summarise what we've learned.	• 12.73°C.	» Highlight the fact that everybody should get the same answer for average temperature as opposed to different possible answers using our earlier method of choosing temperatures at different times.	» Do all students obtain the correct average temperature?

Project Maths Tionscadal Mata Development Team

Teacher Reflections

Project Maths
Tionscadal Mata
Development Team

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Assessing the Learning
» If you were given any continuous function and asked to find its average value – could you do it?	• Yes. • Yes, by finding the sum of the function and dividing by the width of the function. • Yes by finding the area under the graph of the function and dividing by the width of the graph. • Yes by integrating the function.	» Sketch any continuous function on the board naming it between limits a and b.  	» Do all students obtain the correct average temperature? » Do students understand that they've learned how to find the average value of a continuously-changing function?
» Can you describe your approach using mathematical notation?	• $Average\ \ Value = \dfrac{Area}{Width}$ • $Average\ \ Value = \dfrac{\int_a^b f(x)dx}{b-a}$	» Encourage students to use the sketch to describe their approach to finding the average value of any function. » Use the graph of a general function to develop an expression for the average value of any continuous function: $Average\ \ Value = \dfrac{\int_a^b f(x)dx}{b-a}$ $Average\ \ Value = \dfrac{1}{b-a}\int_a^b f(x)dx$ » Encourage students to write their approach to finding the average value of a function into their journals.	» Do students understand that area under the curve may be used to sum the function between two extremes? » Can students describe the process of finding average value of a function? » Do students recognise the average value of a function as being analogous to the average value of a set of discrete data? » Can students represent their approach to finding average value using suitable mathematical notation?

Section A: Student Activity 1

Project Maths Tionscadal Mata Development Team

Match each function to its correct derivative

$\frac{1}{2}x^2$	$\cos(x)$
$\frac{1}{2}x^2 - 0.358$	5
$5x$	$\cos(x)$
$5x + 2$	$\cos(x)$
$5x - 10$	$2x$
$\sin(x) + 9$	x
$\sin(x) - 1.3$	x
$\sin(x)$	$2x$
x^2	5
$x^2\ \frac{1}{2} + \pi$	5

Section A: Student Activity 2

1. Find the anti-derivative of $f(x) = 7x + C$	Check: $\frac{d}{dx}\left[7x + C\right] = 7$
2. Find the anti-derivative of $a(x) = -11$	Check:
3. Find the anti-derivative of $v(x) = x$	Check:
4. Find the anti-derivative of $r(x) = x^2$	Check:
5. Find the anti-derivative of $p(x) = x^3$	Check:
6. Find the anti-derivative of $e(x) = x^4$	Check:
7. Find the anti-derivative of $h(x) = x^5$	Check:
8. Find the anti-derivative of $w(x) = x^{46}$	Check:
9. Find the anti-derivative of $b(x) = x^{12}$	Check:
10. Find the anti-derivative of $f(x) = x^n$	Check:

Section A: Student Activity 3

Project Maths
Tionscadal Mata
Development Team

By referring to page 25 and page 26 of the *Formulae and Tables Booklet* **or otherwise, answer the following questions.**

1. Write down the anti-derivative of $h(x) = 16x^3$

2. Write down the anti-derivative of $c(x) = e^{5x}$

3. Write down the anti-derivative of $r(a) = 9a^{-19}$

4. Write down the anti-derivative of $w(x) = x^{-1}$

5. Write down the anti-derivative of $p(t) = \frac{3}{t^4}$

6. (a) Write down the anti-derivative of $f(x) = \cos(5x)$

   (b) Hence, find the anti-derivative of $g(x) = \sin(5x)$

7. Find the anti-derivative of $t(x) = 4x^3 - 3x$

Section A: Student Activity 4

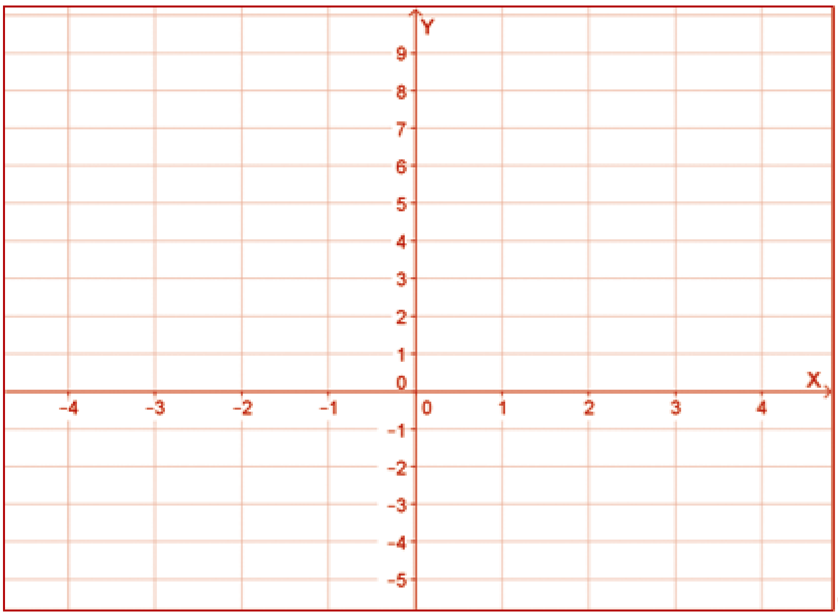
Project Maths Tionscadal Mata Development Team

You are asked to find the anti-derivative of the function $f(x) = 3$ which passes through the point (1, 5).

1. How is this question different to all the previous anti-derivative questions you have encountered?

2. Find the indefinite form of the anti-derivative of $f(x) = 3$

3. Represent the indefinite form of the anti-derivative graphically below by sketching the anti-derivatives for each of the following values of C = {-3, -2, -1, 0, 1, 2, 3}.



4. Identify the distinct anti-derivative you were asked to find.

Section A: Student Activity 5

A hot air balloon is ascending through the atmosphere. When the balloon reaches a height above ground of 80m, a snooker ball is dropped from the basket. The ball's velocity when it is released is 3.6 m/s. The acceleration of a falling object due to gravity is a constant value of -9.8 m/s^2.

1. Write down an expression describing the acceleration of the snooker ball as a function of time?

2. Will the velocity of the ball remain at 3.6 m/s throughout the fall? Explain.

3. Write down an expression describing the ball's velocity as a function of time.

Section A: Student Activity 5 (continued)

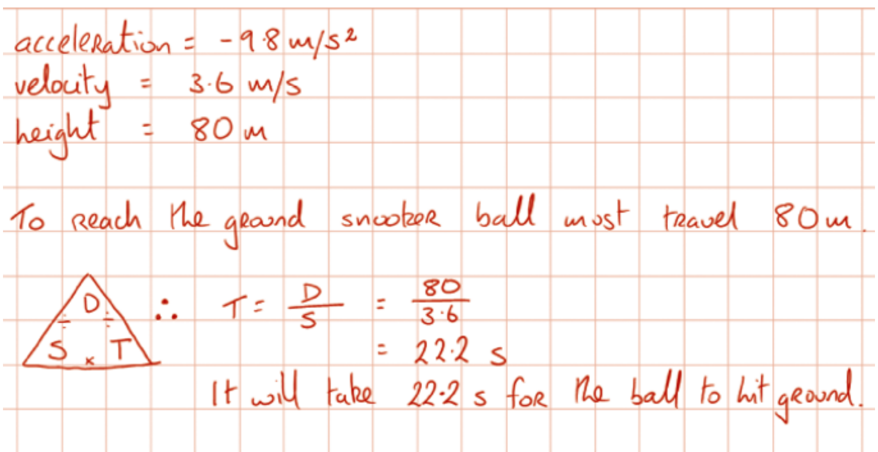
Project Maths
Tionscadal Mata
Development Team

4. Write down an expression describing the ball's height as a function of time.

5. What will the height of the snooker ball be when it hits the ground?

6. How long does it take the snooker ball to hit the ground?

7. The following image shows an example of a student's efforts to calculate how long it took the ball to hit the ground. Explain why this calculation is incorrect.


acceleration = $-9.8\ m/s^2$
velocity = $3.6\ m/s$
height = $80\ m$

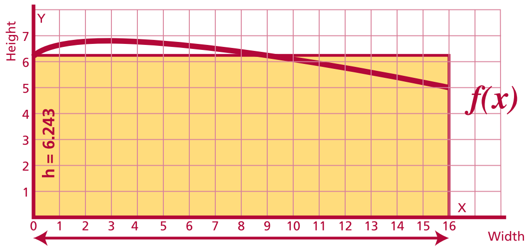
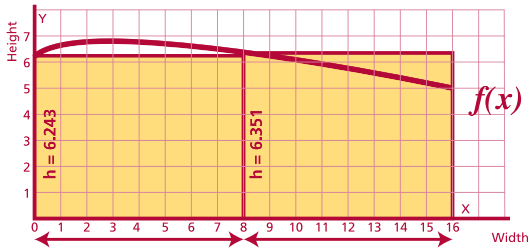
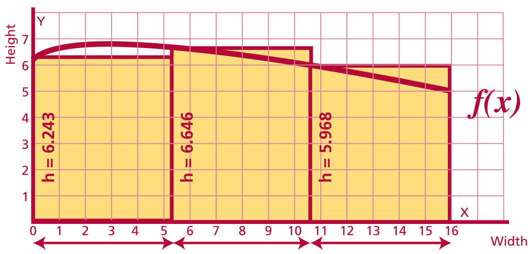
To reach the ground snooker ball must travel 80 m.

$\therefore\ T = \frac{D}{S} = \frac{80}{3.6}$
$= 22.2\ s$

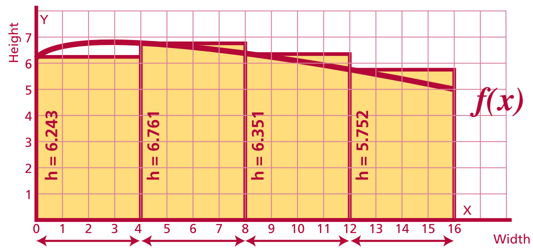
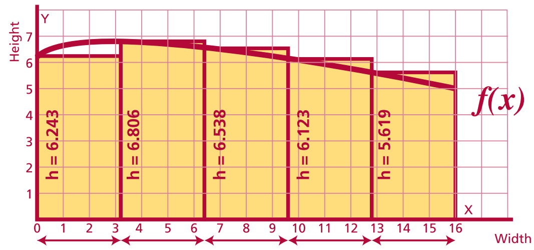
It will take 22.2 s for the ball to hit ground.

Section B: Student Activity 1

Project Maths Tionscadal Mata Development Team

1		Area Calculation
(i)	 $n = 1$ $\Delta x = 16$	
(ii)	 $n = 2$ $\Delta x = 8$	
(iii)	 $n = 3$ $\Delta x = 5.333$	

Section B: Student Activity 1

		Area Calculation
(iv)	 $n = 4$ $\Delta x = 4$	
(v)	 $n = 5$ $\Delta x = 3.2$	

2. Explain what happens to the width of the rectangles (Δx) as the number of rectangles (n) increases. Express this relationship using mathematical notation.

Description in words: As the number of rectangles increases, the width of the rectangles

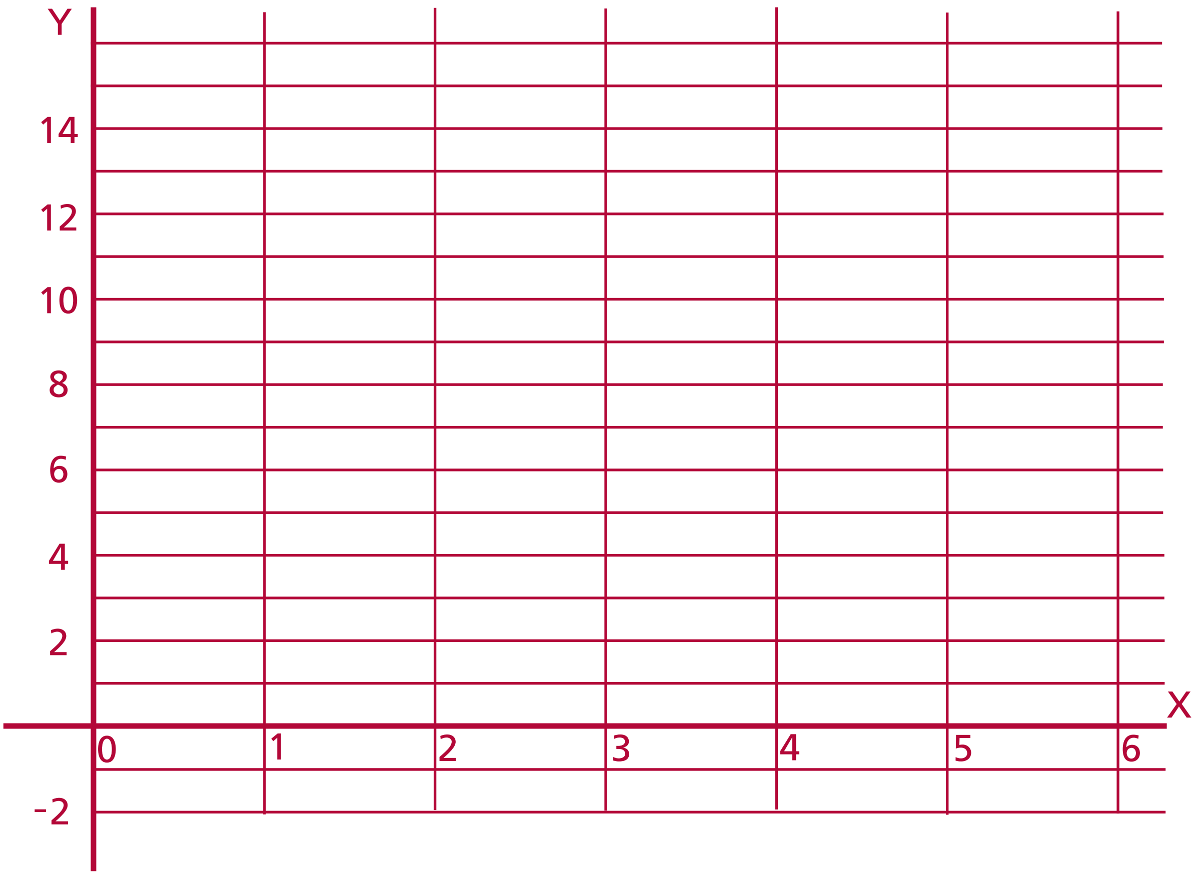
Mathematical Description: As $n \rightarrow$ $\Delta x \rightarrow$

Section B: Student Activity 2

Calculate $\int_2^5 f(x)\,dx$ where $f(x) = 2x + 1$

1. In words describe what you are being asked to do.

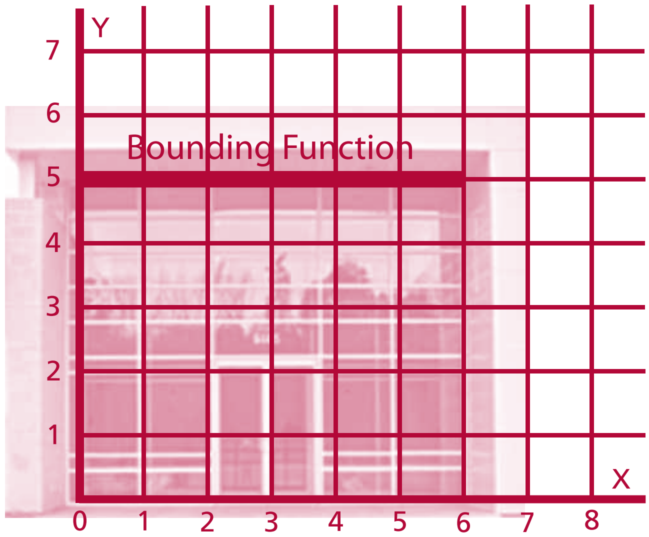
2. Using a suitable approach complete the task.



Section C: Student Activity 1 (Group A)

Project Maths Tionscadal Mata Development Team

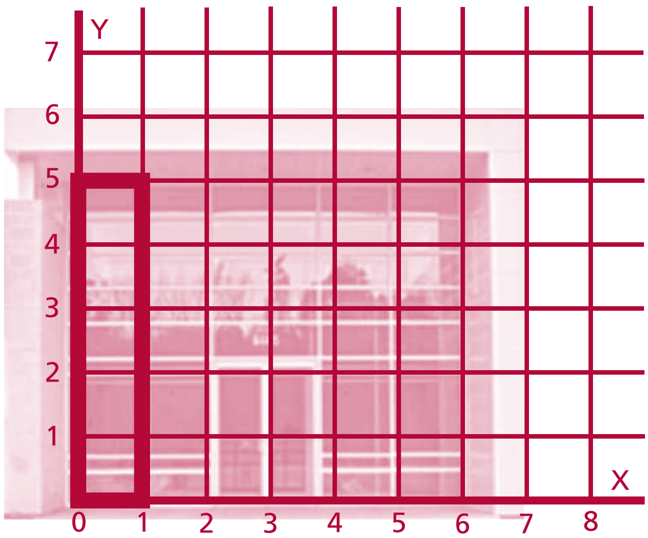
The figure below shows the UCD Student Computer Centre.



1. Write down the function which describes the height of the building as we move from left ($x = 0$) to right ($x = 6$).

$h(x) =$

The area of the building changes as we move from left to right. We will now investigate the relationship between the area of the building and its width.

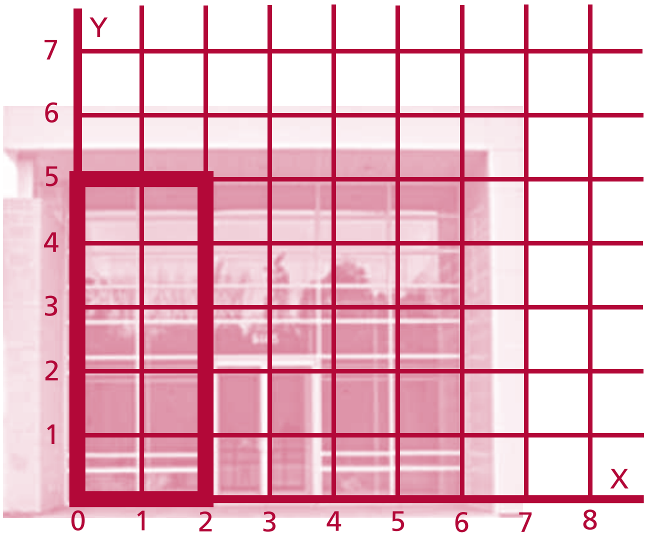


2. Complete the statement below by calculating the area of the rectangular piece of building shown. When the width of the rectangular piece is 1 unit, the area of the rectangle is

$A =$

Section C: Student Activity 1 (Group A)

Project Maths
Tionscadal Mata
Development Team



3. Complete the statement below by calculating the area of the rectangular piece of building shown.

When the width of the rectangular piece is 2 units, the area of the rectangle is

$A =$

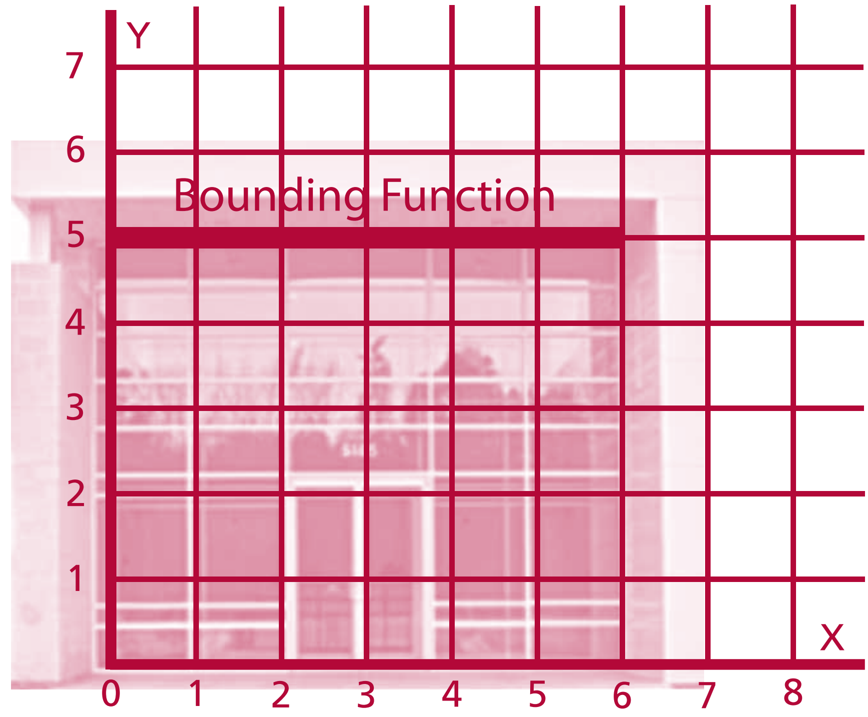
4. Complete the table below using an approach similar to that used in Q2 and Q3.

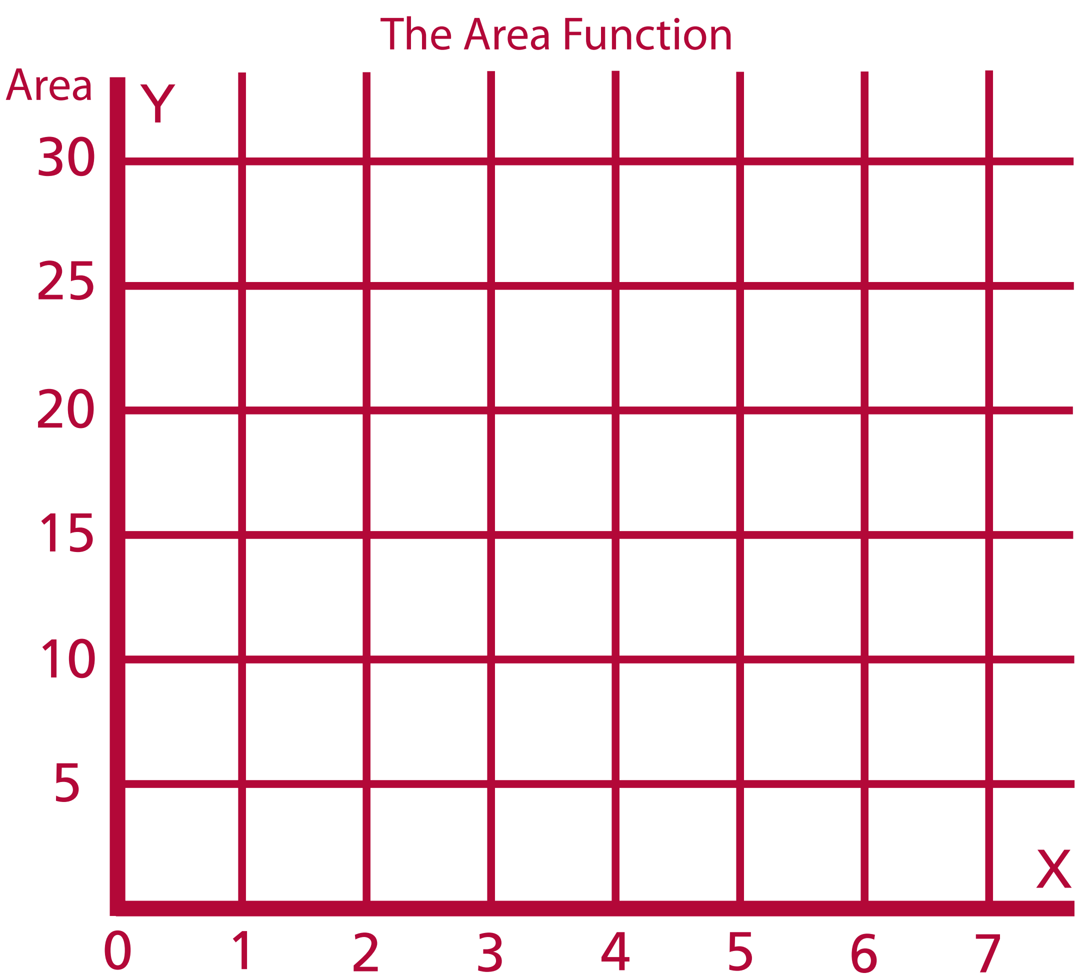
x	Width	Height	Area	Pattern
0				$A =$
1				$A =$
2				$A =$
3				$A =$
4				$A =$
5				$A =$
6				$A =$
	: :	: :	: :	: :
x			$A(x) =$	

Section C: Student Activity 1 (Group A)

Project Maths
Tionscadal Mata
Development Team

5. Sketch the graph of the area function on the empty axes.





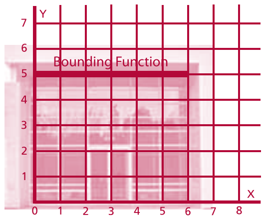
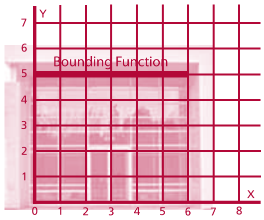
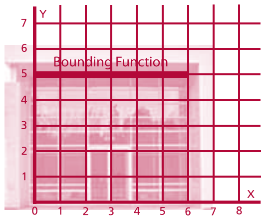
Section C: Student Activity 1 (Group A)

6. For each of the areas in the table below:

(a) Shade in the given area on the diagram.

(b) Use the area function to calculate the given area.

(c) Explain how the area function is used to calculate area.

Section of Building	Diagram	Area Calculation	Explanation
From $x = 0$ up to $x = 2$			
From $x = 0$ up to $x = 5$			
From $x = 2$ up to $x = 5$			

7 In the space below write in the bounding function (from Q1 above) and the area function (from Q3 above).

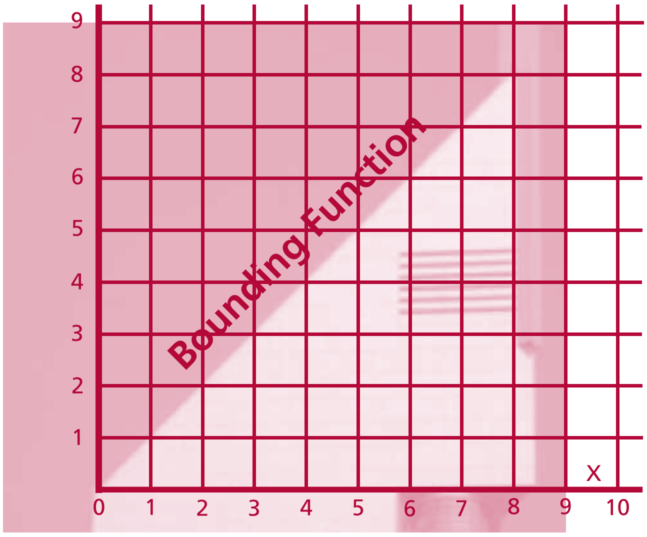
Bounding Function	Area Function
$h(x) =$	$A(x) =$

8 If you were presented only with the bounding function, is there a way in which you could determine the area function? Explain.

Section C: Student Activity 1 (Group B)

Project Maths Tionscadal Mata Development Team

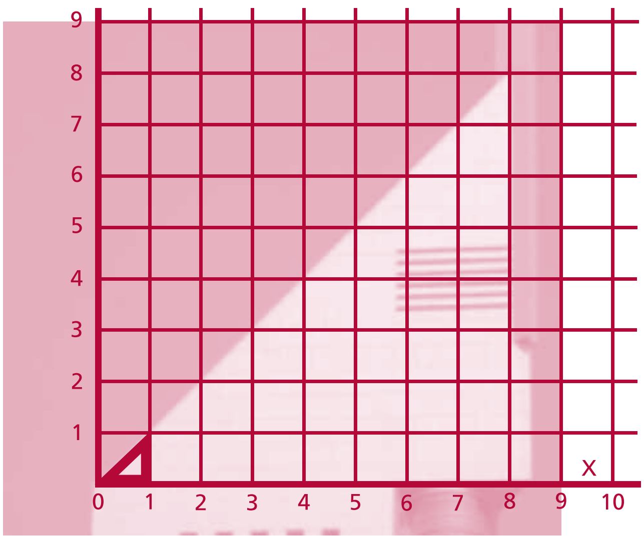
The figure below shows the Vu Bar in Dubai.



1. The height of the building changes as we move from left (x = 0) to right (x = 8). Write down the function which describes the changing height of the building.

$h(x)$ =

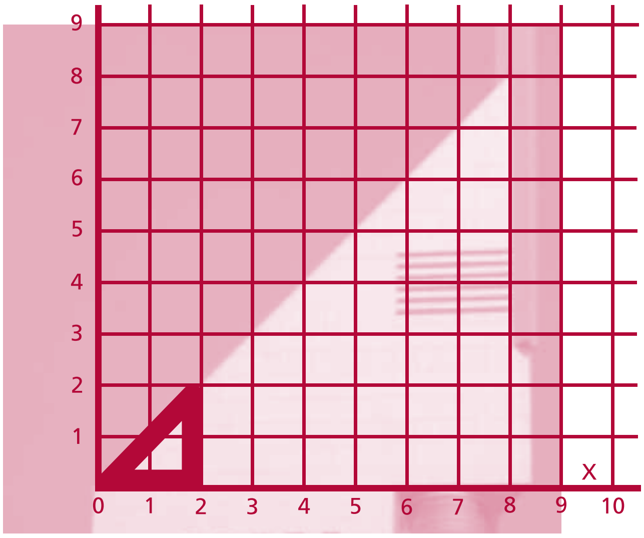
The area of the building also changes as we move from left to right. We will now investigate the relationship between the area of the building and its width.



2. Complete the statement below by calculating the area of the triangular piece of building shown. When the width of the triangular piece is 1 unit, the area of the triangle is

A =

Section C: Student Activity 1 (Group B)



3. Complete the statement below by calculating the area of the triangular piece of building shown. When the width of the triangular piece is 2 units, the area of the triangle is $A =$

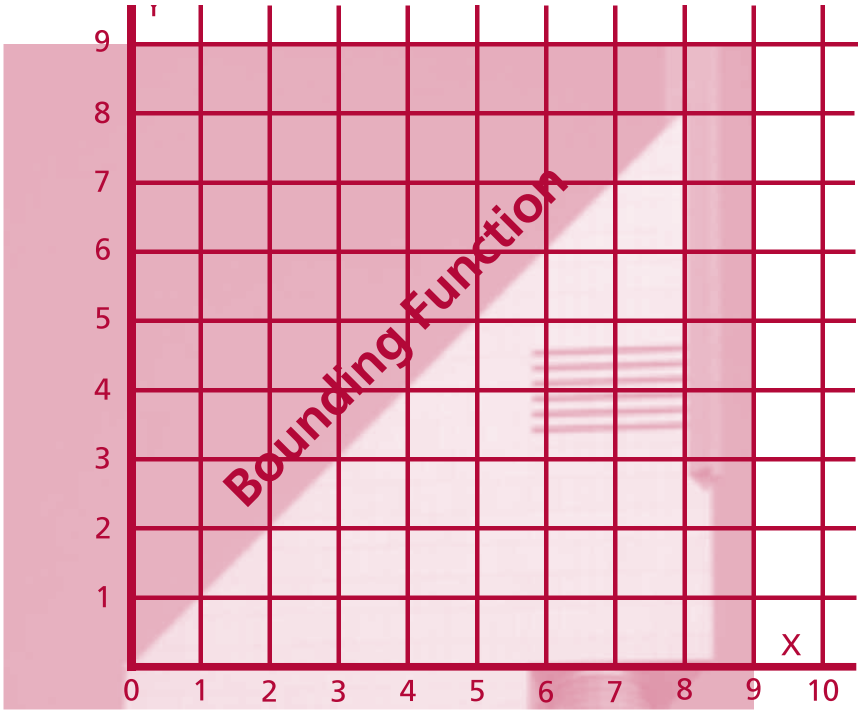
4. Complete the table below using an approach similar to that used in Q2 and Q3.

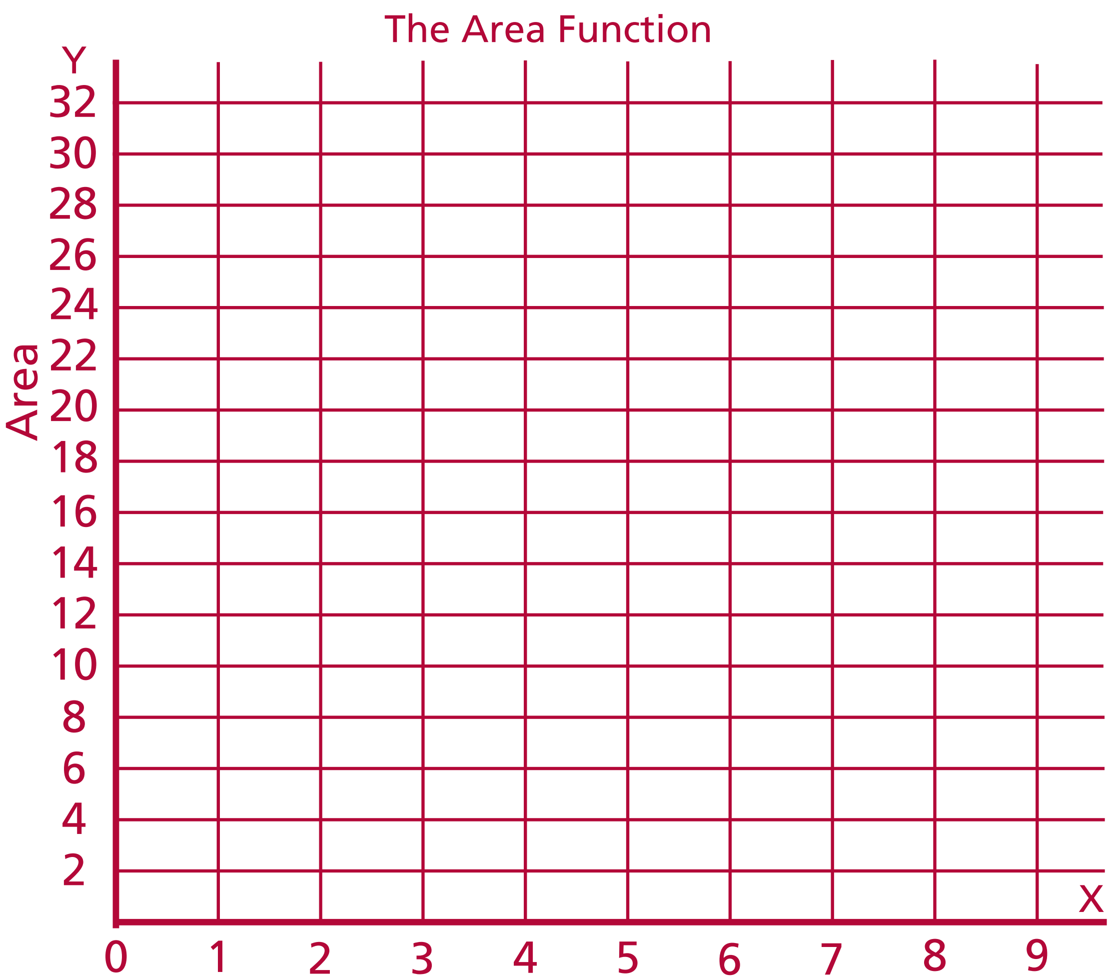
x	Width	Height	Area	Pattern
0				$A =$
1				$A =$
2				$A =$
3				$A =$
4				$A =$
5				$A =$
6				$A =$
7				$A =$
8				$A =$
	: :	: :	: :	: :
x			$A(x) =$	

Section C: Student Activity 1 (Group B)

Project Maths
Tionscadal Mata
Development Team

5. Sketch the graph of the area function on the empty axes.





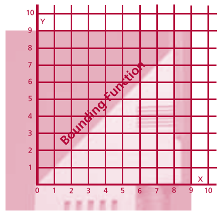
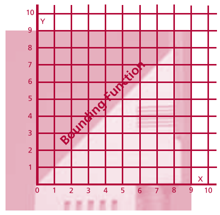
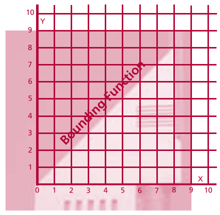
Section C: Student Activity 1 (Group B)

6. For each of the areas in the table below:

(a) Shade in the given area on the diagram.

(b) Use the area function to calculate the given area.

(c) Explain how the area function is used to calculate area.

Section of Building	Diagram	Area Calculation	Explanation
From $x = 0$ up to $x = 3$			
From $x = 0$ up to $x = 5.5$			
From $x = 3$ up to $x = 5.5$			

7. In the space below write in the bounding function from (Q1 above) and the area function from (Q3 above).

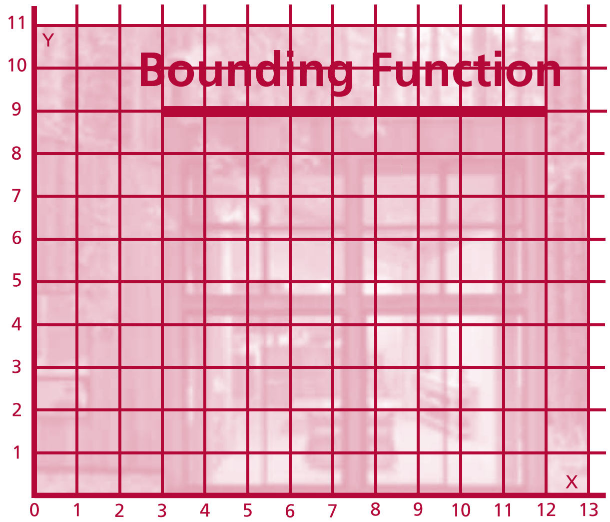
Bounding Function	Area Function
$h(x) =$	$A(x) =$

8. If you were presented only with the bounding function, is there a way in which you could determine the area function? Explain.

Section C: Student Activity 1 (Group C)

Project Maths Tionscadal Mata Development Team

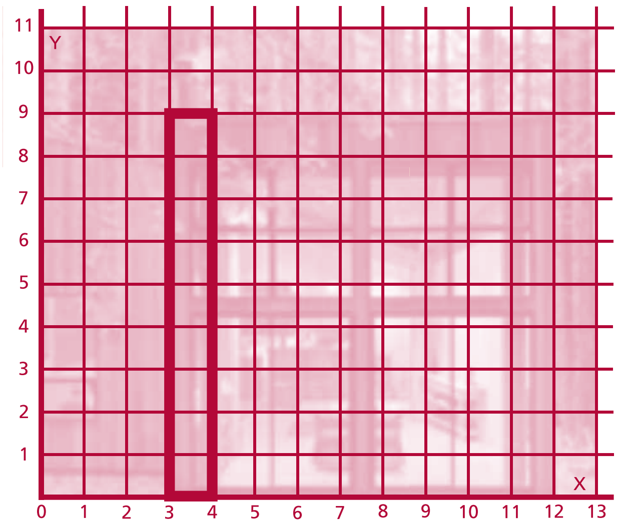
The figure below shows a modern timber dwelling.



1. Write down the function which describes the height of the building as we move from left ($x = 3$) to right ($x = 12$).

$h(x) =$

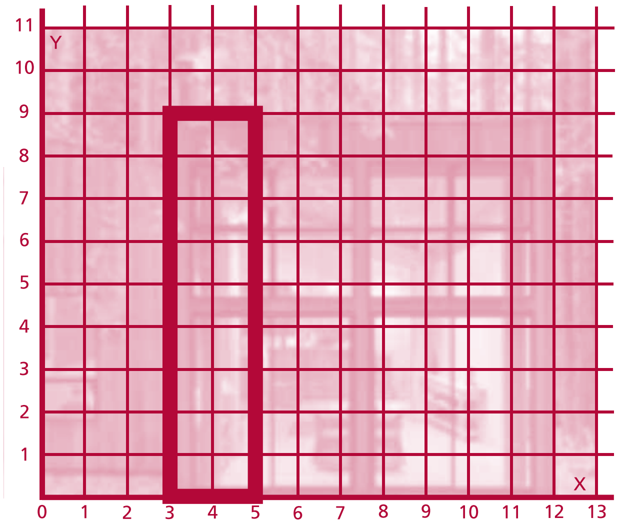
The area of the building changes as we move from left to right. We will now investigate the relationship between the area of the building and its width.



2. Complete the statement below by calculating the area of the rectangular piece of building shown. When the width of the rectangular piece is 1 unit, the area of the rectangle is $A =$

Section C: Student Activity 1 (Group C)

Project Maths Tionscadal Mata Development Team



3. Complete the statement below by calculating the area of the rectangular piece of building shown. When the width of the rectangular piece is 2 units, the area of the rectangle is A =

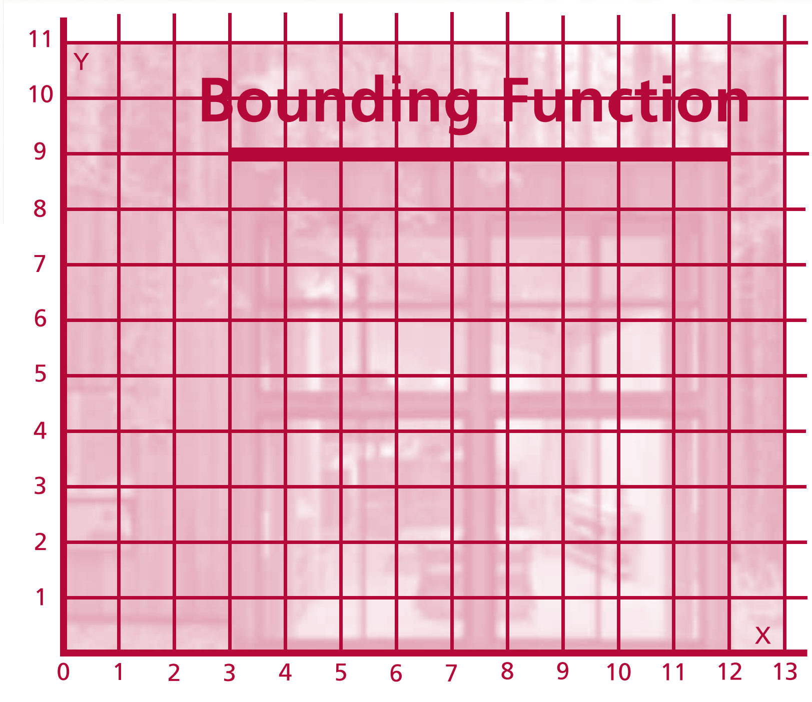
4. Complete the table below using an approach similar to that used in Q2 and Q3.

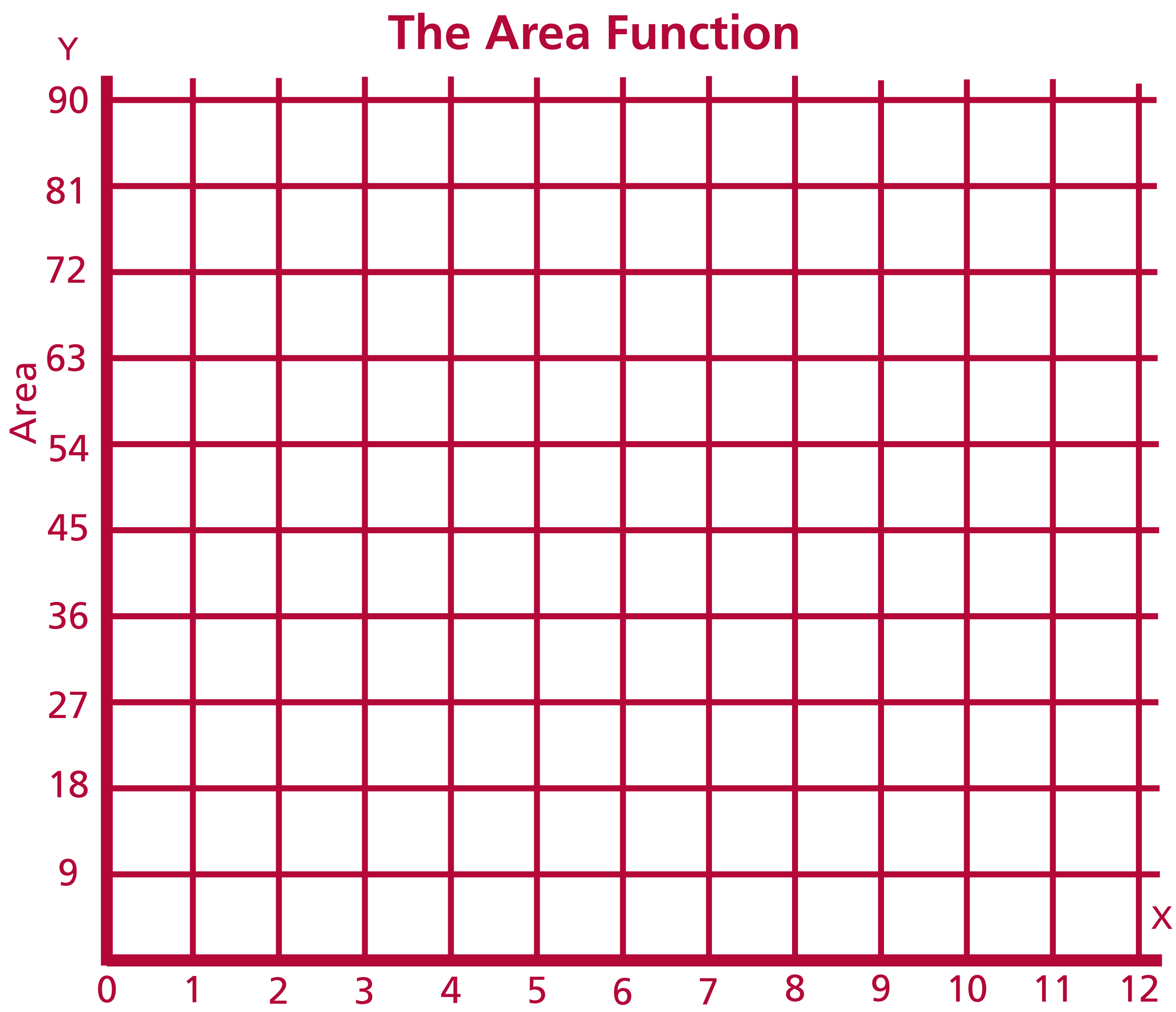
x	Width	Height	Area	Pattern
3				A =
4				A =
5				A =
6				A =
7				A =
8				A =
9				A =
10				A =
11				A =
12				A =
	: :	: :	: :	: :
x			$A(x)$ =	

Section C: Student Activity 1 (Group C)

Project Maths
Tionscadal Mata
Development Team

5. Sketch the graph of the area function on the empty axes.





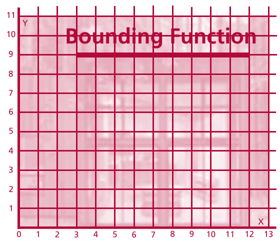
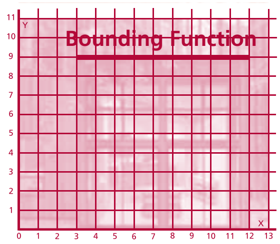
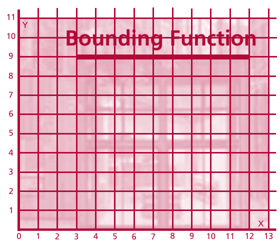
Section C: Student Activity 1 (Group C)

6. For each of the areas in the table below:

(a) Shade in the given area on the diagram.

(b) Use the area function to calculate the given area.

(c) Explain how the area function is used to calculate area.

Section of Building	Diagram	Area Calculation	Explanation
From $x = 3$ up to $x = 11$			
From $x = 3$ up to $x = 6$			
From $x = 6$ up to $x = 11$			

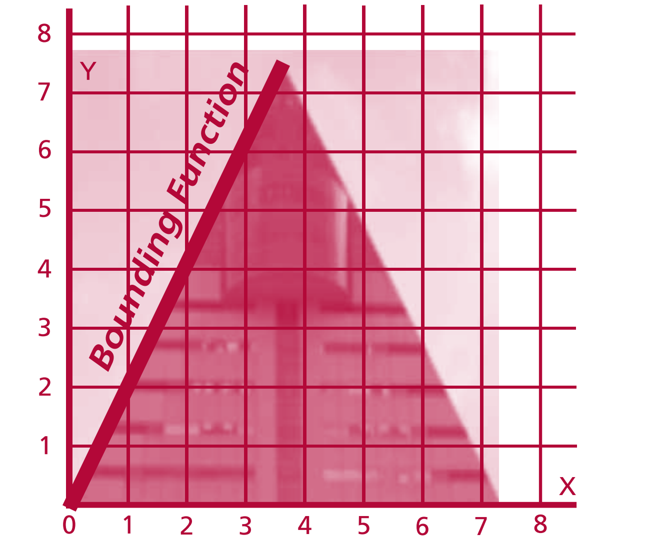
7. In the space below write in the bounding function (from Q1 above) and the area function (from Q3 above).

Bounding Function	Area Function
$h(x) =$	$A(x) =$

8. If you were presented only with the bounding function, is there a way in which you could determine the area function? Explain.

Section C: Student Activity 1 (Group D)

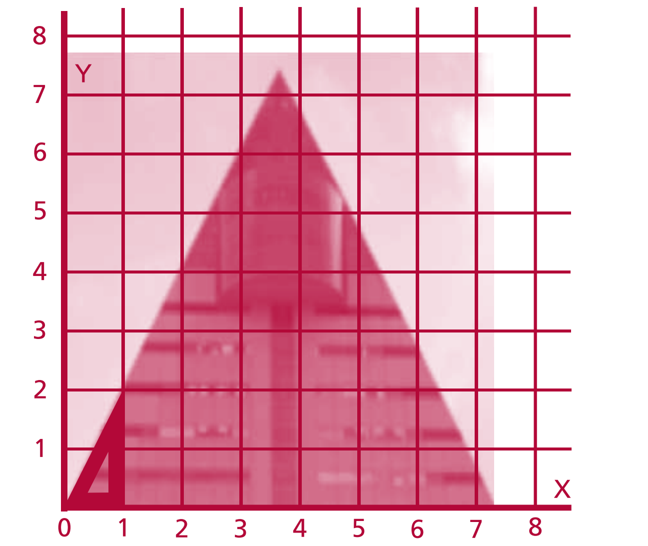
The figure below shows the an office block in Aomori, Japan.



1. The height of the building changes as we move from left ($x = 0$) to right ($x = 3.7$). Write down the function which describes the changing height of the building.

$h(x) =$

The area of the building also changes as we move from left to right. We will now investigate the relationship between the area of the building and its width.

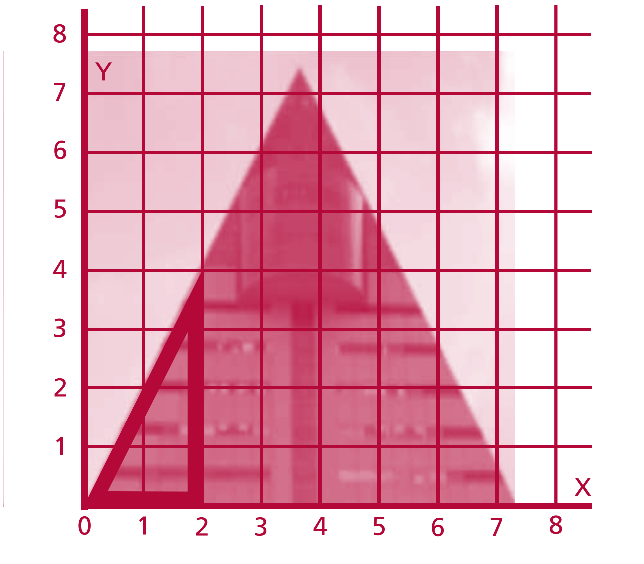


2. Complete the statement below by calculating the area of the triangular piece of building shown. When the width of the triangular piece is 1 unit, the area of the triangle is

$A =$

Section C: Student Activity 1 (Group D)

Project Maths Tionscadal Mata Development Team



3. Complete the statement below by calculating the area of the triangular piece of building shown. When the width of the triangular piece is 2 units, the area of the triangle is A =

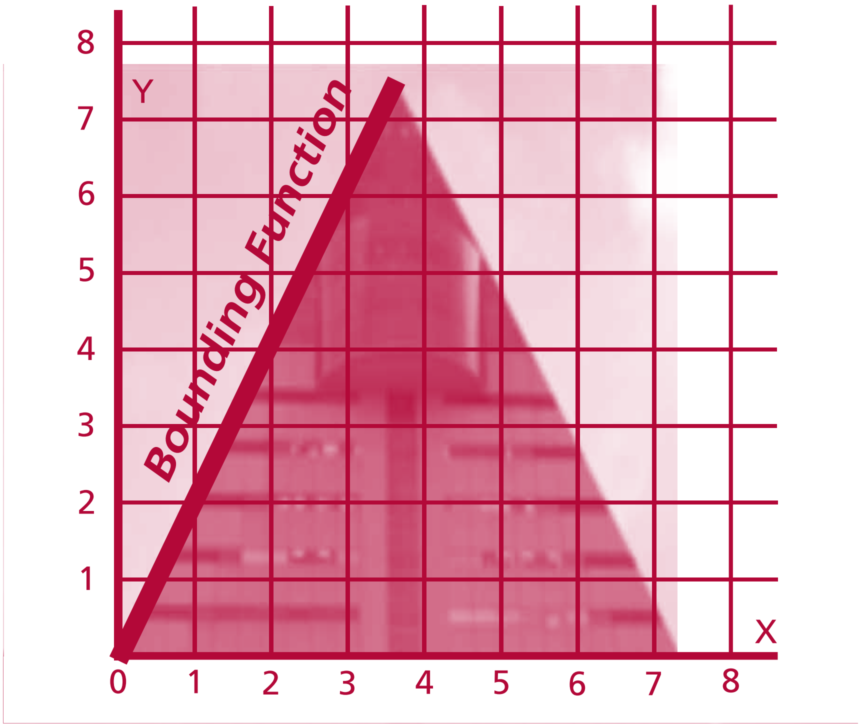
4. Complete the table below using an approach similar to that used in Q2 and Q3.

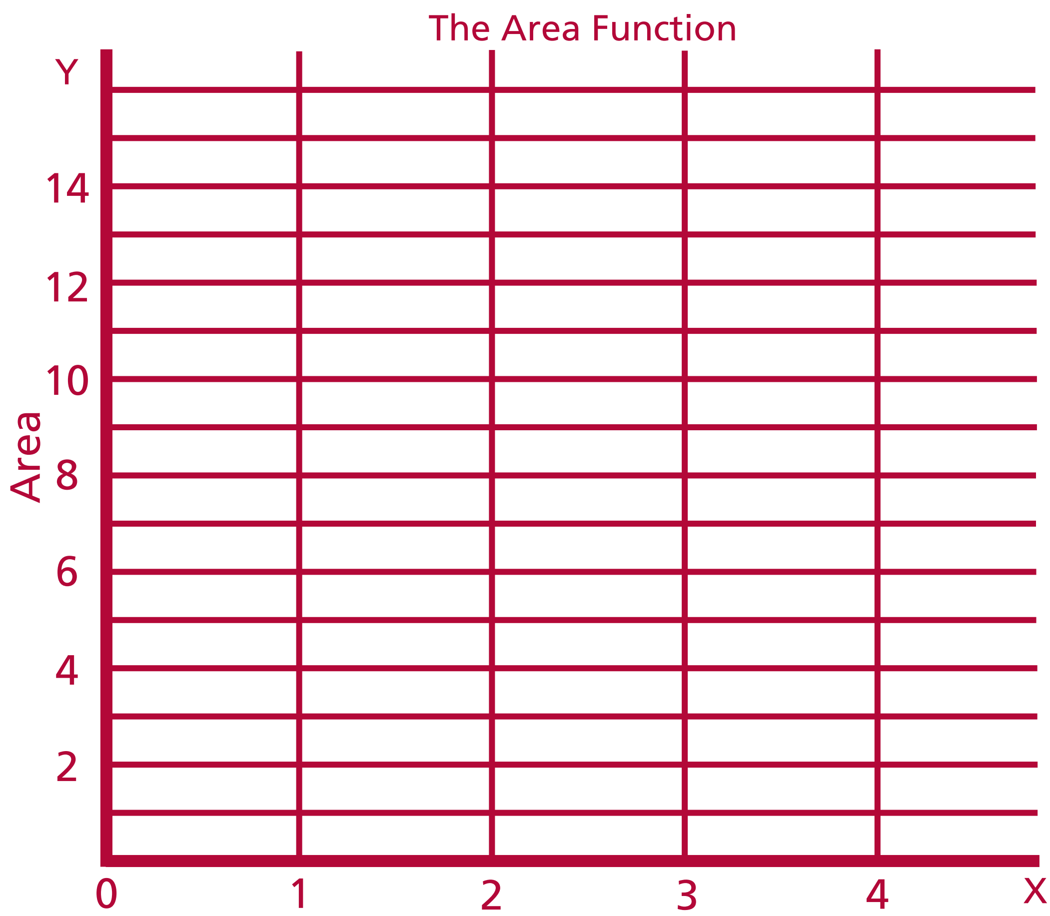
x	Width	Height	Area	Pattern
0				A =
1				A =
2				A =
3				A =
	: :	: :	: :	: :
x			$A(x)$ =	

Section C: Student Activity 1 (Group D)

Project Maths
Tionscadal Mata
Development Team

5. Sketch the graph of the area function on the empty axes.





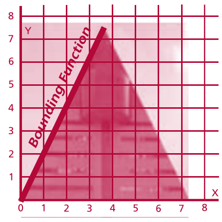
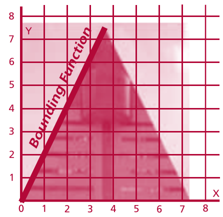
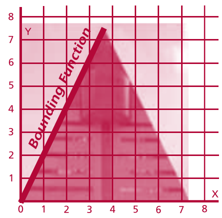
Section C: Student Activity 1 (Group D)

6. For each of the areas in the table below:

(a) Shade in the given area on the diagram.

(b) Use the area function to calculate the given area.

(c) Explain how the area function is used to calculate area.

Section of Building	Diagram	Area Calculation	Explanation
From $x = 0$ up to $x = 1.5$			
From $x = 0$ up to $x = 3.7$			
From $x = 1.5$ up to $x = 3.7$			

7. In the space below write in the bounding function (from Q1 above) and the area function (from Q3 above).

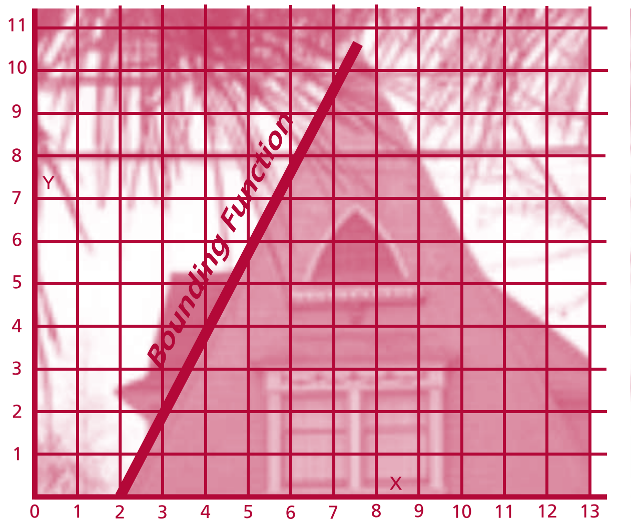
Bounding Function	Area Function
$h(x) =$	$A(x) =$

8. If you were presented only with the bounding function, is there a way in which you could determine the area function? Explain.

Section C: Student Activity 1 (Group E)

Project Maths Tionscadal Mata Development Team

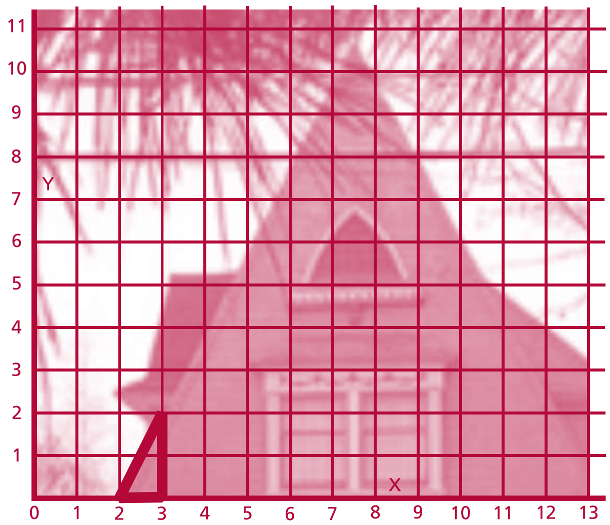
The figure below shows a house in New England.



1. The height of the building changes as we move from left (x = 2) to right (x = 7.4). Write down the function which describes the changing height of the building.

$h(x)$ =

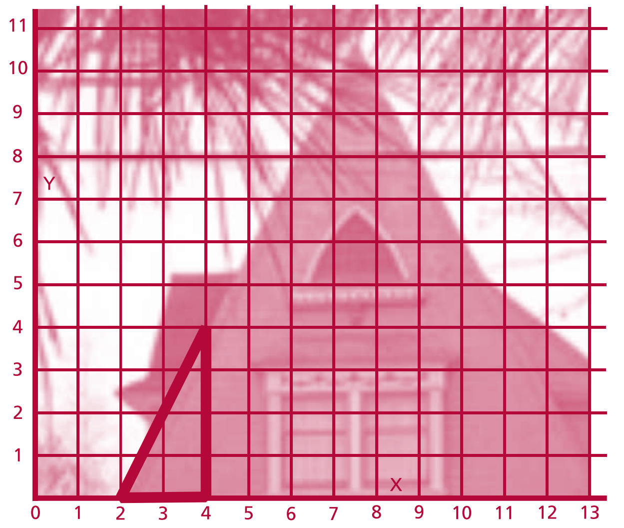
The area of the building also changes as we move from left to right. We will now investigate the relationship between the area of the building and its width.



2. Complete the statement below by calculating the area of the triangular piece of building shown. When the width of the triangular piece is 1 unit, the area of the triangle is

A =

Section C: Student Activity 1 (Group E)



3. Complete the statement below by calculating the area of the triangular piece of building shown. When the width of the triangular piece is 2 units, the area of the triangle is A =

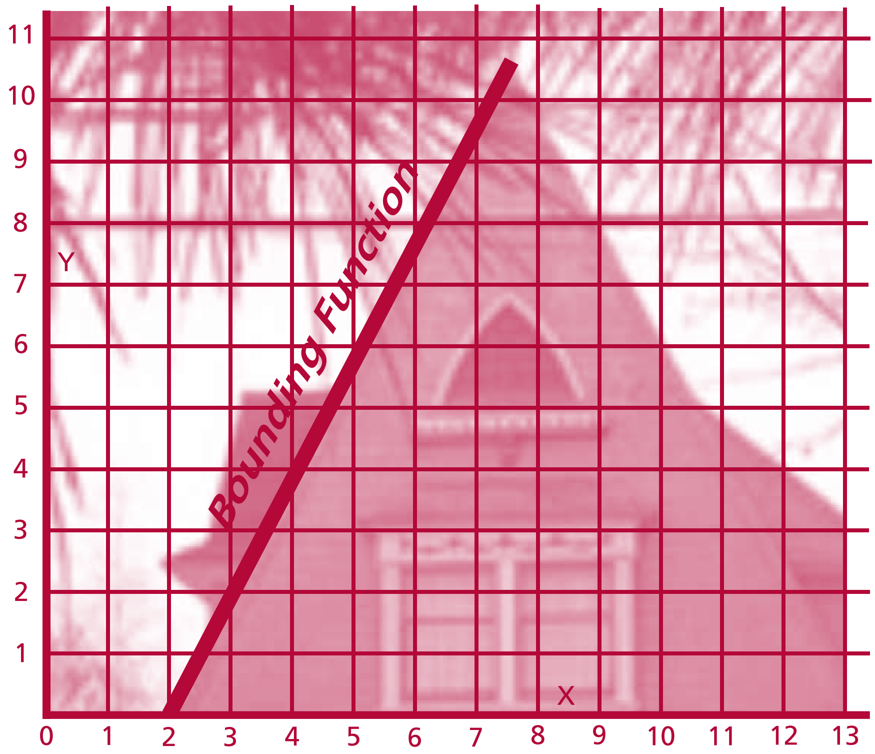
4. Complete the table below using an approach similar to that used in Q2 and Q3.

x	Width	Height	Area	Pattern
2				$A =$
3				$A =$
4				$A =$
5				$A =$
6				$A =$
7				$A =$
	: :	: :	: :	: :
x			$A(x) =$	

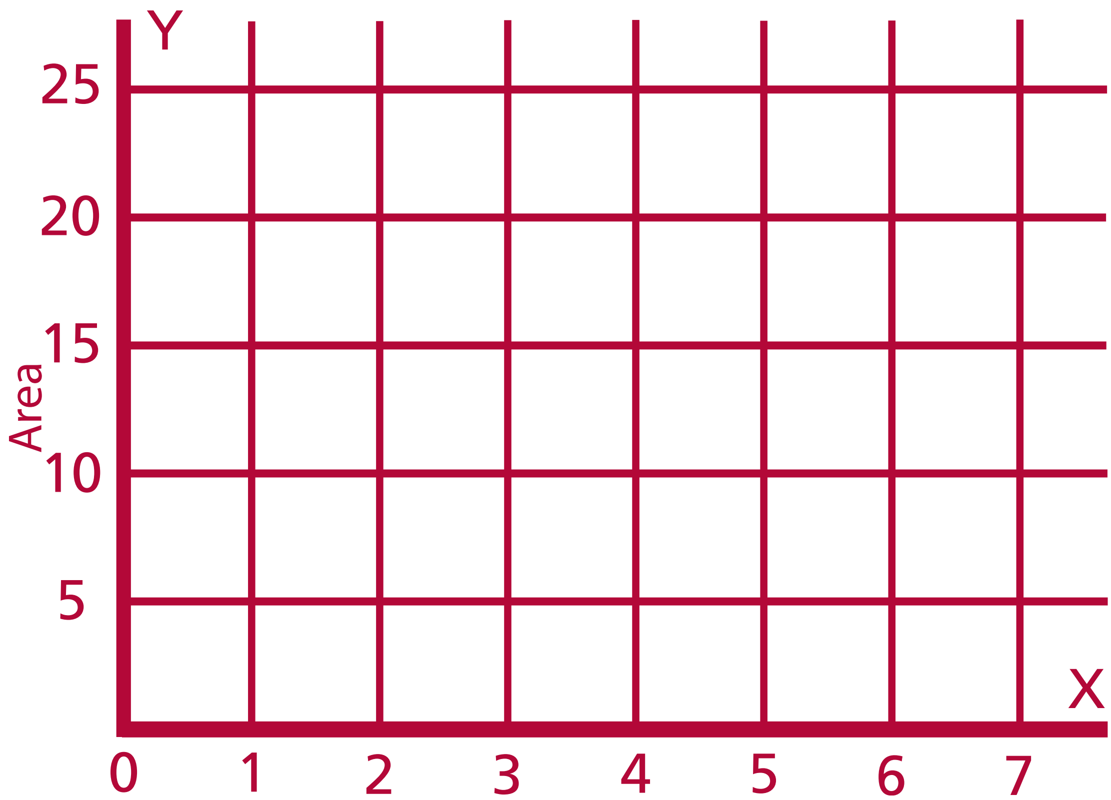
Section C: Student Activity 1 (Group E)

Project Maths
Tionscadal Mata
Development Team

5. Sketch the graph of the area function on the empty axes.



The Area Function



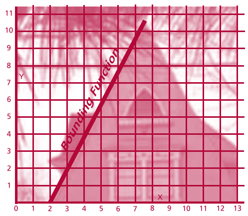
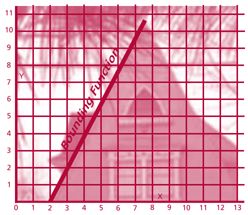
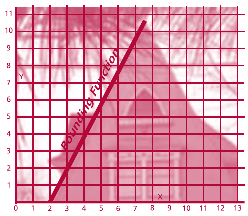
Section C: Student Activity 1 (Group E)

6. For each of the areas in the table below:

(a) Shade in the given area on the diagram.

(b) Use the area function to calculate the given area.

(c) Explain how the area function is used to calculate area.

Section of Building	Diagram	Area Calculation	Explanation
From $x = 2$ up to $x = 3.2$			
From $x = 2$ up to $x = 7$			
From $x = 3.2$ up to $x = 7$			

7 In the space below write in the bounding function (from Q1 above) and the area function (from Q3 above).

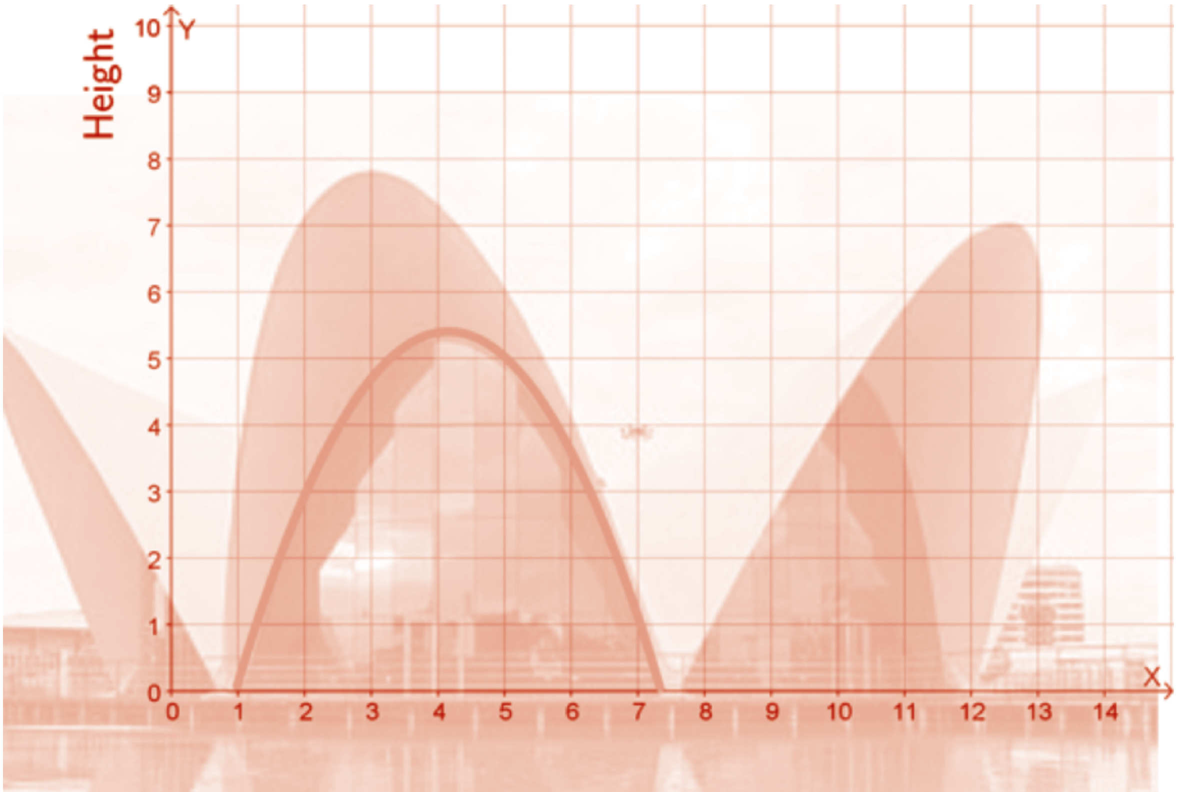
Bounding Function	Area Function
$h(x) =$	$A(x) =$

8 If you were presented only with the bounding function, is there a way in which you could determine the area function? Explain.

Section C: Student Activity 2

1. The glass facade of the building shown is parabolic in shape. The upper boundary of the glass front is described by the function $c(x) = -0.53x^2 + 4.4x - 3.73$.

   Calculate the area of the glass front from $x = 2$ to $x = 6.5$.



Section C: Student Activity 3

Project Maths
Tionscadal Mata
Development Team

1. Using anti-differentiation, calculate the area between the function $k(x) = \sin(x)$ and the x - axis, from $x = 0$ to $x = 2\pi$.

2. The graph of $k(x) = \sin(x)$ is shown on page 101. The shaded portion of the graph is the area which you were asked to calculate in Question 1. From the graph, estimate what the shaded area is? Is your answer for Question 1 consistent with your estimate? Explain.

3. Using anti-differentiation, calculate the area between the function and the x - axis, from $x = 0$ to $x = \pi$.

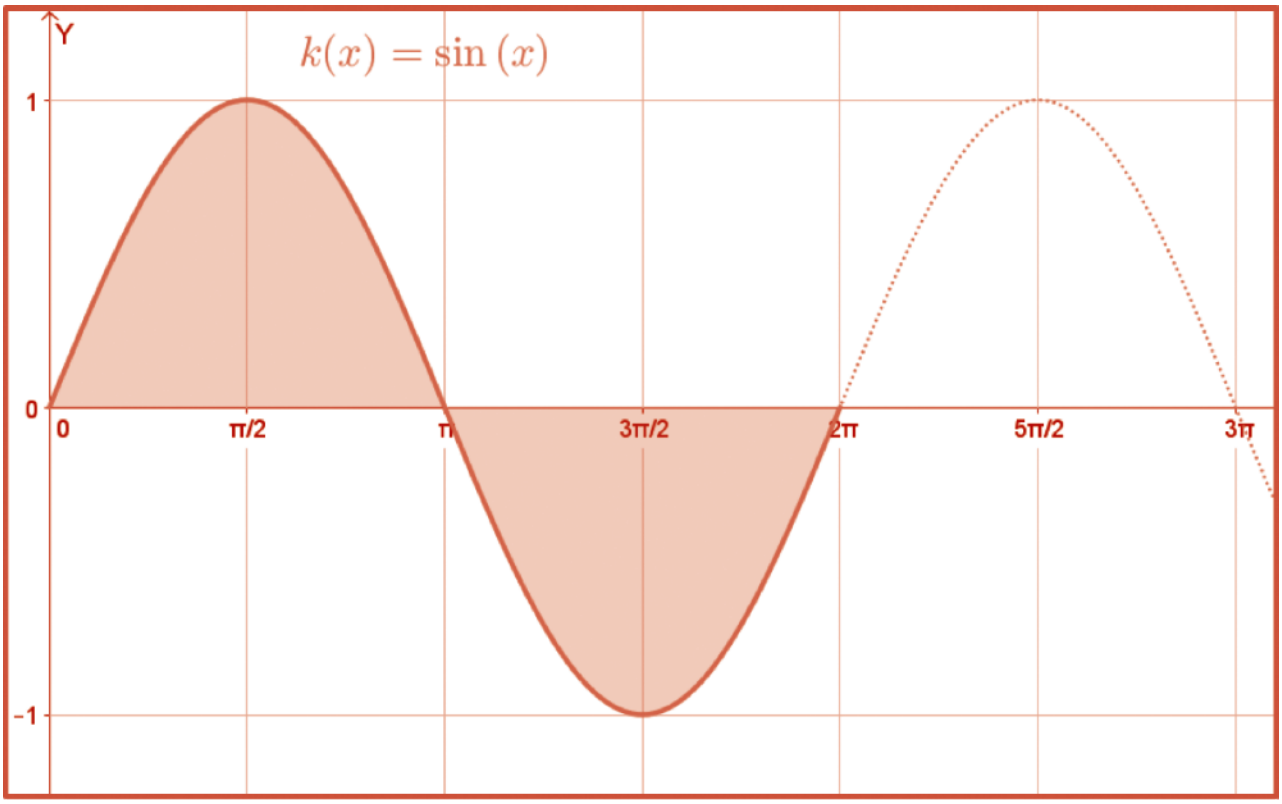
4. Using anti-differentiation, calculate the area between the function and the x - axis, from $x = \pi$ to $x = 2\pi$.

Section C: Student Activity 3

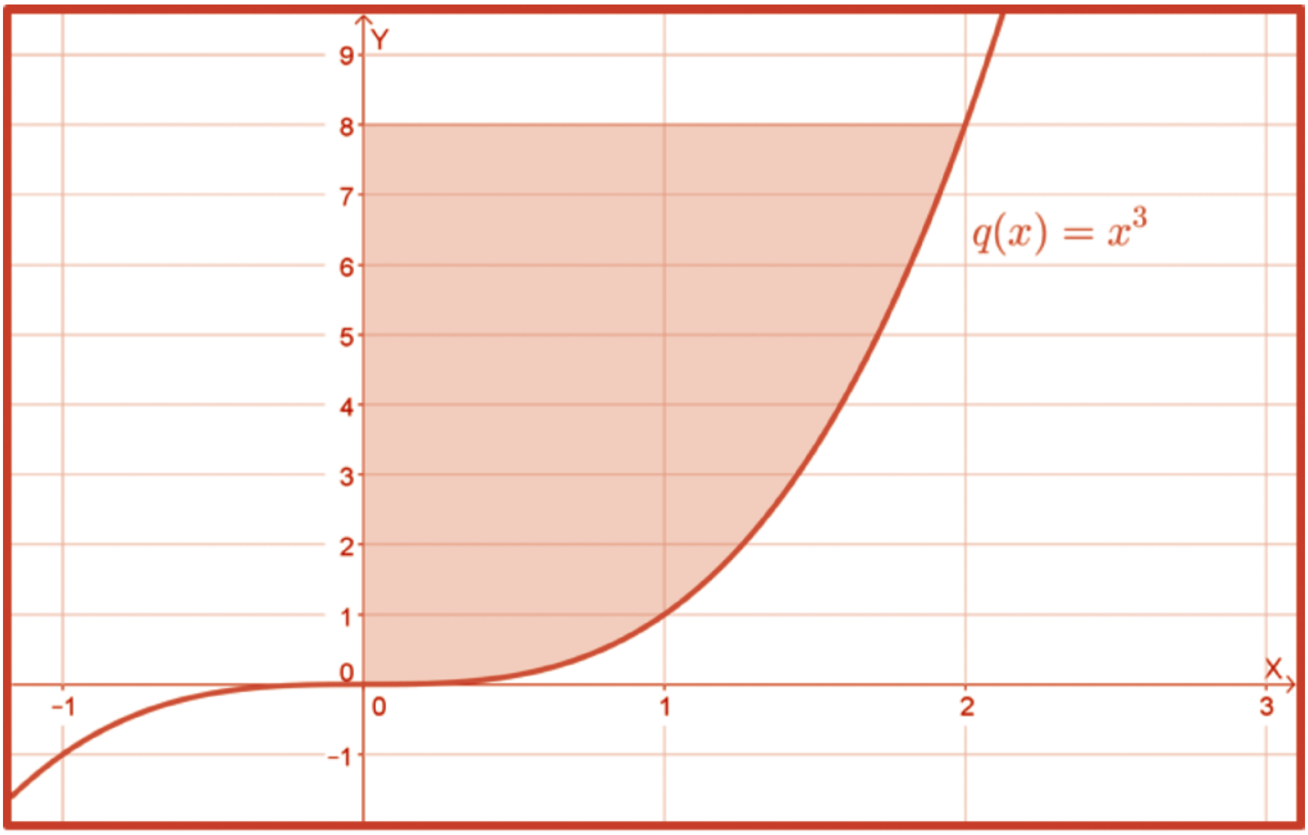
5. Can you now explain why your answer in Question 1 is not the area between the function and the x - axis?

6. Determine the area of the region between the function $w(x) = x^3 - 7x^2 + 10x$ and the x - axis, from $x = 0$ to $x = 5$.

Section C: Student Activity 3



Section C: Student Activity 4



A graph of the function $q(x) = x^3$ is shown above. Calculate the shaded area, that is, the area between $q(x) = x^3$ and the y - axis from $y = 0$ to $y = 8$.

Section D: Student Activity 1

1. Bernie has a savings account which she can add to or withdraw from. The table below shows the activity in the account over a 7 month period:

Time	Savings(€)
April	8000
May	18000
June	16000
July	16000
August	16000
September	12000
October	12000

Calculate the average amount of money in Bernie's account.

2. On a certain day in Cork, air temperature was described by the following function:

$T(t) = -0.2t^2 + 6.4t - 35.2 \; ; \; 9 \leq t \leq 16$

where T is temperature in °C and t is time since midnight in hours.

Calculate the average air temperature between 9 am and 4 pm.

Section D: Student Activity 1