Teaching & Learning Plans

Quadratic Equations

Junior Certificate Syllabus

Project Maths
Tionscadal Mata
Development Team

The Teaching & Learning Plans are structured as follows:

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Aims outline what the lesson, or series of lessons, hopes to achieve.

Prior Knowledge points to relevant knowledge students may already have and also to knowledge which may be necessary in order to support them in accessing this new topic.

Learning Outcomes outline what a student will be able to do, know and understand having completed the topic.

Relationship to Syllabus refers to the relevant section of either the Junior and/or Leaving Certificate Syllabus.

Resources Required lists the resources which will be needed in the teaching and learning of a particular topic.

Introducing the topic (in some plans only) outlines an approach to introducing the topic.

Lesson Interaction is set out under four sub-headings:

i. **Student Learning Tasks – Teacher Input:** This section focuses on possible lines of inquiry and gives details of the key student tasks and teacher questions which move the lesson forward.

ii. **Student Activities – Possible Responses:** Gives details of possible student reactions and responses and possible misconceptions students may have.

iii. **Teacher's Support and Actions:** Gives details of teacher actions designed to support and scaffold student learning.

iv. **Assessing the Learning:** Suggests questions a teacher might ask to evaluate whether the goals/learning outcomes are being/have been achieved. This evaluation will inform and direct the teaching and learning activities of the next class(es).

Student Activities linked to the lesson(s) are provided at the end of each plan.

Teaching & Learning Plan: Junior Certificate Syllabus

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Aim

- To enable students recognise quadratic equations
- To enable students use algebra, graphs and tables to solve quadratic equations
- To enable students form a quadratic equation to represent a given problem
- To enable higher-level students form quadratic equations from their roots

Prior Knowledge

Students have prior knowledge of:

- Simple equations
- Natural numbers, integers and fractions
- Manipulation of fractions
- Finding the factors of $x^2 + bx + c$ where $b, c \in Z$
- Finding the factors of $ax^2 + bx + c$ where $a, b, c \in Q, x \in R$ (Higher Level)
- Patterns
- Basic algebra
- Simple indices
- Finding the factors of $x^2 - a^2$

Learning Outcomes

As a result of studying this topic, students will be able to:

- understand what is meant by a quadratic equation
- recognise a quadratic equation as an equation having as many as two solutions that can be written as $ax^2 + bx + c = 0$
- solve quadratic equations
- represent a word problem as a quadratic equation and solve the relevant problem
- form a quadratic equation given its roots

Catering for Learner Diversity

In class, the needs of all students, whatever their level of ability level, are equally important. In daily classroom teaching, teachers can cater for different abilities by providing students with different activities and assignments graded according to levels of difficulty so that students can work on exercises that match their progress in learning. Less able students, may engage with the activities in a relatively straightforward way while the more able students should engage in more open–ended and challenging activities.

In interacting with the whole class, teachers can make adjustments to suit the needs of students. For example, more challenging material similar to that contained in Question 11 in **Section A: Student Activity 1** can be provided to students where appropriate.

Apart from whole-class teaching, teachers can utilise pair and group work to encourage peer interaction and to facilitate discussion. The use of different grouping arrangements in these lessons should help ensure that the needs of all students are met and that students are encouraged to verbalise their mathematics openly and to share their learning.

Relationship to Junior Certificate Syllabus

Topic Number	**Description of topic** *Students learn about*	**Learning outcomes** *Students should be able to*
4.7 Equations and inequalities	Using a variety of problem solving strategies to solve equations and inequalities. They identify the necessary information, represent problems mathematically, making correct use of symbols, words, diagrams, tables and graphs.	– solve quadratic equations of the form $x^2 + bx + c$ where $b, c \in Z$ and $x^2 + bx + c$ is factorisable $ax^2 + bx + c$ where $a, b, c \in Q$, $x \in R$ (HL only) – form quadratic equations given whole number roots (HL only) – solve simple problems leading to quadratic equations

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Teacher Reflections

Lesson Interaction

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
Section A: To solve quadratic equations of the form $(x-a)(x-b) = 0$ algebraically			
» What is meant by finding the solution of the equation $4x + 6 = 14$? » Why is $x = 1$ not a solution of $4x + 6 = 14$ » How do we find the solution to $4x + 6 = 14$? » Remember to check your answer.	• We find a value for x that makes the statement (that $4x + 6 = 14$) true • Because $4(1) + 6 \neq 14$ • Students write the solution in their copies. -6 : $4x + 6 = 14$: -6 $4x = 8$ ÷4 : $x = 2$: ÷4 or $4x + 6 = 14$ $4x + 6 - 6 = 14 - 6$ $4x = 8$ $x = 2$ $4(2) + 6 = 14$ True	» Write $4x + 6 = 14$ on the board. » Ask students to write the solution on the board.	» Do students remember what is meant by solving an equation? » Can students see why $x = 1$ is not a solution? » Do students know how to check their answer?
» What is 4 x 0? » What is 5 x 0? » What is 0 x 5? » What is 0 x n? » What is 0 x 0? » When something is multiplied by 0 what is the answer?	• 0 • 0 • 0 • 0 • 0 • 0	» Write the questions and solutions on the board. 4 x 0 = 0 5 x 0 = 0 0 x 5 = 0 0 x n = 0 0 x 0 = 0	» Do students understand that multiplication by zero gives zero?

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Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
» If $xy = 0$ what do we know about x and or y? » Compare $(x - 3)(x - 4) = 0$, with $xy = 0$ and what information can we arrive at? » If $x - 3 = 0$, what does this tell us about x? » If $x - 4 = 0$, what does this tell us about y?	• $x = 0$ or y = 0 or both are equal to 0. • $x - 3 = 0$ or $x - 4 = 0$ or both are equal to 0. • $x = 3$ • $x = 4$	» Write $xy = 0$ on the board. » Allow students to discuss and compare answers. » Write on the board: $xy = 0$ $(x - 3)(x - 4) = 0$ $x - 3 = 0$ or $x - 4 = 0$ $x = 3$ or $x = 4$	» Can students find the solution to $(x - 3)(x - 4) = 0$? » Do students understand what is meant by the solution to this equation?
» How do we check if $x = 3$ is a solution to $(x - 3)(x - 4) = 0$? » How do we check if $x = 4$ is a solution to $(x - 3)(x - 4) = 0$?	• Insert $x = 3$ into the equation and check if we get 0. • Insert $x = 4$ into the equation and check if we get 0.	» Write on the board: $(3 - 3)(x - 4) = 0$ $0(x - 4) = 0$ $0 = 0$ » Write on the board: $(x - 3)(4 - 4) = 0$ $(x - 3)\,0 = 0$ $0 = 0$	
» Write in your copies in words what $x = 3$ or $x = 4$ means in the context of $(x - 3)(x - 4) = 0$.	• Students write in words in their copies what this means and discuss with the student beside them.		» Can students write in words what $x = 3$ and $x = 4$ means?
» When an equation is written in the form $(x - a)(x - b) = 0$, what are the solutions?	• $x = a$ or $x = b$	» Write on the board: $(x - a)(x - b) = 0$ $x - a = 0$ $x - b = 0$ $x = a$ $x = b$	» Can students generalise the solution to $(x - a)(x - b) = 0$?

KEY: » next step • student answer/response

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Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
» Solve $(x - 1)(x - 3) = 0$ » How do we check that these are solutions? » Is it sufficient to state $x = 3$ is a solution to $(x - 1)(x - 3) = 0$?	• $(x - 1)(x - 3) = 0$ $(x - 1) = 0$ or $(x - 3) = 0$ $x = 1$ or $x = 3$. $(1 - 1)(x - 3) = 0$ $(1 - 1)(x - 3) = 0$ $0(x - 3) = 0$ $0 = 0$ • $(x - 1)(3 - 3) = 0$ $(x - 1)0 = 0$ $0 = 0$ True • No, both $x = 3$ and $x = 1$ are solutions.	» Write the solution on the board.	» Can students detect that if an equation is of the form $(x - a)(x-b) = 0$, then $x = a$ and $x = b$ are both solutions?
» Answer questions 1 to 11 on **Section A: Student Activity 1**.		» Distribute **Section A: Student Activity 1** » If students are unable to make the jump from $(x - \text{a})(x - \text{b}) = 0$ to $(x + \text{a})(x + \text{b}) = 0$. » If students are having difficulty, allow them to talk through their work so that misconceptions can be identified and addressed.	» Are students able to solve equations of the form $(x - a)(x - b) = 0$?

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Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
» How does the equation $x(x - 5) = 0$ differ from the equation $(x - 0)(x - 5) = 0$? » Why are they the same? » Hence what is the solution?	• It is the same. • Because x and $x - 0$ are the same, i.e. $x = x - 0$. • $x = 0$ or $x = 5$	» Write on the board: $x(x - 5) = 0$ and $(x - 0)(x - 5) = 0$ $x = 0$ and $x - 5 = 0$ $x = 0$ or 5 $0(x - 6) = 0$ True $x(6 - 6) = 0$ True	
» What are the solutions of $x(x - 6) = 0$?	• $x = 0$ and $x = 6$	» Write on the board: $x(x - 6) = 0$ $x = 0$ and $x - 6 = 0$ $x = 0$ or 6 $0(x - 6)= 0$ True $x(6 - 6) = 0$ True	» Do students recognise that the solutions to $x(x-a) = 0$ are $x= 0$ and $x = a$?
» Answer questions 12-16 on **Section A: Student Activity 1.**		» On completion of questions 12-16 those students who need a challenge can be encouraged to do question 17 on **Section A Student Activity 1**.	» Can students solve $x(x - a) = 0$?

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Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
Section B: To solve quadratic equations of the form $(x - a)(x - b) = 0$ making use of tables and graphs			
» Equations of the form $ax^2 + bx + c = 0$ are given a special name, they are called Quadratic Equations. » Give me an example of a Quadratic Equation. » Is $(x - 1)(x - 2) = 0$ a Quadratic Equation? » Why is $(x - 1)(x - 2) = 0$ a Quadratic Equation?	• $2x^2 + 3x + 5 = 0$ • Yes • Because when you multiply it out you get $x^2 - 3x + 2 = 0$, which is in the format $ax^2 + bx + c = 0$.	» Write $ax^2 + bx + c = 0$ and Quadratic Equations on the board. **Note:** The Quadratic Equations that the students list do not have to be factorisable ones at the moment and special cases $ax^2 + bx = 0$ $ax^2 + c = 0$ will be dealt with later.	» Do students recognise a Quadratic Equation? » Do students understand that quadratic equations can be written in the form $(x - a)(x - b)=0$?
» What does it mean to solve the equation $(x - 1)(x - 2) = 0$? » What is meant by the roots of an equation? » It is true that finding the roots of an equation and solving the equation mean the same thing? » Solve the equation $(x - 1)(x - 2) = 0$ using algebra. » So what are the roots of $(x - 1)(x - 2) = 0$	• Find values for x that makes this statement true. • The roots of an equation are the values of x that make the equation true. • Yes • $(x - 1)(x - 2) = 0$ $x - 1 = 0$ or $x - 2 = 0$ $x = 1$ or $x = 2$ $(1 - 1)(x - 2) = 0$ True $(x - 1)(2 - 2) = 0$ True • $x = 1$ and $x = 2$.		» Can students solve the quadratic equations of the form $(x - a)(x - b) = 0$? » Do students understand that finding the roots of an equation and solving an equation are the same thing?

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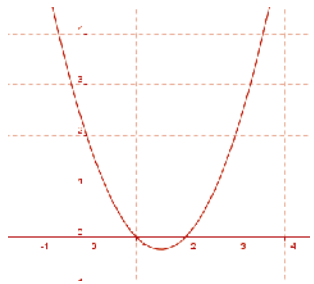
Teacher Reflections

<table>
<tr><th>Student Learning Tasks: Teacher Input</th><th>Student Activities: Possible Responses</th><th>Teacher's Support and Actions</th><th>Assessing the Learning</th></tr>
<tr>
<td>» Copy the table on the board into your exercise book and complete it.

» For what values of x did $(x - 1)(x - 2) = 0$?

» What name is given to the value(s) of x that make(s) an equation true?</td>
<td><table><tr><th>x</th><th>$(x - 1)(x - 2)$</th></tr><tr><td>-2</td><td>12</td></tr><tr><td>-1</td><td>6</td></tr><tr><td>0</td><td>2</td></tr><tr><td>1</td><td>0</td></tr><tr><td>2</td><td>0</td></tr><tr><td>3</td><td>2</td></tr></table>
• $x = 1$ and $x = 2$

• Solution(s) or roots.</td>
<td>» Display the following table on the board:<table><tr><th>x</th><th>$(x - 1)(x - 2)$</th></tr><tr><td>-2</td><td></td></tr><tr><td>-1</td><td></td></tr><tr><td>0</td><td></td></tr><tr><td>1</td><td></td></tr><tr><td>2</td><td></td></tr><tr><td>3</td><td></td></tr></table></td>
<td>» Are students able to complete the table, read from the table the values of x when $(x - 1)(x - 2) = 0$ and hence solve the equation?</td>
</tr>
<tr>
<td>» Draw a graph of the information in the table, letting $y = (x - 1)(x - 2)$.</td>
<td></td>
<td>» Draw the graph on the board</td>
<td>» Are students able to draw the graph and read from the graph the values of x when $(x - 1)(x - 2) = 0$?</td>
</tr>
<tr>
<td>» For what values of x did the graph cut the x axis?

» What is meant by the solution of an equation?

» What was the value of $y = (x - 1)(x - 2)$ when $x = 1$ and $x = 2$?</td>
<td>• $x = 1$ and $x = 2$

• The equation is true for the value of x.

• 0</td>
<td></td>
<td></td>
</tr>
</table>

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Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
» When using algebra to solve, what were the values of x for which $(x - 1)(x - 2) = 0$ called? » How can we get the solution by looking at the table? » How can we get the solution by looking at the graph?	• Solutions, roots • When $(x - 1)(x - 2)$ has the value zero. • Where it cuts the x axis or where the y value is zero	» Write the student's answers on the board.	» Do students understand that: • solving the equation using algebra • finding the value of x when the equation equals zero in the table • finding where the graph of the function cuts the x axis are all methods of finding the solution to the equation?
» Complete the exercises in **Section B: Student Activity 2.**		» Distribute **Section B. Student Activity 2.**	

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Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
Section C: To solve quadratic equations of the form $x^2 + bx + c = 0$ that are factorable			
» What does it mean to find the factors of a number?	• Rewriting the number as a product of two or more numbers.	» Write on the board: $x^2 + 3x + 2 = 0$ a =1, b =3, c =2 Guide number =1 x 2 = 2	» Do students understand the Guide number method of finding the factors of a quadratic?
» What does it mean to find the factors of $x^2 + 3x + 2$?	• It means to rearrange an algebraic expression so that it is a product of its prime factors.	» Write on the board 1 x 2 = 2 and -1x - 2 = 2.	
» What is the Guide number of this equation?	• 2	» Write on the board: $x^2 + 3x + 2 = 0$ $x^2 + 2x + 1x + 2 = 0$	» Can students find the factors of a simple equation?
» How did you get this Guide number?	• Multiplied 1 x 2 because comparing this equation to the form $ax^2 + bx + c = 0$, a = 1 and c = 2.	» Write on the board $x^2 + 1x + 2x + 2 = 0$	» Do students understand why they should use 1 and 2?
» What are the factors of 2?	• 1 and 2 or -1 and -2	» Allow students to compare their work.	
» Which pair shall we use?	• 1 and 2 because added together 1x + 2x gives us +3x.	» Ask two students to come to the board and write down their solution to $x^2 + 2x + 1x + 2 = 0$ and $x^2 + 1x + 2x + 2 = 0$	» Can students connect this to their prior knowledge of Factorising by Grouping?
» Why would I not use the pair -1 and -2?	• They would give you -3x which is not a term in the original equation.		
» Ask students if this looks familiar to any other type of factorising they have done before?	• This is Factorising by Grouping		
» Could I have written 1x plus 2x instead of 2x and 1x?	• Students work on factorising each.		

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Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
» I want you to investigate this for yourselves by Factorising by grouping each of these $x^2 + 2x + 1x + 2 = 0$ $x^2 + 1x + 2x + 2 = 0$ » Why does the first solution have $(x+2)$ in each bracket and the second solution have $(x+1)$ in each bracket? » How would you check that both are correct?	• One student writes on board: $x^2 + 2x + 1x + 2 = 0$ $x(\underline{x + 2}) + 1(\underline{x + 2}) = 0$ $(x + 1)\ (x + 2) = 0$ $x = -1$ or $x = -2$ • Another student writes on the board: $x^2 + 1x + 2x + 2 = 0$ $x(\underline{x + 1}) + 2(\underline{x + 1})$ $(x + 2)\ (x + 1) = 0$ $x = -2$ or $x = -1$		
» Solve the equation: $x^2 + 4x + 3 = 0$	• $(x + 1)\ (x + 3) = 0$ $x + 1 = 0$ or $x + 3 = 0$ $x = -1$ or $x = -3$ $(-1 + 1)\ (x + 3) = 0$ True $(x + 1)\ (-3 + 3) = 0$ True	» Write the solution on the board.	» Are students able to factorise the equation and hence solve it?
» Answer questions 1 - 8 on **Section C: Student Activity 3**.		» Distribute **Section C: Student Activity 3**. » Circulate and see what answers the students are giving and address any misconceptions. » Ask individual students to write their answers on the board.	» Can students factorise an expression of the form and solve an equation of the form $x^2 + bx + c = 0$?

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
» The width of a rectangle is 5cm greater than its length. Could you write this in terms of x?	• $x (x + 5)$	» Get the students to draw a diagram of a suitable rectangle. » Allow students to adopt an explorative approach here before giving the procedure.	» Are students extending their knowledge of quadratic equation?
» If we know the area is equal to $36cm^2$, write the information we know about this rectangle as an equation.	• A = (length) (width) $x (x + 5) = 36$	» Ask a student to write the solution on the board explaining what they are doing in each step.	
» Solve this equation.	• $x^2 + 5x - 36 = 0$ $(x + 9) (x - 4) = 0$ $x + 9 = 0$ $x - 4 = 0$ $x = -9$ or $x = 4$ $(-9 + 9) (x - 4) = 0$ True $(x + 9) (-4 - 4) = 0$ True	» Challenge students to explain why $x= -9$ is rejected.	
» What is the length and width of the rectangle?	• The length is equal to 4cm and width is equal to 9cm.		» Do students understand why $x = -9$ was a spurious solution?
» Is it sufficient to leave this question as $x = 4$?	• No. You must bring it back to the context of the question.		» Do students understand that saying $x = 4$ is not a sufficient answer to the question, but that it must be bought back into context of the question?

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Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
» Complete the remaining exercises on **Section C: Student Activity 3.**		» Teacher may select a number of these questions or get students to complete this activity sheet for homework. » Ask individual students to do questions on the board when the class has done some of the work. Students should explain their reasoning in each step.	» Can students answer the problems posed in the student activity?

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Teacher Reflections

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Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
Section D: To solve quadratic equations of the form $ax^2 + bx + c$ that are factorable (Higher Level only)			
» Solve the equation $2x^2 + 5x + 2 = 0$.	• $2x^2 + 5x + 2 = 0$ Guide number is: 4 [coefficient of x^2 and 2 the constant] 2 x 2 = 4 Factors of 4 are 4 x 1 or 2 x 2 • Use 4 x 1 because this gives you a sum of 5 $2x^2 + 4x + 1x + 2 = 0$ $2x(x + 2) + 1(x + 2) = 0$ $(2x + 1)(x + 2) = 0$ $2x + 1 = 0$ or $x + 2 = 0$ $x = -1/2$ or $x = -2$	» Ask a student to come to the board and explain how they factorised this. $2x^2 + 4x + 1x + 2 = 0$ $2x(x + 2) + 1(x + 2) = 0$ $(2x + 1)(x + 2) = 0$ $2x + 1 = 0$ or $x + 2 = 0$ $x = -1/2$ or $x = -2$ or $2x^2 + 1x + 4x + 2 = 0$ $x(2x + 1) + 2(2x + 1) = 0$ $(x + 2)(2x + 1) = 0$ $x + 2 = 0$ or $2x + 1 = 0$ $x = -2$ or $x = -1/2$	» Do students understand that $2x^2 + 5x + 2 = (2x + 1)(x + 2)$?
» Find the factors of $3x^2 + 4x + 1$ and hence solve $3x^2 + 4x + 1 = 0$	• $(3x + 1)(x + 1)$ $3x + 1 = 0$ $x + 1 = 0$ $3x = -1$ $x = -1$ $x = -⅓$ $x = -1$	» Circulate and see what answers the students are giving and address any misconceptions.	
» Answer questions contained in **Section D: Student Activity 4**.		» Distribute **Section D: Student Activity 4.** » Circulate and check the students' work ensuring that all students can complete the task. » Ask individual students to do questions on the board when some of the work is done using the algorithm, table and graph.	» Can students factorise expressions of the form $ax^2 + bx + c$ and hence solve equations of the form $ax^2 + bx + c = 0$? » Can students solve equations of the form $ax^2 + bx + c = 0$ by algorithm, table and graphically?

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Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
Section E: To solve quadratic equations of the form $x^2 - a^2 = 0$			
» Are $x^2 + 6x + 5 = 0$ and $x^2 + 6x = 0$ quadratic equations? Explain why. » Give a general definition of a quadratic equation? » Are $ax^2 + c = 0$? $ax^2 + bx = 0$ $ax^2 = 0$ quadratic equations? » Do you have a quadratic equation if $a = 0$? If not what is it called?	• Yes because the highest power of the unknown in these examples is 2. • $ax^2 + bx + c = 0$. • Yes, because the highest power of the unknown is still 2. • No, because the highest power is no longer 2. This is called a linear equation.		» Do students understand that quadratic equations all take the form $ax^2 + bx + c = 0$ and b and/or c can be zero
» What would the general form of the equation look like if (i) $b = 0$, (ii) $c = 0$, (iii) $a = 0$, (iv) $b = 0$ and $c = 0$?	• $ax^2 + c = 0$ • $ax^2 + bx = 0$ • $bx + c = 0$ • $ax^2 = 0$		» Do students understand the difference between a quadratic and a linear equation?
» So is $x^2 - 16$ a quadratic expression? » What are the factors of $x^2 - 16$? » What was this called? » What is the solution of $x^2 - 16 = 0$? » How can we prove these values are the factors of $x^2 - 16$?	• Yes, because the highest power of the unknown is 2. • $(x + 4)(x - 4)$ • The difference of two squares. • $x = -4$ and $x = 4$ • $(-4)^2 - 16 = 0$ True	» Give students time to find the factors. » Ask a student to write the expression and its factors on the board explaining his/her reasoning.	» Can students get the factors of $x^2 - b^2 = 0$ and hence solve equations of the form $x^2 - b^2 = 0$?

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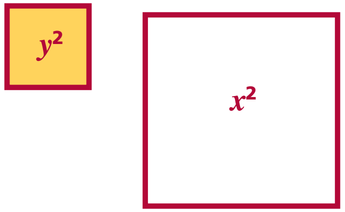
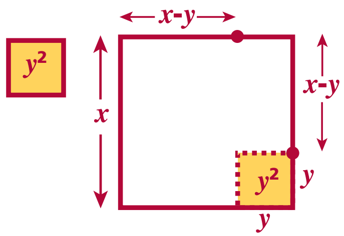
Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
» How can you use the following diagrams of two squares to graphically show that $x^2 - y^2 = (x + y)(x - y)$? 	• Area of the small square is y^2 and the area of the big square is x^2. • Area of the unshaded region is $x^2 - y^2$. • The area of the unshaded region is also $x(x - y) + y(x - y)$. • Hence $x^2 - y^2 = x(x - y) + y(x - y)$. • Hence $x^2 - y^2 = (x + y)(x - y)$?	» Draw the squares on the board. 	» Can students see how this is a physical representation of $x^2 - y^2 = (x + y)(x - y)$?
» Complete **Section E: Student Activity 5**.		» Distribute **Section E: Student Activity 5**. » Circulate the room and address any misconceptions.	» If students are having difficulties, allow them to talk through their work.This will help them identify their misunderstandings and misconceptions. Difficulties identified can then be addressed.

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Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
Section F: To solve quadratic equations using the formula (Higher Level only)			
» Is $x^2 + 3x + 1$ a quadratic equation? » Can you find the factors of $x^2 + 3x + 1$?	• Yes, because the highest power of the unknown is 2. • No	» Write the expression on the board. » Give students time to consider if they can get the factors and justify their answers.	» Do students see that not all quadratics are factorable?
» It is not always possible to solve quadratic equations through the use of factors, but there are alternative methods to solve them. » Comparing $x^2 + 3x + 1 = 0$ to the general form of a quadratic equation $ax^2 + bx + c = 0$. What are the values of a, b and c?	• $a = 1$, $b = 3$ and $c = 1$	» Write $x^2 + 3x + 1 = 0$ on the board. » Write $a = 1$, $b = 3$ and $c = 1$ on the board.	
» Mathematicians use the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to find the solutions to quadratic equations when they are unable to find factors. It is worth noting however that the formula can be used with all quadratic equation. » We will now try it for $x^2 + 3x + 2 = 0$, which we already know has $x = -2$ and $x = -1$ as its solutions.		» Write $x^2 + 3x + 2 = 0$ and $(x + 2)(x + 1) = 0$ $x = -2$ and $x = -1$ on the board. » Write the formula on the board. $x = \frac{-3 \pm \sqrt{3^2 - 4(1)(2)}}{2(1)}$ $x = \frac{-3 \pm \sqrt{1}}{2}$ $x = \frac{-3 \pm 1}{2}$ $x = -1$ or $x = -2$	» Can students complete the formula?

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Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
» Solve $x^2 + 3x + 1 = 0$ using the formula.	• $x = -2.816$ and $x = 0.382$	» Challenge students to find the factors on their own before going through the algorithm. » Write on the board $x = \frac{-3 \pm \sqrt{3^2 - 4(1)(2)}}{2(1)}$ $x = \frac{-3 \pm \sqrt{5}}{2}$ $x = \frac{-3 \pm 2.236}{2}$ $x = -2.618$ or $x = -0.382$	
» Complete the exercises in **Section F: Student Activity 6**.		» Distribute **Section F: Student Activity 6**.	» Can students use the formula to solve quadratic equations?

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Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
Section G: To solve quadratic equations given in rational form			
» Simplify $\frac{x^2}{2} = \frac{x}{3}$ » Now solve the equation.	• $3x^2 - 2x = 0$ $x(3x - 2) = 0$ $x = 0$ or $x = 2/3$	» Get individual students to write the solution on the board and explain their work.	» Can students convert equations given in simple fraction form to the form $ax^2 + bx + c = 0$?
» Write $x + 3 = 10/x$ in the form $ax^2 + bx + c = 0$ » How did you do this? » Hence solve $x + 3 = 10/x$	• $x + 3 = 10/x$ $x^2 + 3x - 10 = 0$ • Multiplied both sides by x. • $(x + 5)(x - 2) = 0$ $x = -5$ or $x = 2$ (-5) 2 + 3 (-5) -10 = 0 True (2) 2 + 3 (2) -10 = 0 True	» Get an individual student to write the solution on the board and explain their reasoning.	» Can students simplify the equation and then solve it?
» Attempt the questions on **Section G: Student Activity 7**.		» Distribute **Section G: Student Activity 7**. » Circulate the room and address any misconceptions.	» Can students correctly answer the questions on the activity sheet?

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Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
Section H: To form quadratics given whole number roots (Higher Level Only)			
» What are the factors of $x^2 + 6x + 8$? » What is the solution of $x^2 + 6x + 8 = 0$? » If we were told the equation had roots $x = -4$ and $x = -2$, how could we get an equation?	• $(x + 4)\ (x + 2)$ • $x = -4$ and $x = -2$ • $(x - (-4))\ (x- (-2)) = 0$ $(x + 4)\ (x + 2) = 0$ $x^2 + 4x + 2x + 8 = 0$ $x^2 + 6x + 8 = 0$	» Challenge the students to find the equation for themselves. » Get individual students to write the solution on the board and explain their work.	
» Given that an equation has roots 2 and 5, write the equation in the form $(x - s)\ (x - t) = 0$. » Then write it in the form $ax^2 + bx + c = 0$.	• $(x -2)\ (x -5) = 0$ • $x^2 -2x -5x + 10 = 0$ $x^2 -7x + 10 = 0$	» Write students' responses on the board.	» Given the roots, can students write the equation in the form $ax^2 + bx + c = 0$?
» Given that an equation has roots 1 and -3, write the equation in the form $ax^2 + bx + c = 0$.	• $(x -1)\ (x + 3) = 0$ $x^2 - 1x + 3x - 3 = 0$ $x^2 + 2x -3 = 0$	» Write students' responses on the board.	
» What do you notice about the following equations: $x^2 + 2x - 8 = 0$ $2x^2 + 4x - 16 = 0$ $8 - 2x - x^2 = 0$?	• They are the same equations but are arranged in a different order.	» Write each equation on the board.	» Can students recognise that the equations are the same?
» Now complete **Section H: Student Activity** 8.		» Distribute **Section H: Student Activity 8**.	

Section A: Student Activity 1

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Note: It is always good practice to check solutions. The roots of a quadratic equation are the elements of its solution set. For example if $x = 1$, $x = 2$ are the root $\Rightarrow \{1, 2\}$ = solution set. The roots of a quadratic equation are another name for its solution set.

1. If $xy = 0$, what value must either x or y or both have?
2. Write in your own words what solving an equation means.
3. Solve the following equations:

a. $(x - 1)(x - 2) = 0$ b. $(x - 4)(x - 5) = 0$

c. $(x - 3)(x - 5) = 0$ d. $(x - 2)(x - 5) = 0$

4. What values of x make the following statements true:

a. $(x - 2)(x - 5) = 0$ b. $(x - 4)(x + 5) = 0$ c. $(x - 2)(x + 4) = 0$

5. Find the roots of $(x - 4)(x + 5) = 0$.
6. Solve the equation $(x - 3)(x + 2) = 0$.
Hence state what the roots of $(x - 3)(x + 2) = 0$ are.
7. Find a positive value for x that makes the statement $(x - 4)(x + 2) = 0$ true.
8. Solve the following equations:

a. $x(x - 1) = 0$ b. $x(x - 2) = 0$ c. $x(x + 4) = 0$

9. a. These students each made at least one error, explain the error(s) in each case:

Student A		Student B		Student C	
$(x - 8)(x - 9) = 0$		$(x - 7)(x - 9) = 0$		$(x + 5)(x + 9) = 0$	
$x - 8 = 0$	$x - 9 = 0$	$x - 7 = 0$	$x - 9 = 0$	$x + 5 = 0$	$x + 9 = 0$
$x = -8$	$x = 9$	$x = -7$	$x = -9$	$x = 5$	$x = 9$

b. Solve each equation correctly showing all the steps clearly.

10. If $x = 5$ is a solution to the equation $(x - 4)(x - b) = 0$, what is the value of b?
11. Is $x = 3$ a solution to the equation $(x - 3)(x - 2) = 2$. Explain your reasoning. Solve this equation.

Section B: Student Activity 2 (continued)

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1. a. Compete the following table:

x	$x + 2$	$x + 1$	$y = f(x) = (x + 2)(x + 1)$
–4			
–3			
–2			
–1			
0			
1			
2			

b. From the table above determine the values of x for which the equation is equal to 0.

c. Solve the equation $(x + 2)(x + 1) = 0$ by algebra.

d. What do you notice about the answer you got to parts b. and c. in this question?

e. Draw a graph of the data represented in the above table.

f. Where does the graph cut the x axis? What is the value of $f(x) = (x + 2)(x + 1)$ at the points where the graph cuts the x axis?

g. Can you describe three methods of finding the solution to $(x + 2)(x + 1) = 0$.

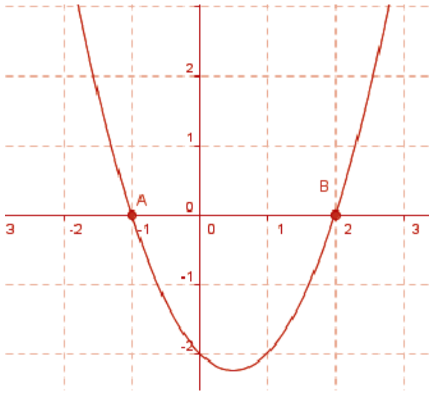
2. Solve the equation $(x - 1)(x - 4) = 0$ a) by table, b) by graph and c) algebraically.

3. Write the equation represented in this table in the form $(x–a)(x–b)=0$.

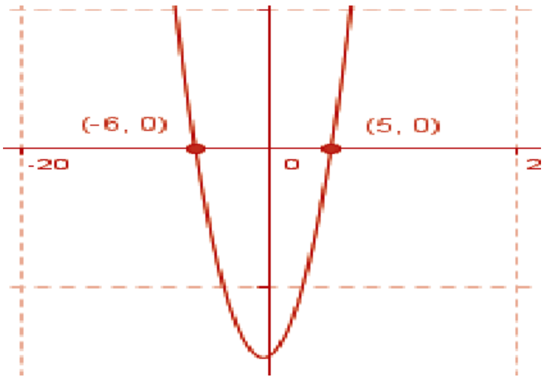
x	$f(x)$
–2	20
–1	12
0	6
1	2
2	0
3	0

Section B: Student Activity 2

4. The graph of a quadratic function $f(x) = ax^2 + bx + c$ is represented by the curve in the diagram below. Find the roots of the equation $f(x) = 0$ and so identify the function.



5. The graph of a quadratic function $f(x) = ax^2 + bx + c$ is represented by the curve in the diagram below. Find the roots of the equation $f(x) = 0$ and so identify the function.



6. Where will the graphs of the following functions cut the x axis?

a. $f(x) = (x - 7)(x - 8)$

b. $f(x) = (x + 7)(x + 8)$

c. $f(x) = (x - 7)(x + 8)$

d. $f(x) = (x + 7)(x - 8)$

7. For what values of x does $(x - 7)(x - 8) = 0$?

Section C: Student Activity 3

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Note: It is always good practice to check solutions.
It is recommended you use the Guide number method to find the factors.

1. Solve the following equations:
   a. $x^2 + 6x + 8 = 0$ b. $x^2 + 5x + 4 = 0$
   c. $x^2 - 9x + 8 = 0$ d. $x^2 - 6x + 8 = 0$

2. Solve the equations:
   a. $x^2 + 2x = 3$ b. $x(x - 1) = 0$ c. $x(x - 1) = 6$

3. Are the following two equations different $x(x - 1) = 0$ and $x(x - 1) = 6$? Explain.

4. When a particular Natural number is added to its square the result is 12. Write an equation to represent this and solve the equation. Are both solutions realistic? Explain.

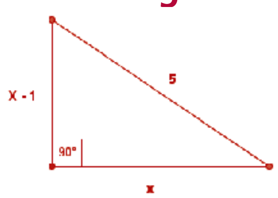
5. A number is 3 greater than another number. The product of the numbers is 28. Write an equation to represent this and hence find two sets of numbers that satisfy this problem.

6. The area of a garden is 50cm^2. The width of the garden is 5cm less than the breadth. Represent this as an equation. Solve the equation. Use this information to find the dimensions of the garden.

7. A garden with an area of 99m^2 has length xm. Its width is 2m longer than its length. Write its area in term of x. Solve the equation to find the length and width of the garden.

8. The product of two consecutive positive numbers is 110. Represent this as an algebraic equation and solve the equation to find the numbers.

9. Use Pythagoras theorem to generate an equation to represent the information in the diagram below. Solve this equation to find x.



Section C: Student Activity 3 (continued)

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10. One number is 2 greater than another number. When these two numbers are multiplied together the result is 99. Represent this problem as an equation and solve the equation.

11. Examine these students' work and spot the error(s) in each case and solve the equation fully:

Student A	Student B	Student C
$x^2 - 6x - 7 = 0$	$x^2 - 6x - 7 = 0$	$x^2 - 6x - 7 = 0$
$(x + 7)\quad(x + 1) = 0$	$(x - 7)\quad(x + 1) = 0$	$(x - 1)\quad(x + 7) = 0$
$x = -7\quad x = -1$	$x = 7\quad x = 1$	$x = 1\quad x = -7$

12. a. Complete a table for $x^2 - 2x + 1$ for integer values between -2 and 2.

b. Draw the graph of $x^2 - 2x + 1$ for values of x between -2 and 2. Where does this graph cut the x axis?

c. Factorise $x^2 - 2x + 1$ and solve $x^2 - 2x + 1 = 0$.

d. What do you notice about the values you got for parts a), b) and c)?

13. a. Complete a table for $x^2 + 3x + 2$ for integer values between -3 and 3.

b. Draw the graph of x^2+3x+2 for values of x between -3 and 3. Where does this graph cut the x axis?

c. Factorise $x^2+3x +2$ and solve $x^2+3x +2=0$.

d. What do you notice about the values you got for parts a), b) and c)?

Section D: Student Activity 4

Note: It is always good practice to check solutions.
It is recommended you use the Guide number method to find the factors.

1. Find the factors of $2x^2 + 7x + 3$. Hence solve $2x^2 + 7x + 3 = 0$.

2. Find the factors of $3x^2 + 4x + 1$. Hence solve $3x^2 + 4x + 1 = 0$.

3. Find the factors of $4x^2 + 4x + 1$. Hence solve $4x^2 + 4x + 1 = 0$.

4. Find the factors of $3x^2 - 4x + 1$. Hence solve $3x^2 - 4x + 1 = 0$.

5. Find the factors of $2x^2 + x - 3$. Hence solve $2x^2 + x - 3 = 0$.

6. a. Is $2x^2 + x = 0$ a quadratic equation? Explain your reasoning.

   b. Find the factors of $2x^2 + x$. Hence solve $2x^2 + x = 0$.

7. Factorise $4x^2 - 1x^2 + 9$. Hence solve $4x^2 - 1x^2 + 9 = 0$.

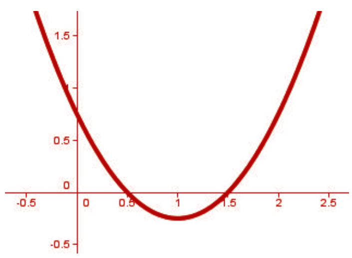
8. Twice a certain number plus four times the same number less one is 0. Find the numbers.

9. a. Complete the following table and using your results, suggest solutions to $2x^2 + 3x + 1 = 0$.

x	$2x^2$	$3x$	1	$2x^2 + 3x + 1$
-2			1	
-1			1	
0			1	
1			1	
2			1	

b. Using the information in the table above, draw a graph of $f(x) = 2x^2 + 3x + 1$, hence solve the equation.

c. Did your results for a. agree with your results in b?



10. Find the function represented by the curve in the diagram opposite in the form $f(x) = ax^2 + bx + c = 0$. Then solve the equation.

Section E: Student Activity 5

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1. Factorise:
   a. x^2 - 25 b. x^2 - 49
2. Solve:
   a. x^2 - 36 = 0 b. 36 - x^2 = 0 c. n2 - 625 = 0
   d. x^2 - x = 0 e. 3x^2 - 4x = 0 f. 3x^2 = 11x
   g. (a + 3)^2 - 25 = 0
3. Think of a number, square it, and subtract 64. If the answer is 0, find the number(s).
4. Given an equation of the form x^2 - b = 0, write the solutions to this equation in terms of b.
5. Solve the equation x^2 - 16 = 0 graphically. Did you get the results you expected? Explain your answer.
6. Calculate:
   a. $99^2 - 101^2$
   b. $103^2 - 97^2$

Higher Level Only

7. Solve the following equations:
   a. 4x^2 - 36 = 0
   b. 4x^2 - 9 = 0
   c. 4x^2 - 25 = 0
   d. 16x^2 - 9 = 0
8. A man has a square garden of side 20m. He builds a pen for his dog in one corner. If the area of the remaining part of his garden is 144m^2, find the dimensions of the dog's pen.

Section F: Student Activity 6

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1. Using the formula solve the following equations:
   a. $x^2 + 5x + 4 = 0$
   b. $x^2 + 4x + 3 = 0$
   c. $x^2 + 4x -3 = 0$
   d. $x^2 - 4x + 3 = 0$
   e. $x^2 - 4x -3 = 0$
   f. $x^2 - 4 = 0$
   g. $x^2 + 3x = 4$
   h. $x^2 + 2x -3 = 0$
2. Solve the equation $x^2 -5x - 2 = 0$. Write the roots in the form $a\pm\sqrt{b}$.
3. Given $2+\sqrt{3}$ as a solution to the equation $ax^2 + bx + c = 0$ find the other solution.
4. When using the quadratic formula to solve an equation and you know $x = 3$ is a solution, does that mean that $x = -3$ is definitely the other solution? Explain your reasoning with examples.
5.
   a. Solve the equation $x^2 + x^2 + 1 = 0$ by using a:
      i. Table.
      ii. Graph.
      iii. Factors.
      iv. Formula.
   b. Did you get the same solutions using all four methods?

Section G: Student Activity 7

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Solve the following equations:

$\frac{x^2}{2}+4x-\frac{9}{2}=0$	$\frac{2}{5}x^2-\frac{4}{5}x=-\frac{2}{5}$
$\frac{x^2+3}{5}+\frac{x}{2}=0.4$	$\frac{x^2+4}{2}-\frac{x^2+6}{3}+6x=0$
$\frac{x^2-3}{1}+2=\frac{x+1}{3}$	$\frac{4}{5}-\frac{x^2}{2}+\frac{2x+1}{4}$

2. **Square a number add 9, divide the result by 5. The result is equal to twice the number. Write an equation to represent this and solve the equation.**

3. **A prize is divided equally among five people.**
   If the same prize money is divided among six people each prize winner would get €2 less than previously. Write an equation to represent this and solve the equation.

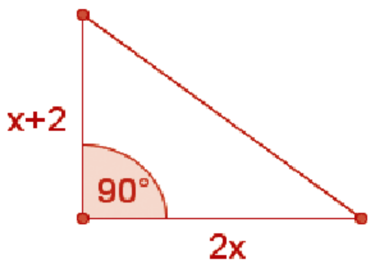
Section G: Student Activity 7

Higher Level Only

1. If the roots of the following quadratic equations are as follows write the equations in the form $(x - s)(x - t) = 0$:
   a. 2 and 3 b. 2 and -3 c. -2 and -3
   d. -2 and 3 e. 2 and ¾

2. What is the relationship between the roots of a quadratic equation and where the graph of the same quadratic cuts the x axis?

3. The area of the triangle below is $8cm^2$. Find the length of the base and the height of the triangle.



4. A field is in the shape of a square with side equal to x metres. One side of the field is shortened by 5 metres and the other side is lengthened by 6 metres. If it is known the area of the new field is $126m^2$, write an equation that represents the area of the new field. Solve the equation and find the dimensions of the original square field.

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5. a. Draw the graph of $f(x) = 3x^2 - 10x + 8$ for values between -3 and 3. Where does this graph cut the x axis?
   b. Factorise and solve $3x^2 - 10x + 8 = 0$
   c. What do you notice about the values you got for part a and part b?

6. Find the factors of $2x^2 - 5x + 3$. Hence solve $2x^2 - 5x + 3 = 0$