Teaching & Learning Plans

Plan 4: Outcomes of Coin Tosses

Junior Certificate Syllabus
Leaving Certificate Syllabus

Project Maths ©
Tionscadal Mata

Development Team

The Teaching & Learning Plans are structured as follows:

Aims outline what the lesson, or series of lessons, hopes to achieve.

Prior Knowledge points to relevant knowledge students may already have and also to knowledge which may be necessary in order to support them in accessing this new topic.

Learning Outcomes outline what a student will be able to do, know and understand having completed the topic.

Relationship to Syllabus refers to the relevant section of either the Junior and/or Leaving Certificate Syllabus.

Resources Required lists the resources which will be needed in the teaching and learning of a particular topic.

Introducing the topic (in some plans only) outlines an approach to introducing the topic.

Lesson Interaction is set out under four sub-headings:

i. **Student Learning Tasks – Teacher Input:** This section focuses on teacher input and gives details of the key student tasks and teacher questions which move the lesson forward.

ii. **Student Activities – Possible and Expected Responses:** Gives details of possible student reactions and responses and possible misconceptions students may have.

iii. **Teacher's Support and Actions:** Gives details of teacher actions designed to support and scaffold student learning.

iv. **Checking Understanding:** Suggests questions a teacher might ask to evaluate whether the goals/learning outcomes are being/have been achieved. This evaluation will inform and direct the teaching and learning activities of the next class(es).

Student Activities linked to the lesson(s) are provided at the end of each plan.

Teaching & Learning Plan 4: Outcomes of Coin Tosses

Aims

- To learn that by tossing two coins and observing the results (two heads, two tails, one head and one tail) not all results are equally likely

Prior Knowledge

Students should have prior knowledge (from T&L Plan 1 and/or from primary school) of some terms associated with chance and uncertainty. They should be familiar with probability expressed as fraction or decimal in the range 0 to 1, or as a percentage in the range 0% to 100%.

Learning Outcomes

As a result of studying this topic, students will be able to

- list the two possible outcomes when tossing a coin and calculate the probability of getting a head or a tail

- list all the possible outcomes when tossing 2 coins

- relate the number of outcomes to the fundamental principle of counting

- determine the probability of an event using the results of an experiment and use this to predict the result of a repetition of the experiment, for equally likely outcomes

- use the term 'independent events'

Relationship to Junior Certificate Syllabus

Sub-topics	Ordinary Level	Higher Level
1.5 Counting	List all possible outcomes of an experiment. Apply the fundamental principle of counting.	
1.6 Concepts of probability	Estimate probabilities from experimental data. Recognise that, if an experiment is repeated, there will be different outcomes and that increasing the number of times an experiment is repeated generally leads to better estimates of probability.	
1.7 Outcomes of simple random processes	Apply the principle that, in the case of equally likely outcomes, the probability is given by the number of outcomes of interest divided by the total number of outcomes.	Use binary/counting methods to solve problems involving successive events where only two possible outcomes apply to each event.

Relationship to Leaving Certificate Syllabus

Sub-topics	Foundation Level	Ordinary Level
1.1 Counting	List outcomes of an experiment. Apply the fundamental principle of counting.	
1.2 Concepts of probability		Estimate probability from experimental data; appreciate that, if an experiment is repeated, there will be different outcomes and that increasing the number of times an experiment is repeated generally leads to better estimates of probability. Associate the probability of an event with its long run frequency. Calculate expected value.

Resources Required

Coins for students.

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Lesson Interaction

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» Toss a coin. How many outcomes are there and what are they? » Fill in the Table on **Student Activity 1**.	• 2 – a head and a tail	» Distribute some coins to a few students first. » Distribute **Student Activity 1**.	
» Are the outcomes equally likely and why?	• Yes because of the symmetry of the coin.	» Emphasise that students must think about this before calculating probabilities.	
» Given that there are 2 possible outcomes, heads and tails, what is the probability of getting a head when you toss a coin? Probability of a tail?	• ½	» Circulate to see if students are filling in the table correctly and ask a student for the answer.	» Can all students fill out their tables correctly?
» Fill in on **Student Activity 1A.**	• ½		
» If you were to toss a coin 30 times predict and write down how many heads you would expect to get and how many tails. » Fill in on **Student Activity 1B.**	• 15 heads, 15 tails	» Circulate checking written predictions and then ask a student for an answer.	» Can students work out the expected value?

KEY: » next step   • student answer/response

Teaching & Learning Plan 4: Outcomes of Coin Tosses

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» Now, working in pairs, toss a coin 30 times and fill in the table on **Student Activity 1C**. You have a time limit of _____ .		» Distribute coins to the rest of the class. » Circulate and remind students about the time limit, checking progress.	» Are students keeping tallies correctly?
» Compare your result with your prediction and fill in **Student Activity 1 D**.	» Results should be close to predicted values.	» Circulate and look at tables of results, asking different groups how many heads and tails they got. » Enter results from the whole class on the board.	» Were most students able to make correct predictions?
» Fill results into your own master table **Student Activity 1E** and complete the table. » Check against your prediction. Complete in **Student Activity 1F**.	» Results should show closer agreement with predicted values.		
» Why are the results now closer to the predicted values?	• Because of the larger number of trials the relative frequency is approx. equal to probability.		» Do students see the connection between relative frequency and probability?
» If I get four heads in a row does that increase the chances of getting a tail next time? » Can you think of any other real life situations which have only 2 possible outcomes?	• No because the outcomes are independent – the outcome of one trial does not influence the outcome of any other trial.		» Do students understand that the events are independent of each other?

KEY: » next step   • student answer/response

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» Given 2 coins make a list of all the possible outcomes you could get using the table in **Student Activity 2A**.	• HH,TT,HT,TH	» Distribute **Student Activity 2**. » Circulate and check the list of outcomes, questioning where necessary.	» Do students see HT as a different outcome to TH? » Have they got 4 possible outcomes?
» Are all outcomes equally likely?	• Yes, as the coins are symmetrical	• Emphasise equally likely outcomes	

» Using the information from the table:
 	
  » what is the probability of getting 2 heads?
 	
  » the probability of getting 2 tails?
 	
  » the probability of getting a head and a tail?
 	
  » Fill in on **Student Activity 2B**.
	• Probability of 2 heads =¼
	
	• Probability of 2 tails = ¼
	
	• Probability of head and tail = 2/4 =1/2
			» Can students calculate the theoretical probability given equally likely outcomes?

• Predict and write down the outcomes you might get if you were to toss 2 coins simultaneously 28 times. (Why didn't I pick 30?)	• HH – 7 times • TT- 7 times • A head and a tail (HT or TH) – 14 times	» Circulate and check predictions asking for justifications.	

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» Working in pairs, toss 2 coins simultaneously 28 times and record your results in table (**Student Activity 2C**). » Your time limit is _____.	» Students should get approx twice as many H and T combinations as HH or TT.		» Can students see that there are 2 possible ways of getting a head and a tail?
» Do your results agree with your predictions?	» Results should be close to predictions.	» Ask a number of groups to call out their results while walking around check others.	» Have students been able to make correct predictions and justify them?
» Fill in results on master table **Student Activity 2D** from the board. » Are these results in agreement with your predictions?	» Results should now be closer to predictions.	» Enter results from the whole class on the board.	» Can students see that as the number of trials increases the relative frequency approaches the probability?
» The table only gives one option for heads and tails combination, while the original table showed HT and TH. Will this make a difference to the results? Why?	• Since one head and one tail can occur as HT or TH, the number of outcomes for HH and TT should be half that for a head and a tail.		
» How does this fit in with what you learned in the fundamental principle of counting?	• Coin 1 has 2 possible outcomes and coin 2 has 2 possible outcomes so total number of possible outcomes is 2x2=4.		» Can students relate back to the fundamental principle of counting?

KEY: » next step   • student answer/response

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» If you tossed 2 coins simultaneously 1000 times how many of each of the possible outcomes would you expect?	• HH—250 • TT—250 • A head and a tail – 500		» Are students able to calculate the expected value given the probability?
» Is a game 'fair' or 'unfair' if A wins if he gets two heads and B wins if the outcome is one head and one tail. » How would you make the game 'fair'?	• Unfair! B has twice the probability of winning. • Make it fair by allowing A to win for HH or TT.		» Do students recognise the need for a numeric representation of the phrase "probably won't"?
Reflection » Write down 3 things you learned about probability today. » Write down anything you found difficult. » Write down any questions you may have.	1. How to list all possible outcomes for tossing 1 coin and tossing 2 coins 2. How to use the list of all possible outcomes to judge fairness. 3. Calculate expected value for a large number of trials. 4. Note that the outcome of any one trial does not affect the outcome of any other trial – independent events.	» Circulate and take note particularly of any questions students have and help them to answer them.	» Have all students learned and understood these items? » Are they using the terminology with understanding and communicating with each other using these terms?

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Student Activity 1

Student Activity 1A
Tossing a coin

Possible outcome	Number of times it can appear

Probability of getting a head_____   Probability of getting a tail_____

Student Activity 1B
Tossing a coin 30 times

I predict that I will get _____ heads and _____ tails.

Student Activity 1C
Results of the experiment

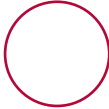| Outcome	Tally	Frequency	Relative frequency

Student Activity 1D
Do the results agree with my predictions?

_____

Student Activity 1E
Master table of class results

Outcome	Counts from each group	Frequency	Relative frequency
Heads			
Tails			

Student Activity 1F
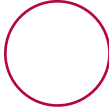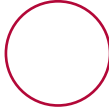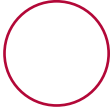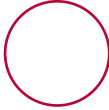## Do the results agree with my predictions?

_____
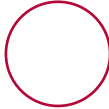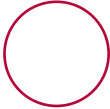
Student Activity 2

Student Activity 2C

Tossing 2 coins simultaneously: List all the possible outcomes in the table.

To help you visualise what's happening list the possible outcomes on the diagrams below.

Coin 1	Coin 2

Coin 1	H	◯	◯	◯
Coin 2	◯	◯	◯	◯

Student Activity 2C

Using the information from the table,

What is the probability of getting 2 heads?_____

What is the probability of getting 2 tails?_____

What is the probability of getting one head and one tail?_____?

Student Activity 2C
Table for experiment

Outcome	Tally	Frequency
HH		
HT or TH		
TT		

Student Activity 2C
Master table for experiment

Outcome	Frequency (from each group)	Total	Relative frequency
HH			
HT or TH			
TT			