

# Teaching & Learning Plans

## Plan 6: Planes and Points

Junior Certificate Syllabus

# The Teaching & Learning Plans are structured as follows:



**Aims** outline what the lesson, or series of lessons, hopes to achieve.

**Prior Knowledge** points to relevant knowledge students may already have and also to knowledge which may be necessary in order to support them in accessing this new topic.

**Learning Outcomes** outline what a student will be able to do, know and understand having completed the topic.

**Relationship to Syllabus** refers to the relevant section of either the Junior and/or Leaving Certificate Syllabus.

**Resources Required** lists the resources which will be needed in the teaching and learning of a particular topic.

**Introducing the topic** (in some plans only) outlines an approach to introducing the topic.

**Lesson Interaction** is set out under four sub-headings:

- i. **Student Learning Tasks – Teacher Input:** This section focuses on teacher input and gives details of the key student tasks and teacher questions which move the lesson forward.
- ii. **Student Activities – Possible and Expected Responses:** Gives details of possible student reactions and responses and possible misconceptions students may have.
- iii. **Teacher's Support and Actions:** Gives details of teacher actions designed to support and scaffold student learning.
- iv. **Checking Understanding:** Suggests questions a teacher might ask to evaluate whether the goals/learning outcomes are being/have been achieved. This evaluation will inform and direct the teaching and learning activities of the next class(es).

**Student Activities** linked to the lesson(s) are provided at the end of each plan.

# Teaching & Learning Plan 6: Planes and Points

## Aims

- To introduce students to the concepts: plane; points; lines; line segments; rays and length of line segments
- To enable them to develop an understanding of, and use, mathematical instruments

## Prior Knowledge

No prior knowledge of geometry assumed. Students should however know metric units of distance and be able to measure lengths.

## Learning Outcomes

As a result of studying this topic, students will be able to

- recall the origins of geometry
- recognise and have an intuitive knowledge of the plane, points, lines, line segments, rays and length of line segments
- have an interest in the subject and a desire to learn more

## Relationship to Junior Certificate Syllabus

Sub-topics	Ordinary Level
2.1 Synthetic geometry	<p>The geometrical results should be first encountered through discovery and investigation.</p> <p>(Refer to the appendix on geometry) Concepts (introduced as required) Constructions: 1, 2, 3, 4, 5, 6, 8, 9, 10, 11 Theorems: non-rigorous treatment of theorems 1 – 6.</p>

## Resources Required

A geometry set and a sheet of drawing paper for each student

## Introducing the Topic

Geometry is one of the oldest branches of mathematics, having arisen in response to such practical problems as those found in surveying, and its name comes from Greek words meaning “Earth measurement”. It is concerned with the shape of individual objects, the way that shapes relate to other objects and the properties of the surrounding space. So, we will be looking at lines, triangles, quadrilaterals and so on, which you may have already studied in Primary School.

The geometry which we will be doing is mostly concerned with FLAT surfaces. This is called PLANE GEOMETRY. We will look into work which is over 2000 years old and is called Euclidean Geometry, after a mathematician called Euclid (c. 300 B.C.) One of the first things you should do is to look up “Euclid” on the internet or in the library and see what you find. Other famous people who contributed to this before Euclid were Thales, Pythagoras, Plato and Aristotle.



Lesson Interaction			
Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
<ul style="list-style-type: none"> <li>» The first concept we meet in geometry is a 'plane'. This is just a flat surface like the top of the desk, or the teacher's board, except that it goes on and on forever in every direction. If you could imagine this, it would slice right through space and cut it in two. It is also a bit like the vast plains of Africa, but spelled differently.</li> <li>» Try to imagine your drawing sheet going on forever in every direction: this is what a plane is. Just imagine- going on forever...</li> <li>» Does anyone know a word we use for "going on forever"?</li> <li>» We use this symbol for infinity <math>\infty</math></li> <li>» You do not need to remember this just now, but do get used to symbols in maths and do not be afraid to use them.</li> <li>» Look around the room and see if you can pick out examples of a plane. What examples can you see?</li> </ul>		<ul style="list-style-type: none"> <li>» Begin a column headed "New Words" on the board (RHS) and write in the word "PLANE".</li> </ul>	
<ul style="list-style-type: none"> <li>» Try to imagine your drawing sheet going on forever in every direction: this is what a plane is. Just imagine- going on forever...</li> <li>» Does anyone know a word we use for "going on forever"?</li> <li>» We use this symbol for infinity <math>\infty</math></li> <li>» You do not need to remember this just now, but do get used to symbols in maths and do not be afraid to use them.</li> <li>» Look around the room and see if you can pick out examples of a plane. What examples can you see?</li> </ul>	<ul style="list-style-type: none"> <li>• Infinity.</li> </ul>	<ul style="list-style-type: none"> <li>• Give a sheet of drawing paper to each student.</li> <li>• Write the word 'infinity' on the board in the "New Words" list.</li> <li>• Write the <math>\infty</math> symbol on the board.</li> </ul>	
<ul style="list-style-type: none"> <li>» Look around the room and see if you can pick out examples of a plane. What examples can you see?</li> </ul>	<ul style="list-style-type: none"> <li>» Students look for examples of a plane in the room.</li> <li>• The ceiling, floor, wall....</li> </ul>		

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
<ul style="list-style-type: none"> <li>» This drawing sheet is the plane you will be working on.</li> <li>» Next we want to talk about the things which make up the plane. These are called 'points'.</li> <li>» A point is like a dot that you would make with your pencil, only even smaller.</li> <li>» On your sheet, mark in 10 dots anywhere at all on the page (well spread out).</li> <li>» We will call these points, even though strictly speaking a point has only got position; it does not have length, width or thickness.</li> <li>» Have you heard of a word for little elements which make up a digital photograph?</li> <li>» The word PIXEL stands for "picture elements". Our points are a bit like that; there are millions and millions of them in the plane. This is not something which you need to be concerned about, but just remember that a point has no size.</li> </ul>	<ul style="list-style-type: none"> <li>» Students spread out drawing sheet and get pencil and ruler ready for drawing.</li> <li>» Students mark in 10 dots on their sheets.</li> </ul>	<ul style="list-style-type: none"> <li>» Move around observing students' progress on the task.</li> <li>» Add the word "Points" to the "new words" column on the board.</li> </ul>	<ul style="list-style-type: none"> <li>» Do all the students know how to place points on the page?</li> </ul>
<ul style="list-style-type: none"> <li>» The word PIXEL stands for "picture elements". Our points are a bit like that; there are millions and millions of them in the plane. This is not something which you need to be concerned about, but just remember that a point has no size.</li> </ul>	<ul style="list-style-type: none"> <li>• Pixels.</li> </ul>	<ul style="list-style-type: none"> <li>» Now put 10 points on the board (main section) for extra clarity and further demonstration.</li> </ul>	

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
<ul style="list-style-type: none"> <li>» We denote points by capital letters A, B, C, etc.</li> </ul>			
<ul style="list-style-type: none"> <li>» Put "names" on your 10 points with capital letters (say, the 1st 10 letters of the alphabet).</li> </ul>	<ul style="list-style-type: none"> <li>» Students denote their 10 points by capital letters (A to J).</li> </ul>	<ul style="list-style-type: none"> <li>» Circulate observing students' progress on the task.</li> </ul>	<ul style="list-style-type: none"> <li>» Are students labelling points correctly with capital letters?</li> </ul>
<ul style="list-style-type: none"> <li>» Now, get your ruler and join up points A and B, and let your pencil go beyond the two points at each end until you come to the edge of the sheet at each end. This is what we call the line AB as you might have imagined.</li> </ul>	<ul style="list-style-type: none"> <li>» Students draw the line AB.</li> </ul>	<ul style="list-style-type: none"> <li>» Move about the room and observe the students at work, all the time checking for understanding.</li> </ul>	<ul style="list-style-type: none"> <li>» Are students drawing the line correctly?</li> </ul>
<ul style="list-style-type: none"> <li>» How far can this line go?</li> </ul>	<ul style="list-style-type: none"> <li>• It can go on forever in both directions. To infinity both ways.</li> </ul>		
<ul style="list-style-type: none"> <li>» Now write in "line AB" on this line.</li> </ul>	<ul style="list-style-type: none"> <li>» Students mark "line AB" or "BA" on their line.</li> </ul>	<ul style="list-style-type: none"> <li>» Move about the room evaluating the work visually.</li> </ul>	<ul style="list-style-type: none"> <li>» Are students using capital letters?</li> </ul>
<ul style="list-style-type: none"> <li>» Notice that the name of the line does not have any brackets, just two letters naming two points on the line. You may reverse the order and call it line BA if you wish.</li> </ul>		<ul style="list-style-type: none"> <li>» Draw the line AB on the board so that the pupils can compare it with their work.</li> </ul>	<ul style="list-style-type: none"> <li>» Do students understand that line AB is the same as line BA?</li> </ul>

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
<ul style="list-style-type: none"> <li>» The line can be extended in either direction for as far as we wish. (We call this infinity). We name a line by any two points through which it passes.</li> <li>» Could I draw a different line through A and B?</li> <li>» So this is the only line which can be drawn through A and B.</li> </ul>	<ul style="list-style-type: none"> <li>• No.</li> </ul>	<ul style="list-style-type: none"> <li>» Mark "line AB" on the line on the board and add it also to the new words list.</li> </ul>	<ul style="list-style-type: none"> <li>» Are students actively participating in drawing and showing understanding?</li> </ul>
<ul style="list-style-type: none"> <li>» Now mark in the point P anywhere on this line. It can be between A and B or it need not be.</li> </ul>	<ul style="list-style-type: none"> <li>» Students mark in point P on the line AB.</li> </ul>	<ul style="list-style-type: none"> <li>» Move about the room and observe the students at work, all the time checking for understanding.</li> </ul>	
<ul style="list-style-type: none"> <li>» Now it is possible to name this line in several other ways. Can you name them?</li> </ul>	<ul style="list-style-type: none"> <li>• AP or PA.</li> <li>• BP or PB.</li> </ul>	<ul style="list-style-type: none"> <li>» Mark in point P on the line AB on the board.</li> </ul>	
<ul style="list-style-type: none"> <li>» Since the points A, P and B are in the same (straight) line, we call them COLLINEAR points.</li> </ul>		<ul style="list-style-type: none"> <li>» Add the words "COLLINEAR POINTS" to the new words list on the board.</li> </ul>	



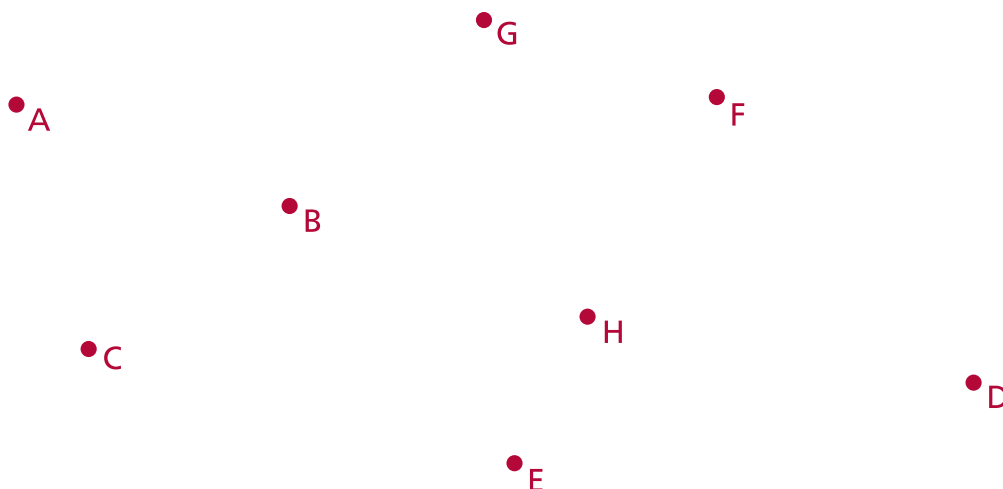
Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
<ul style="list-style-type: none"> <li>» Now with your pencil and ruler, starting at C, draw in a line passing through D and going on as far as you like.</li> <li>» This is called the HALF-LINE CD or simply the RAY [CD].</li> <li>» Notice how the square bracket indicates a starting point and there is no bracket at the other side, indicating that there is no end- it keeps going on and on (to infinity).</li> <li>» Write in the name on the 'ray' on your page.</li> <li>» Is the ray [DC the same set of points?</li> <li>» So we must be careful of the order in which we write the name of a ray.</li> </ul>	<ul style="list-style-type: none"> <li>» Students draw the ray [CD.</li> <li>» Students write in "ray [CD" on the ray.</li> <li>• No, because this starts at D and goes through C in the opposite direction.</li> </ul>	<ul style="list-style-type: none"> <li>» Move about the room and observe the students at work, all the time checking for understanding.</li> </ul>	<ul style="list-style-type: none"> <li>» Do the students' drawings and labelling of points show any misconceptions?</li> </ul>

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
<ul style="list-style-type: none"> <li>» Now draw some more rays starting at C going in different directions. (Say 5 more). Remember your rays do not need to pass through any of your points.</li> <li>» Can you think why these might be called rays?</li> </ul>	<ul style="list-style-type: none"> <li>• Students draw in 5 rays emanating from C.</li> <li>• They look a bit like rays of the sun.</li> </ul>	<ul style="list-style-type: none"> <li>» Add the words "RAY [CD]" to the new words list on the board.</li> <li>» Draw the ray [CD] on the board.</li> <li>» Write in "ray [CD]" on the ray on the board.</li> <li>» Having observed progress, draw in 5 rays emanating from C on the blackboard.</li> </ul>	<ul style="list-style-type: none"> <li>» Are students able to draw and label all lines, rays and points successfully?</li> </ul>
<ul style="list-style-type: none"> <li>» Now with your pencil and ruler join up the points E and F, this time starting at E and stopping at F.</li> </ul>	<ul style="list-style-type: none"> <li>» Students draw the line segment [EF].</li> </ul>		
<ul style="list-style-type: none"> <li>» This is part of the line EF and is called the line segment [EF], since it is really part of the full line EF. A segment is a "part of", like a segment of an orange is part of it.</li> <li>» We name this [EF]. Notice how the square brackets show a "beginning" and an "end".</li> <li>» Write "line segment [EF]" on it.</li> </ul>	<ul style="list-style-type: none"> <li>» Students write "line segment [EF]" on the line segment.</li> </ul>		

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
<ul style="list-style-type: none"> <li>» Now, for extra practice, draw the lines GH and JI. Remember to extend your lines beyond the naming points.</li> </ul>	<ul style="list-style-type: none"> <li>» Students draw the lines GH and JI.</li> </ul>	<ul style="list-style-type: none"> <li>» Draw [EF] on the board and write "line segment [EF]" on the line segment on the board.</li> <li>» Add the words "LINE SEGMENT [EF]" to the new words list on the board.</li> </ul>	
<ul style="list-style-type: none"> <li>» When you have that done, draw in the line segment [BJ] and the ray [DE].</li> </ul>	<ul style="list-style-type: none"> <li>» Students draw the line segment [BJ] and the ray [DE].</li> </ul>	<ul style="list-style-type: none"> <li>» Observe the students' progress on this task, add these extra elements to the diagram on the board.</li> </ul>	<ul style="list-style-type: none"> <li>» Can all students see the difference between Line AB, Ray [AB and line segment [AB] and  AB , the length of [AB] and the way brackets are used to name them?</li> </ul>
<ul style="list-style-type: none"> <li>» Now one last task. Use your ruler to measure the length of line segment [AB].</li> <li>» This is written as  AB .</li> <li>» Write in <math> AB  = \text{"value"}</math> on the [AB].</li> </ul>	<ul style="list-style-type: none"> <li>» Students measure the length of the line segment [AB] and write <math> AB  = \text{"value"}</math> on it.</li> </ul>	<ul style="list-style-type: none"> <li>» Measure the line segment [AB] on the board and write <math> AB  = \text{"value"}</math> on it.</li> </ul>	
<ul style="list-style-type: none"> <li>» Fill into your copies the list of new words from the board.</li> <li>» Fill in <b>Student Activity 1</b>.</li> </ul>	<ul style="list-style-type: none"> <li>» Students write the list of new words into their copies.</li> </ul>	<ul style="list-style-type: none"> <li>» Distribute <b>Student Activity 1</b>.</li> </ul>	

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
<p><b>Reflection</b></p> <ul style="list-style-type: none"> <li>» Write down 3 things you learned about planes and points today.</li>   <li>» Write down anything you found difficult.</li>   <li>» Write down any questions you may have.</li> </ul>	<ol style="list-style-type: none"> <li>1. Understand the words plane and points.</li> <li>2. How to label points with capital letters.</li> <li>3. How to draw a Ray [AB and a line segment [AB].</li> <li>4. How to use brackets for a Ray [AB, a line segment [AB] and  AB , the length of [AB].</li> </ol>	<ul style="list-style-type: none"> <li>» Circulate and take note particularly of any questions students have and help them to answer them. Have all students learned and understood these items?</li> </ul>	

# Student Activity 1



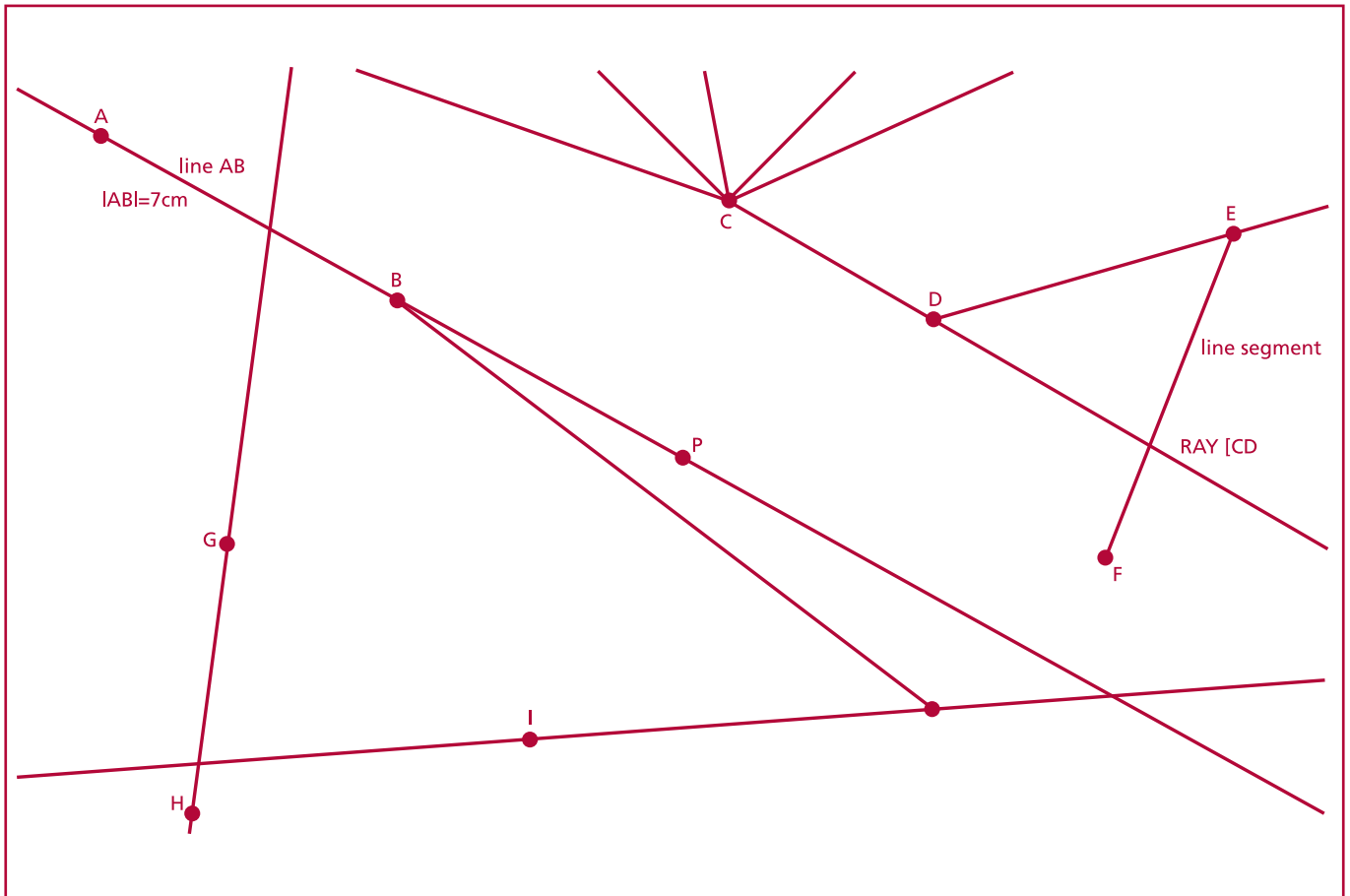
1. Join A to C. This is [AC]. Measure the length of [AC]. Write your answer in cm. and mm. \_\_\_\_\_. How do we name the length of [AC]? \_\_\_\_\_
2. Join D to E. This is [DE]. Measure the length of [DE]. Write your answer in cm. and mm. \_\_\_\_\_. How do we name the length of [DE]? \_\_\_\_\_
3. Draw line AB.
4. Draw ray [DE.
5. How big is a point? \_\_\_\_\_
6. Write down the symbol for infinity. \_\_\_\_\_
7. How many points are on a line? \_\_\_\_\_
8. How many points do you need to know to define a line? \_\_\_\_\_

Answer true/false to the following questions.

9. Line AB contains H. \_\_\_\_\_
10. Line CD contains B \_\_\_\_\_
11. Line AB and F lie in the same plane \_\_\_\_\_
12. A, B and H are collinear \_\_\_\_\_
13. C,D and B are collinear. \_\_\_\_\_
14. More than one line can be drawn through E and H? \_\_\_\_\_
15. More than one line can be drawn through B and D? \_\_\_\_\_
16. CG intersects EG at G \_\_\_\_\_
17. Points A,B,C,D,E,F,G,H all lie in the same plane which I will call X \_\_\_\_\_
18. Points A,B,C,D,E,F,G,H are the only points in plane X. \_\_\_\_\_
19. [CH is the same as [HC. \_\_\_\_\_
20. Line AB is the same as line BA. \_\_\_\_\_

# Appendix

## An example of the Teacher's Board at the end of class



## NEW WORDS

**Plane**

**Line AB**

**Line segment [EF]**

**infinity**

**Collinear points**

**Points**

**Ray [CD]**