Teaching & Learning Plans

Plan 9: The Unit Circle

Leaving Certificate Syllabus

Project Maths
Tionscadal Mata
Development Team

The Teaching & Learning Plans are structured as follows:

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Aims outline what the lesson, or series of lessons, hopes to achieve.

Prior Knowledge points to relevant knowledge students may already have and also to knowledge which may be necessary in order to support them in accessing this new topic.

Learning Outcomes outline what a student will be able to do, know and understand having completed the topic.

Relationship to Syllabus refers to the relevant section of either the Junior and/or Leaving Certificate Syllabus.

Resources Required lists the resources which will be needed in the teaching and learning of a particular topic.

Introducing the topic (in some plans only) outlines an approach to introducing the topic.

Lesson Interaction is set out under four sub-headings:

i. **Student Learning Tasks – Teacher Input:** This section focuses on teacher input and gives details of the key student tasks and teacher questions which move the lesson forward.

ii. **Student Activities – Possible and Expected Responses:** Gives details of possible student reactions and responses and possible misconceptions students may have.

iii. **Teacher's Support and Actions:** Gives details of teacher actions designed to support and scaffold student learning.

iv. **Checking Understanding:** Suggests questions a teacher might ask to evaluate whether the goals/learning outcomes are being/have been achieved. This evaluation will inform and direct the teaching and learning activities of the next class(es).

Student Activities linked to the lesson(s) are provided at the end of each plan.

Teaching & Learning Plan 9: The Unit Circle

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Aims

- To enable students to become familiar with the unit circle
- To use the unit circle to evaluate the trigonometric functions sin, cos and tan for all angles

Prior Knowledge

Students should be able to plot and read coordinates on a Cartesian plane. They should recognise that the circle, as the locus of all points equidistant from a given point, the centre, is uniquely defined by its centre and radius. Students should be familiar with the concept of a function as a one to one or many to one mapping, and with the domain and range of a function. Students should recall the theorem of Pythagoras and be able to calculate sin x, cos x, and tan x from the right-angled triangle, i.e. the trigonometric ratios (SOHCAHTOA) and know that $\tan x = \frac{\sin x}{\cos x}$

Learning Outcomes

As a result of studying this topic, students will be able to

- associate the coordinates of points on the circumference of the unit circle with the cos and sin of the angle made by the radius containing these points, with the positive direction of the x-axis
- use the reference angle to calculate the sin, cos and tan of any angle θ, where $0° \leq \theta \leq 360°$
- find values of sin, cos and tan of negative angles and of angles >360° from the unit circle
- solve equations of the type $\cos x = \pm\frac{1}{2}$

Relationship to Leaving Certificate Syllabus

Sub-topics	Ordinary Level	Higher Level
2.2 Coordinate Geometry	Recognise that $x^2+y^2=r^2$ represents the relationship between the x and y co-ordinates of points on a circle centre (0,0) and radius r.	
2.3 Trigonometry	Define sin x and cos x for all values of x. Define tan x.	Solve trigonometric equations such as $a \sin n\theta=0$ and $\cos n\theta=½$, giving all solutions.

Resources Required

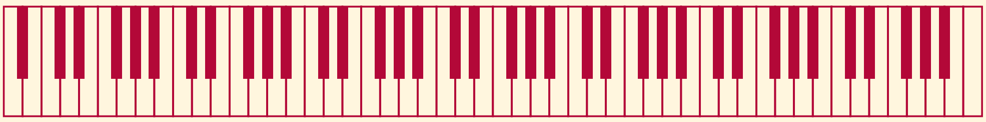
Compasses, protractors, clear rulers, pencils, formulae and tables booklet, Geogebra, Autograph, Perspex Model of Unit Circle (last 3 desirable but not essential).

Introducing the Topic

Trigonometric functions are very important in science and engineering, architecture and even medicine. Surveying is one of the many applications. Bridge builders and road makers all use trigonometry in their daily work.

The mathematics of sine and cosine functions describes how many physical systems behave. As a playground Ferris wheel rotates, the height above a central horizontal line can be modelled by a sine function. The displacement of the prongs of a vibrating tuning fork from the rest position is described by a sine function. The resulting displacement with time of a weight attached to a spring when it is displaced slightly from its rest position is described by a sine function. Tidal movements and sound waves are two other vibrations described by sine functions. Anything that has a regular cycle (like the tides, temperatures, rotation of the earth, etc) can be modelled using a sine or cosine function. The piston engine is the most commonly used engine in the world. Its motion can be described using a sine curve. http://www.intmath.com/Trigonometric-graphs/2_Graphs-sine-cosine-period.php (Scroll to the bottom of the web page to get the piston applet.)

When students have drawn the graph of sin x, it would then be useful to show the applet on the piston as it may not make much sense initially. Alternatively, show it initially and again after **Student Activity 7** when it should make more sense.



The fundamental background of trigonometry finds usage in an area which is a passion for many people i.e. music. Sound travels in waves and in developing computer generated music, combinations of sine and cosine functions are used to generate the sounds of different musical instruments. Hence sound engineers and technologists who research advances in computer music and hi-tech music composers have to relate to the basic laws of trigonometry.

Techniques in trigonometry are used in navigation particularly satellite systems and astronomy, naval and aviation industries, oceanography, land surveying, and in cartography (creation of maps).

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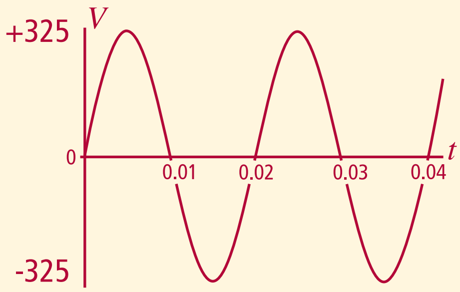

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Lesson Interaction

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» What 2 items of information do we need to define a circle?	• Centre and radius	» Distribute **Student Activity 1**. (Unit Circle)	
» Given a unit circle, what distinguishes the unit circle from all other circles? » Note that the radius is 1 unit; watch out for a reason why this might be useful.	• It has a radius of 1 and a centre (0, 0), and is drawn on a Cartesian plane.		
» Identify the 4 quadrants. What direction do we move in going from the first to the fourth quadrant?	• Anticlockwise.		
» How would you describe points on the circumference of the circle?	• Points on the circumference can be described by an ordered pair (x, y)	• Encourage use of correct terminology.	
» Read the Cartesian coordinates of points in each quadrant. Note the signs of the coordinates in each quadrant, and fill in **Student Activity 1A**.	• Students fill in the coordinates and note the signs in the different quadrants.		
Note: Emphasise to students that it is important to note the sign of x and y in each quadrant for future reference in the lesson.			

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Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» An angle in standard position has its vertex at the origin, the positive direction of the x-axis as the initial ray and the other ray forming the angle which intersects the circumference of the circle is the terminal ray. » When the terminal ray is rotated in an anticlockwise direction from the initial ray a positive angle is formed.			
» On the Unit Circle **Student Activity 1A**, using a ruler and protractor, draw an angle of 30° in standard position, with the terminal ray longer that the radius. Mark the point Q where the terminal ray intersects the circumference.	» Students draw an angle of 30° on the unit circle.	» Demonstrate on the board if students have difficulty with the terminology. Important to get used to it as it simplifies descriptions later on.	
» How would you draw an angle of -30°?	• By rotating the terminal ray by 30° in a clockwise direction.		
» Answer the questions on **Student Activity 2A**.		» Distribute **Student Activity 2.**	» Can students apply trigonometric ratios?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» What have you discovered about the x and y coordinates for an angle of 30° on the unit circle?	• The x and y coordinates represent cos 30° and sin 30°.	» Walk around and check that **Student Activity 2A** is being filled. When all the sheets are filled ask individual students for their answers.	
» What is the significance now of the radius of the circle being 1?	• The sin 30° =opp/hyp and cos 30° = adj/hyp • hypotenuse = radius, and, $\lvert r \rvert = 1$, sin 30°= opp = y and cos 30° = adj = x		
» Can you generalise this for any angle θ<90°? » Complete **Student Activity 2B**.	» Students complete **Student Activity 2B** for any angle θ in the first quadrant and show that sin θ =y/1, cos θ=x/1, tan θ =y/x		
» The coordinates of any point on the unit circle may be written as (x, y) the Cartesian coordinates, or as (cos θ, sin θ).			» Do students understand that the x and y coordinates of points on the unit circle are equal to the cos and sin of the angle θ made by the radius containing these points, with the positive direction of the x-axis?

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Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» What are the signs for sin, cos and tan of an angle in the first quadrant and why? Fill in **Student Activity 2C**.	• All positive, since the x and y coordinates are both positive in the first quadrant.	» Ask the class and then an individual to explain. Ask the class if they agree with the answers and if not to explain why not.	» Do students know that sin, cos and tan are positive for angles in the first quadrant?
» Using the unit circle, how would you get the cos 60° and sin 60°? Check using a calculator.	» Students make an angle of 60° in standard position and read the coordinates. cos 60°= x and sin 60°= y	» Distribute **Appendix A** (Unit Circle).	» Did students use trig. ratios first, or were they now confident in using (x, y) for (cos A, sin A)?
» What are the sin, cos and tan of 0° and 90° from what you have learned? Fill in **Student Activity 2D**.	» Students read the coordinates on the circumference for 0° and 90° from the Unit Circle and fill in the table on **Student Activity 2D**.		
» What did you notice about tan 90°?	• It's undefined, due to division by zero.		
» Using the calculator, find the tan 89°, tan 89.999°, tan 89.99999°. What do you notice?	• Tan increases very rapidly as the angle tends to 90°.		
» Working in pairs write in short points a summary of what you have learned.		» Ask different students to read out points made in their summaries - concept of the unit circle; angle in standard position; initial ray and terminal ray; forming a right angled triangle, in the first quadrant with the origin as a vertex. » Write the points on the board. » Distribute **Student Activity 3**.	» Were students able to summarise what they had done?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» How do we define sin and cos for angles between 90° and 180° as we don't form right angled triangles using these angles?	• Use the unit circle coordinates.		
» Fill in **Student Activity 3A**. » Using the unit circle, find the sin 150°, cos 150° and tan 150°. Check answers using a calculator	» Students read sin 150° and cos 150° from the unit circle. Confirm using the calculator. Compare the results with sin 30° and cos 30°.	» Check answers. Ask individuals.	» Did everyone get the same answers for sin 150°, cos 150° and tan 150° from the coordinates as from the calculator?
» Can you form a right angled triangle in the second quadrant with an angle at the origin with the same values of sin and cos as 150°?	• Drop a perpendicular from Q' to the x-axis forming an acute angle A at the origin in the second quadrant. (**Student Activity 3B**)		
» Fill in **Student Activity 3B**.	» Students fill in **Student Activity 3B**.	» Check values students are filling in on **Student Activity 3B**.	
» Find the sin A and cos A of the angle A (not in standard position) using trig ratios. What do you notice?	• Angle A has the same sin and cos as 150°.		
» How do you calculate the size of A?	• $A = 180° – 150° = 30°$		

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» Angle A is called the reference angle. Can you describe it? » Fill in **Student Activity 3C**.	• It is the acute angle between the terminal ray of an angle in standard position and the x-axis.		
» What transformation could you use to form a congruent triangle to OB'Q' in the first quadrant?	• Axial symmetry in the y-axis gives congruent triangle OBQ, with angle A at the vertex.		
» Hence what is the relationship between the ratio of sides in triangle OB'Q' and the ratio of the sides of its image in the y- axis ?	• They are numerically equal.		
» What is the relationship between the trig ratios for triangle OB'Q' and triangle OBQ?	• They are numerically equal.		
» What is the same and what is different about sin 30° and cos 30° and sin 150° and cos 150°?	• sin 150° = sin 30° and cos 150° = – cos 30°		» Are students able to calculate the value of sin 150° and cos 150° using the reference angle and the appropriate signs?
» Follow this line of reasoning for any angle in the second quadrant, complete **Student Activity 3D**.	» Students fill in **Student Activity 3D**.	» Circulate and check **Student Activity 3D** as it is being filled. When filled, ask different students to read out the answers and ask class if they agree and if not to justify their answer.	» Are students able to calculate the sin, cos and tan of any angle in the second quadrant using the reference angle and correct signs?

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Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» Fill in **Student Activity 4**.	» Students fill in **Student Activity 4**. » Students should avoid errors in calculating the reference angle if they refer to the drawing of the unit circle rather than just memorising a formula.	» Distribute **Student Activity 4**.	» Are students able to calculate the sin, cos and tan of any angle in the third quadrant using the reference angle and correct signs?
» Fill in **Student Activity 5**.	» Students fill in **Student Activity 5**.	» Remind students to always draw a small diagram of the unit circle with the summary information when dealing with the trig. functions.	» Are students able to calculate the sin, cos and tan of any angle in the fourth quadrant using the reference angle and correct signs?
» http://www.wou.edu/~burtonl/trig.html http://www.mathsisfun.com/geometry/unit-circle.html (Scroll to the bottom of the web page to see the applet on reference angle)		» Show students Java applets.	» Do Java applets contribute to understanding?
» Fill in the summary on **Student Activity 6A**.	• Students fill in **Student Activity 6A**.	» Distribute **Student Activity 6**.	
» Find the sin (–210°). We only dealt with positive values for angles before. What is the difference between positive and negative angles?	• Positive angles represent anticlockwise rotations and negative angles are clockwise rotations.		

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Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» Can you suggest a strategy for dealing with evaluating the trig functions for negative angles? » In which quadrant is – 210°? » What is the sign of sin in the second quadrant? » What positive angle is equal to – 210°? » What is the reference angle for – 150°? » What is the sin (– 210°) equal to?	• Draw the unit circle. • Second quadrant • Positive • 360° – 210° = 150° • 30° • ∴sin (– 210)° = + sin 30°.	» Ask students to come up with the strategy themselves first. » Having given them a few minutes, ask students for the steps and show each step on the board.	» Can students come up with the strategy by themselves, given that they are used to drawing the unit circle? » Can students calculate sin, cos and tan for negative angles?
» Do the exercises on **Student Activity 6B** on negative angles.			
» Can you suggest a strategy for finding trig functions of angles greater than 360°? For example cos 910°? » Do the exercise in **Student Activity 6C** on angles greater than 360°.	• Draw the unit circle. • Each rotation of 360° is equivalent to an angle of 0°. • 910°= 2(360°) + 190° = 190°, which is in the third quadrant where cos is negative and the reference angle • 190° – 180°= 10°. • cos 910° = – cos 10°.	» Ask students to come up with the strategy themselves first. » Given the students a few minutes then asks students for the steps and show each step on the board.	» Can students calculate sin, cos and tan for angles greater than 360°?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» In which quadrants can sin θ be positive? » In which quadrants can sin θ be negative? » In which quadrants can cos θ be positive? » In which quadrants can cos θ be negative? » The previous activities concentrated on finding the reference angle A, for a given angle θ in each of the 4 quadrants. We will now try to find θ, knowing the value of A.	• First and second • Third and fourth • First and fourth • Second and third	» Refer students back to the summary diagram, **Student Activity 6A**. » Ask individual students.	• Are students clear on the signs of the trigonometric functions in the different quadrants?
» Given a reference angle A, how many values of θ could this reference angle have?	• 4	» Distribute **Student Activity 7**. » Ask an individual student and ask the whole class if they agree.	
» Refer to the diagrams on **Student Activity 7A**.	• Students fill in the formulae in **Student Activity 7A**.	» Walking around. Check each student's work and then ask individuals to call out his/her answers for class agreement/disagreement with justification.	

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Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» We are now going to solve the equation in **Student Activity 7B** using the summary of the unit circle and formulae on **Student Activity 7A**.	• Students work in pairs, discussing the problem.		
$\sin\theta = \frac{1}{\sqrt{2}}, 0° \le \theta \le 360°$ » What are we trying to find out here?	• We are trying to find all the angles θ where $\sin\theta = \frac{1}{\sqrt{2}}$	» Teacher refers students to their summary on **Student Activity 6A**.	» Have students been able to write θ in terms of A in **Student Activity 7B**?
» How do you know which quadrants the angle θ could belong to?	• Sin is positive, therefore θ could be in the first or the second quadrant.	» Refer students back to **Student Activity 6A**.	
» How will you find the reference angle A?	• The reference angle is the acute angle whose sin is $\frac{1}{\sqrt{2}}$ i.e. 45°		» When students get a problem, are they first clarifying what the problem is asking?
» Could the reference angle have a negative trigonometric function associated with it? Explain.	• Reference angle cannot have a negative trig function associated with it as it is always an acute angle.		» Are students using the unit circle effectively to solve this problem?
» Knowing the reference angle, how do you find the angle θ?	• Use the formulae for each quadrant. First quadrant: $\theta = A = 45°$, Second quadrant: $180° - \theta = A$ $\therefore \theta = 180° - A = 135°$		
» Solve the above equation i.e. $\sin\theta = \frac{1}{\sqrt{2}}$, for all values of θ. Use what you learned about trig functions of angles > 360°.	• $\therefore \theta = 45° + n360°$ or $\theta = 135° + n360°, n \in Z$		» Are students using what they have learned previously about angles greater than 360°?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
Reflection » Write down 3 things you learned about trigonometry today. » Write down anything you found difficult. » Write down any questions you may have.	1. Concept of the Unit Circle 2. Angle in standard position 3. Initial ray and terminal ray 4. Forming a right angled triangle in the first quadrant with the origin as a vertex 5. Using trig ratios, and discovering that the x and y coordinates represent the cos and sin of an angle 6. cos 150° and sin 150° can be got from the x and y coordinates of where the terminal ray of the angle meets the circumference of the circle. 7. A right angled triangle with the terminal ray as the hypotenuse in the second quadrant gives angle A at the origin in the right angled triangle, which has the same cos and sin as 150°. 8. 150° in the second quadrant has a reference angle in the first quadrant with the same magnitude of sin and cos but with a different sign for cos. 9. The same line of reasoning for angles in the third and fourth quadrants. 10. Solving trig equations for example $\sin\theta = \frac{1}{\sqrt{2}}$.	» Circulate and take note particularly of any questions students have and help them to answer them.	» Have all students learned and understood these items? » Are they using the terminology with understanding and communicating with each other using these terms?

Student Activity 1

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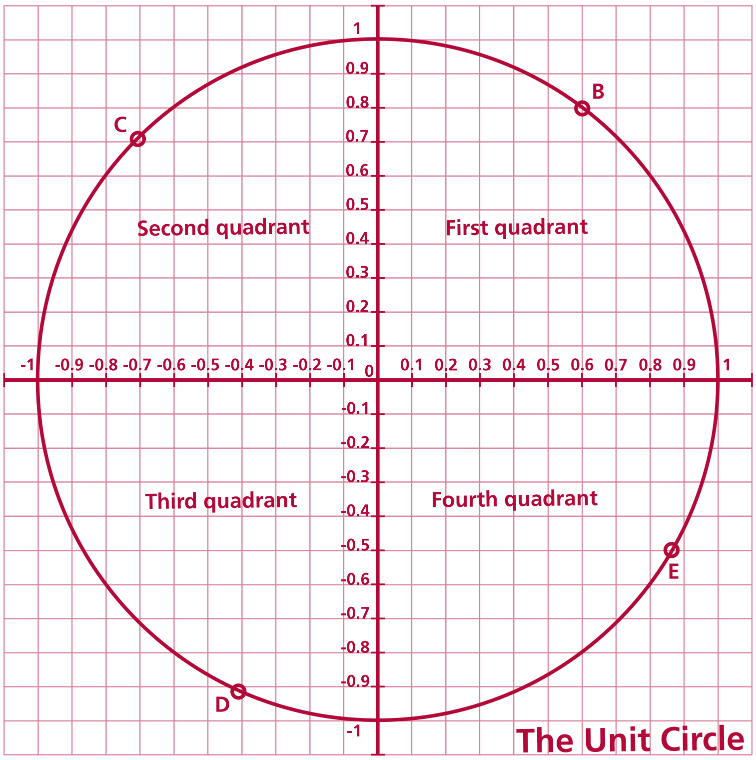
Student Activity 1A



The unit circle – A circle whose centre is at (0,0) and whose radius is 1

Any point on the circumference of the circle can be described by an ordered pair (x,y).

The coordinates of B are (0.6, 0.8)

What are the coordinates of C, D, and E?

C = ________________, D = ________________, E = ________________.

In which quadrant are both x and y positive? ________________________________

In which quadrant is x negative and y positive? ________________________________

In which quadrant is x positive and y negative? ________________________________

In which quadrant is x negative and y negative? ________________________________

Angles in standard position - the vertex is at the origin, with the initial ray as the positive direction of the x-axis, and the other ray forming the angle is the terminal ray.

Student Activity 1B

Angles in the first quadrant

Draw an angle of 30° in standard position on the unit circle on Student Activity Sheet 1A. Mark the initial ray and the terminal ray. Label the point where the terminal ray meets the circumference as θ.

The coordinates of θ are ____ .

Student Activity 2

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Student Activity 2A

Drop a perpendicular from Q to the x-axis to construct a right angled triangle, centred at (0, 0).

What is the length of the hypotenuse? ______________________

What is the length of the opposite? ______________________

What is the length of the adjacent? ______________________

Using trigonometric ratios, (not a calculator), calculate the sin 30°, cos 30°and the tan 30°.

sin 30° =________ cos 30° =________ tan 30° =________

Compare these with the values of the x and y coordinates of Q. What do you notice about the x and y coordinates of Q and the trigonometric functions sin 30°,cos 30° and tan 30°?

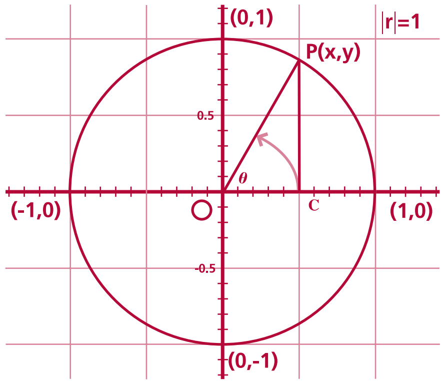
Check the answers using a calculator.

sin 30° =________ cos 30° =________ tan 30° =________

Student Activity 2B

Application to any angle in the first quadrant $0° < \theta < 90°$

sin θ =________ cos θ =________ tan θ =________



Student Activity 2C

What signs will sin, cos and tan have in the first quadrant ? Why have sin, cos and tan got these signs in the first quadrant?

Trigonometric function	Sign in the first quadrant
sin	
cos	
tan	

____

Student Activity 2D

Mark angles of 0° and 90° degrees in standard position on the unit circle, and from what you have just learned, without using a calculator, write down the sin, cos, and tan of 0° and 90°.

A/°	0°	90°
Coordinates on the unit circle		
cos A		
sin A		
tan A		

Student Activity 3

Student Activity 3A

Angles in the second quadrant $90° < \theta < 180°$

On the unit circle on Appendix A, mark an angle of 150° in standard position. Read the x and y coordinates of the point Q', where the terminal ray intersects the circumference.

(x, y) of point Q' are ______________________________

Using what you have learned about the coordinates of points on the circumference of the unit circle, fill in the following:

cos 150° = ________ sin 150° = ________ tan 150° = ________

Check these values using the calculator.

cos 150° = ________ sin 150° = ________ tan 150° = ________

Compare with

cos 30° = _________ sin 30° = _________ tan 30° = _________

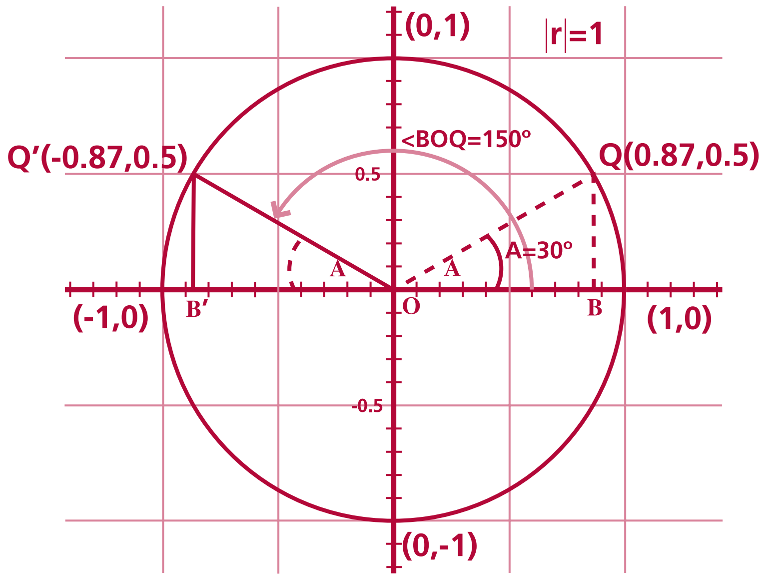
Student Activity 3B

Drop a perpendicular from point Q', to the negative direction of the x- axis, to make a right angled triangle, with angle A at the origin.

What is the value of A in degrees?

____

____



Using the trigonometric ratios on triangle OB'Q', what is sin A =________ cos A =________

Student Activity 3C

A is called the reference angle. Describe the reference angle?______________________

____

____

Student Activity 3D

What do you notice about the sin and cos of the reference angle and the sin 150° and cos 150°? ____________________

What is the image of triangle OB'Q' by reflection in the y- axis? ____________________

What is the relationship between triangle OB'Q' and its image in the y- axis? ____________________

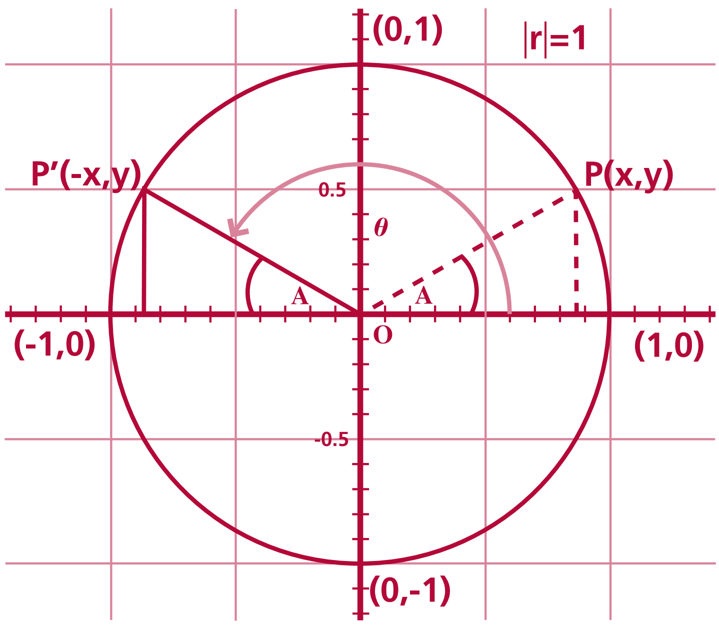
____

Hence what is the relationship between ratio of sides in triangle OB'Q' and the ratio of the sides of its image in the y- axis? ____________________

Therefore 150° in the second quadrant has a reference angle of ___ in the first quadrant

sin 150° =________ sin 30° =________ cos 150° =________ cos 30° =________

Application to any angle in the second quadrant



sin θ = ______________

cos θ = ______________

tan θ = ______________

sin A = opp/hyp = _____ (A in the 2nd quadrant)

cos A = adj/hyp = _____ (A in the 2nd quadrant)

sin A = _____ (A in the 1st quadrant)

cos A = _____ (A in the 1st quadrant)

Express A in terms of θ and 180° ____________________ Equation (i)

Write down the relationship between sin θ in the second quadrant and sin A in the first quadrant ____________________

Rewrite the answer using equation (i) above ____________________

Write down the relationship between cos θ and the second quadrant cos A in the first quadrant ____________________

Rewrite the answer using equation (i) above ____________________

Write down the relationship between tan θ in the second quadrant and the tan A in the first quadrant____________________

Rewrite the answer using equation (i) above ____________________

Student Activity 3D

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Fill in the signs for cos and sin and tan of an angle in the second quadrant.

Trigonometric function	Sign in the second quadrant
sin	
cos	
tan	

Using the reference angle, how would you calculate the sin 130°, cos 130°, tan 130°?

Using the reference angle, how would you calculate the sin 110°, cos 110°, tan 110°?

Using the reference angle, how would you calculate the sin 170°, cos 170°, tan 170°?

Mark an angle of 180° degrees in standard position on the unit circle, and using coordinates, not a calculator, write down the sin, cos, and tan of 180°. Check the answers using a calculator.

A/°	180°
Coordinates on the unit circle	
cos A	
sin A	
tan A	

Student Activity 4

Angles in the third quadrant $180° < \theta < 270°$

On the unit circle on Appendix A, mark an angle of 210º in standard position. Read the x and y coordinates of the point Q", where the terminal ray intersects the circumference.

(x, y) of point Q" are _____ Hence: cos 210° =______ sin 210° =______ tan 210° =______

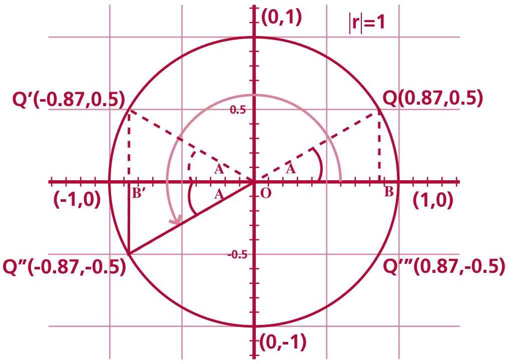
Check these values using the calculator. cos 210° =_____ sin 210° =_____ tan 210° =_____

Compare with cos 30° =________ sin 30° =________ tan 30° =________

Is there a relationship between trig functions of angles in the first and third quadrants?

____

Explanation of the relationship



Drop a perpendicular from point Q", to the negative direction of the x- axis, to make a right angled triangle OB'Q". What is the value of A in degrees?_

Using the trigonometric ratios on triangle OB'Q", what is sin A =________ cos A =________

A is called the Reference angle. Describe the reference angle? ________________________________

What do you notice about the sin and cos of the reference angle and sin 210º and cos 210°? ____

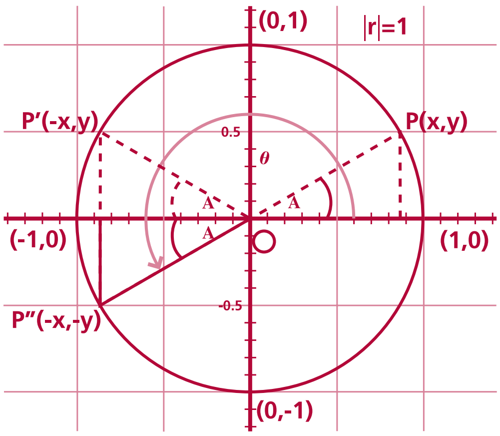
What is the image of triangle OB'Q" by S_0? ______________________________

What is the relationship between triangle OB'Q" and its image by S_0?_______________

Hence what is the relationship between ratio of sides in triangle OB'Q" and the ratio of the sides of its image by S_0? ______________________________

Therefore 210º in the third quadrant has a reference angle of_____in the first quadrant.

sin 210º = _______ sin 30º= _______, cos 210º = _______ cos 30º= _______



Application to any angle in the third quadrant.

sin θ = ______________

cos θ = ______________

tan θ = ______________

sin A = opp/hyp = _____ (A in the 3rd quadrant)

cos A = adj/hyp = _____ (A in the 3rd quadrant)

sin A = _____ (A in the 1st quadrant)

cos A = _____ (A in the 1st quadrant)

Student Activity 4

Express A in terms of θ and 180°. ____equation (i)

Write down the relationship between sin θ and the sin A in the third quadrant and then rewrite the answer using equation (i) above.____

____

____

____

Write down the relationship between cos θ and the cos A in the third quadrant and then rewrite the answer using equation (i) above. ________________________________

____

____

____

Write down the relationship between tan θ and the tan A in the third quadrant and then rewrite the answer using equation (i) above.________________________________

____

____

____

Fill in the signs for cos and sin and tan of an angle in the third quadrant.

Trigonometric function	Sign in the third quadrant
sin	
cos	
tan	

Using the reference angle, calculate the sin, cos and tan of 220°. ____________________

____

____

____

Mark an angle of 270° degrees in standard position on the unit circle, and using coordinates, not a calculator, write down the sin, cos, and tan of 270°. Check the answers using a calculator.

A/°	270°
Coordinates on the unit circle	
cos A	
sin A	
tan A	

Student Activity 5

Angles in the fourth quadrant $270° < \theta < 360°$

On the unit circle on Appendix A, mark an angle of 330° in standard position. Read the x and y coordinates of the point Q‴, where the terminal ray intersects the circumference.

(x, y) of point Q‴ are _______________

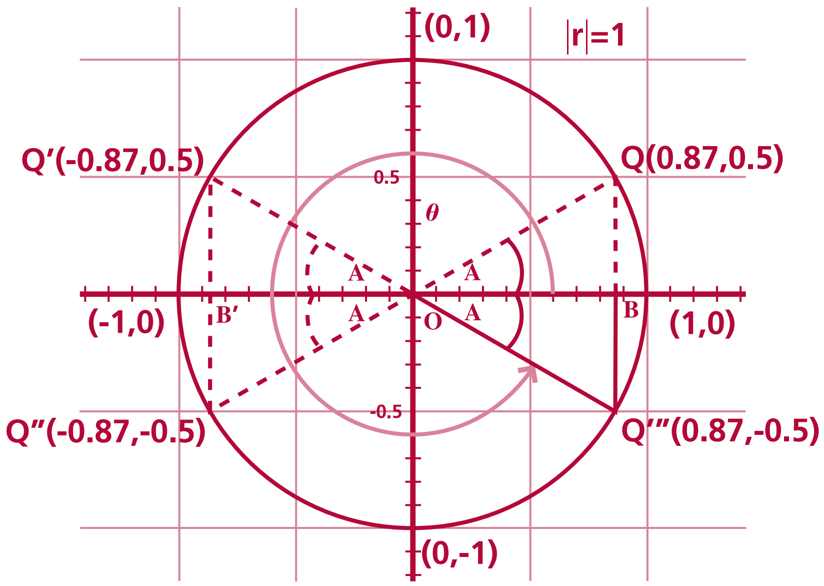
Hence: cos 330° =_______ sin 330° =_______ tan 330° =_______

Using the calculator: cos 330° =_______ sin 330° =_______ tan 330° =_______

Compare with: cos 30° =_______ sin 30° =_______ tan 30° =_______

Is there a relationship between trig functions of angles in the first and fourth quadrants? ____

Explanation of the relationship



Drop a perpendicular from point Q‴, to the negative direction of the x- axis, to make a right angled triangle OBQ‴. What is the value of A in degrees?_____________

Using the trigonometric ratios on triangle OBQ‴,

sin A =__________

cos A =__________

A is called the reference angle. Describe the reference angle?_______________________

____

What do you notice about the sin and cos of the reference angle and that of 330°?____________

What is the image of triangle OBQ‴ by S_x?________________________

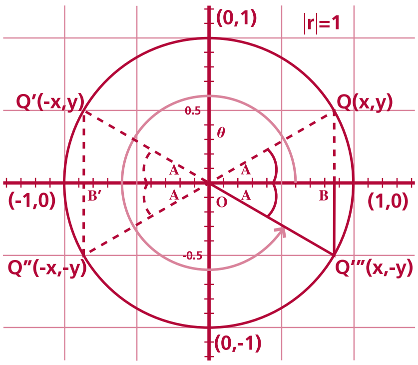
What is the relationship between triangle OBQ‴ and its image by S_x?_______________

Hence what is the relationship between ratio of sides in triangle OBQ‴ and the ratio of the sides of its image in the y- axis?____

Therefore 330° in the fourth quadrant has a reference angle of _____ in the first quadrant

sin 330° = _______ sin 30°= _______ cos 330° = _______ cos 30°= _______

Student Activity 5



Application to any angle in the fourth quadrant.

sin θ = ______________

cos θ = ______________

tan θ = ______________

sin A = opp/hyp = ____ (A in the 4th quadrant)

cos A = adj/hyp = ____ (A in the 4th quadrant)

sin A = ____ (A in the 1st quadrant)

cos A = ____ (A in the 1st quadrant)

Express A in terms of θ and 360°.____equation (i)

Write down the relationship between sin θ and the sin A in the fourth quadrant and then rewrite the answer using equation (i) above ____________________________________

____

Write down the relationship between cos θ and the cos A in the fourth quadrant and then rewrite the answer using equation (i) above.____________________________________

____

Write down the relationship between tan θ and the tan A in the fourth quadrant and then rewrite the answer using equation (i) above.____________________________________

____

Fill in the signs for cos and sin and tan of an angle in the fourth quadrant.

Trigonometric function	Sign in the fourth quadrant
sin	
cos	
tan	

Using the reference angle, calculate the sin, cos and tan of 315°, writing the answers in surd form.____

____

Mark an angle of 360° degrees in standard position on the unit circle, and using coordinates, not a calculator, write down the sin, cos, and tan of 360°. Check the answers using a calculator.

A/°	360°
Coordinates on the unit circle	
cos A	
sin A	
tan A	

Student Activity 6

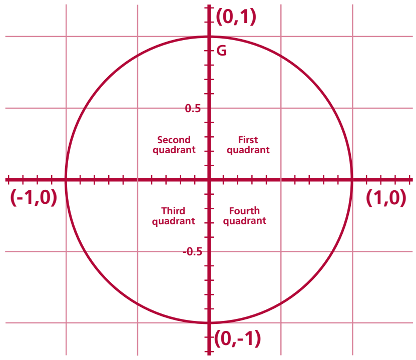
Student Activity 6A

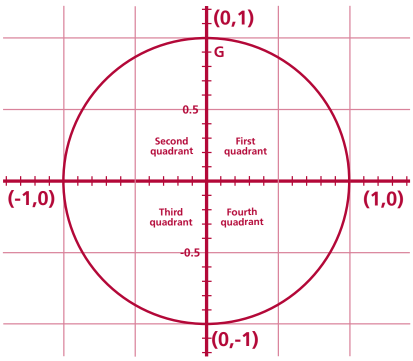
Summary on finding trig functions of all angles

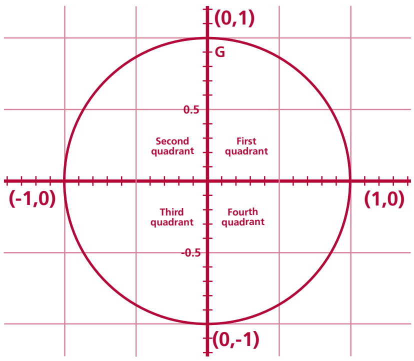
Fill in on the unit circle, in each quadrant, the first letter of the trigonometric function which is positive in each quadrant.

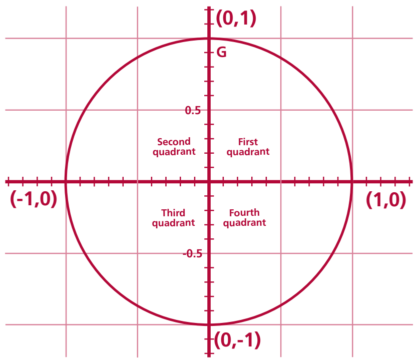
Mark in an angle θ and its reference angle A for each quadrant. Use a different unit circle for each situation.

Fill in also in each quadrant, the formula for the reference angle given an angle θ in that quadrant.









Student Activity 6B

Negative angles

Find, without using the calculator. Show steps.

sin (–120º) ____

cos (–120º) ____

tan (–120º) ____

Student Activity 6C

Angles greater than 360º

Find, without using the calculator. Show steps.

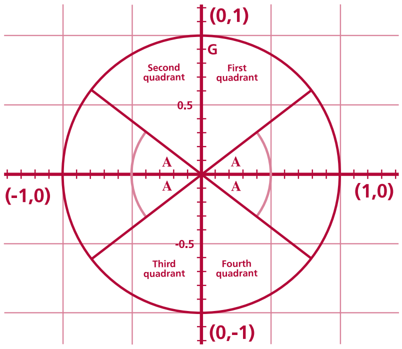
sin 450º ____________________ **sin 1250º** ____________________

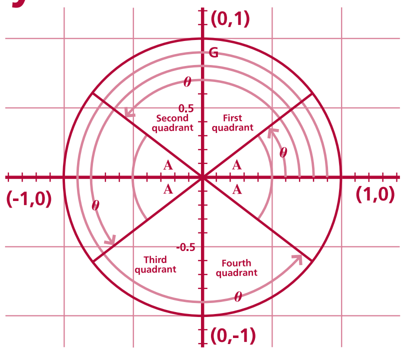
cos 450º ____________________ **cos 1250º** ____________________

tan 450º ____________________ **tan 1250º** ____________________

Student Activity 7

Student Activity 7A





If we know A, how do we calculate θ? Always draw a unit circle.

Quadrant in which terminal ray of θ lies	Ist	2nd	3rd	4th
Formula for A (reference angle)	A=	A=	A=	A=
Formula for θ in terms of A	θ=	θ=	θ=	θ=

Student Activity 7B

Solve the equation cos $\theta = \frac{1}{\sqrt{2}}$ where $0° < \theta < 360°$ (use Tables)

What are we trying to find? ____

____

In what quadrants could θ be located and why? ______________________________

____

As the reference angle is acute what sign will the trig functions of reference angles have? ____

____

What is the value of the reference angle (the acute angle with this value of cos)?

____

____

Knowing the reference angle, what are the possible values of θ? ____________________

____

____

Solve the equation sin $\theta = -\frac{\sqrt{3}}{2}$

Appendix A

Project Maths
Tionscadal Mata
Development Team

The Unit Circle