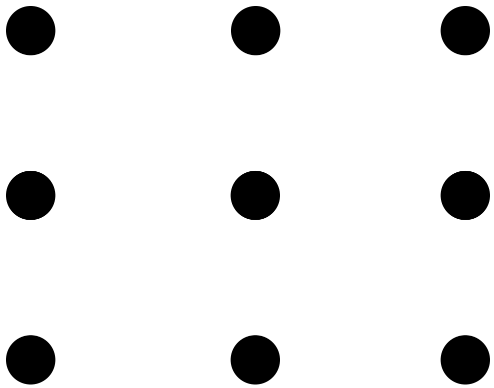
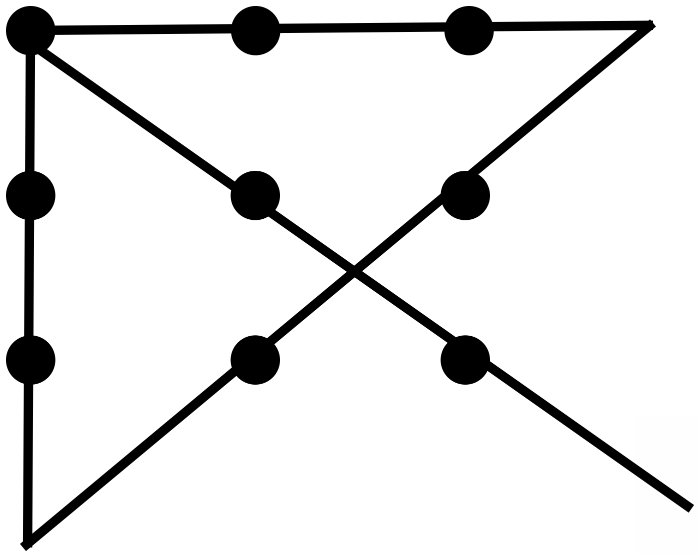
Warm up

- Connect these nine dots with only four straight lines without lifting your pencil from the paper.

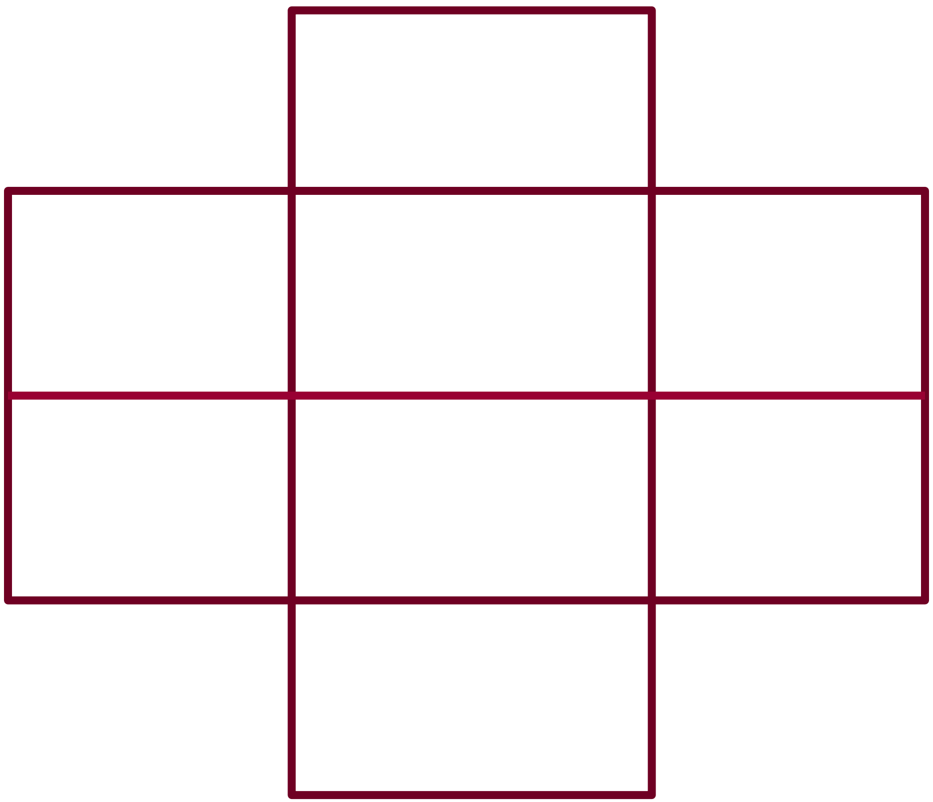


Warm up Solution

Sometimes we need to think outside the box!



Warm up

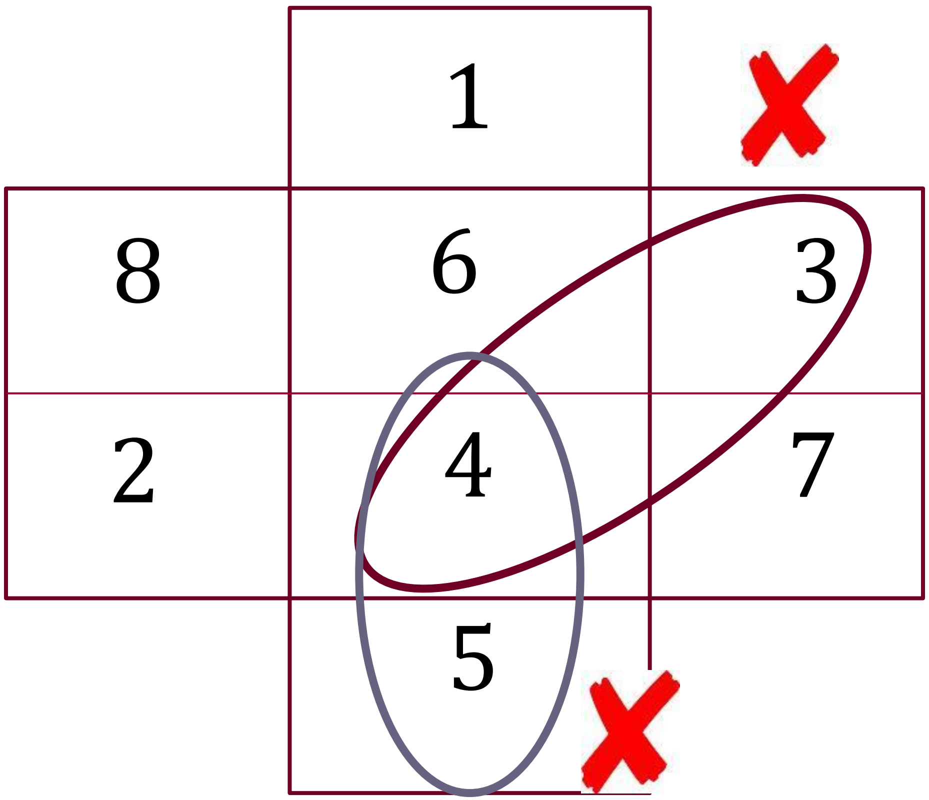


Insert the Numbers 1 – 8 into the boxes provided.

Consecutive numbers cannot be beside, adjacent, or diagonal to each other.

Example:

Warm up

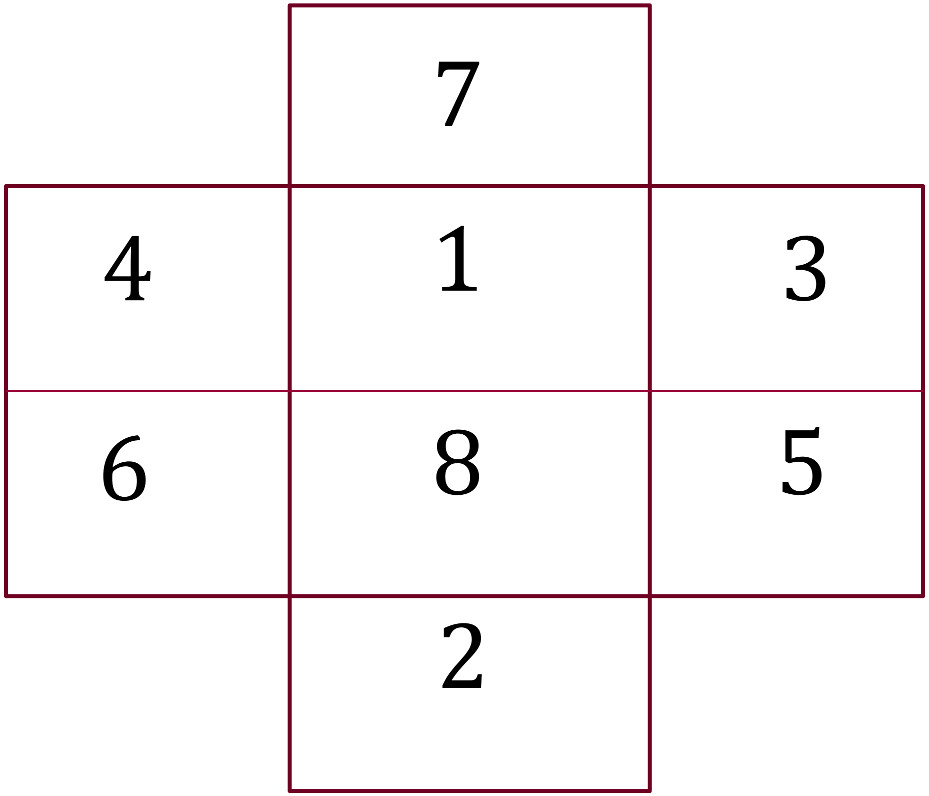


Insert the Numbers 1 – 8 into the boxes provided.

Consecutive numbers cannot be beside, adjacent, or diagonal to each other.

Example:

Warm up Solution



Insert the Numbers 1 – 8 into the boxes provided.

Consecutive numbers cannot be beside, adjacent, or diagonal to each other.

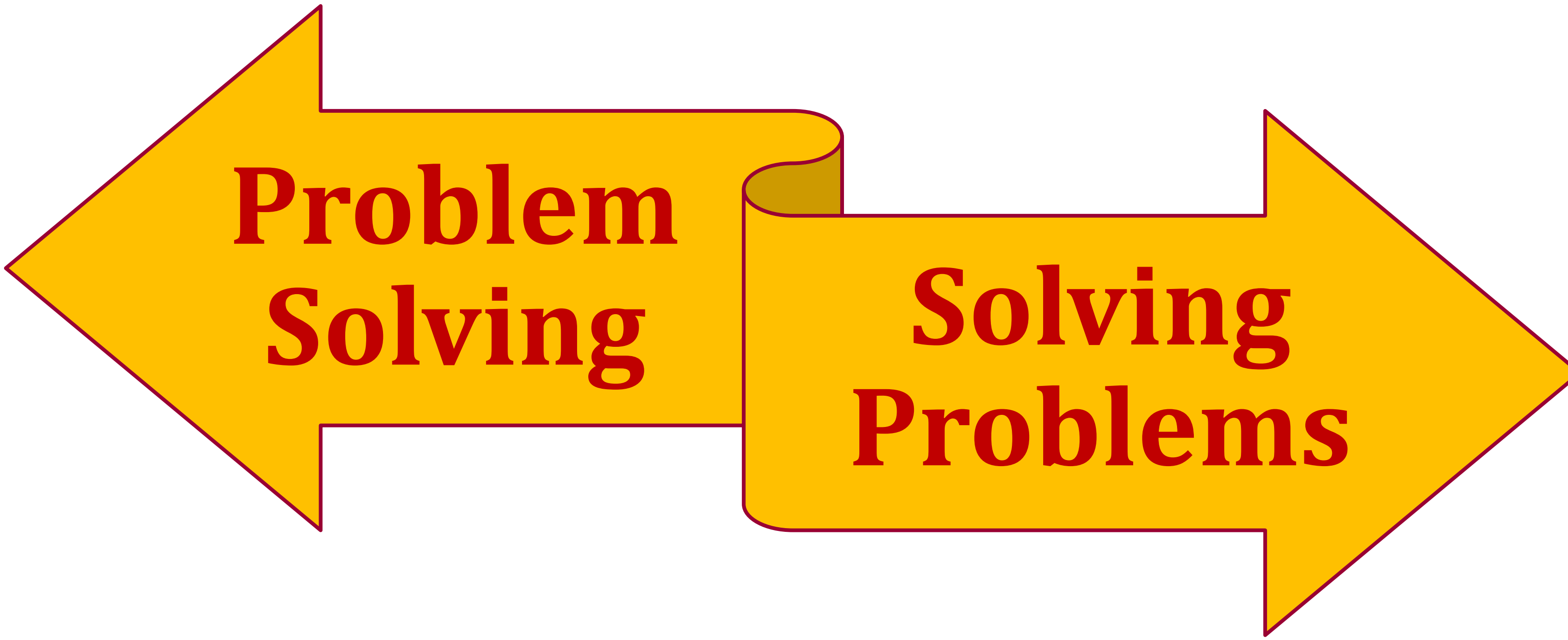
Problem Solving

Addressing misconceptions during teaching does actually improve achievement and long-term retention of mathematical skills and concepts.

Drawing attention to a misconception before giving the examples was less effective than letting the pupils fall into the 'trap' and then having the discussion.

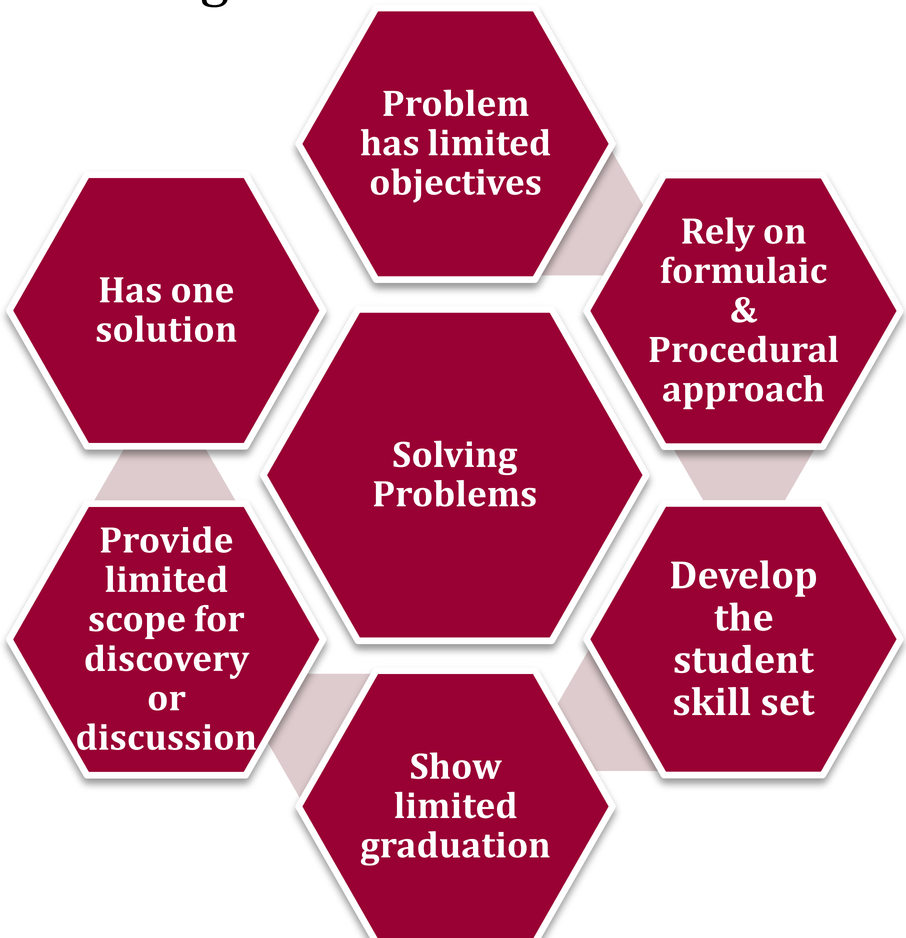
(Askew and Wiliam 1995)

What's the difference?



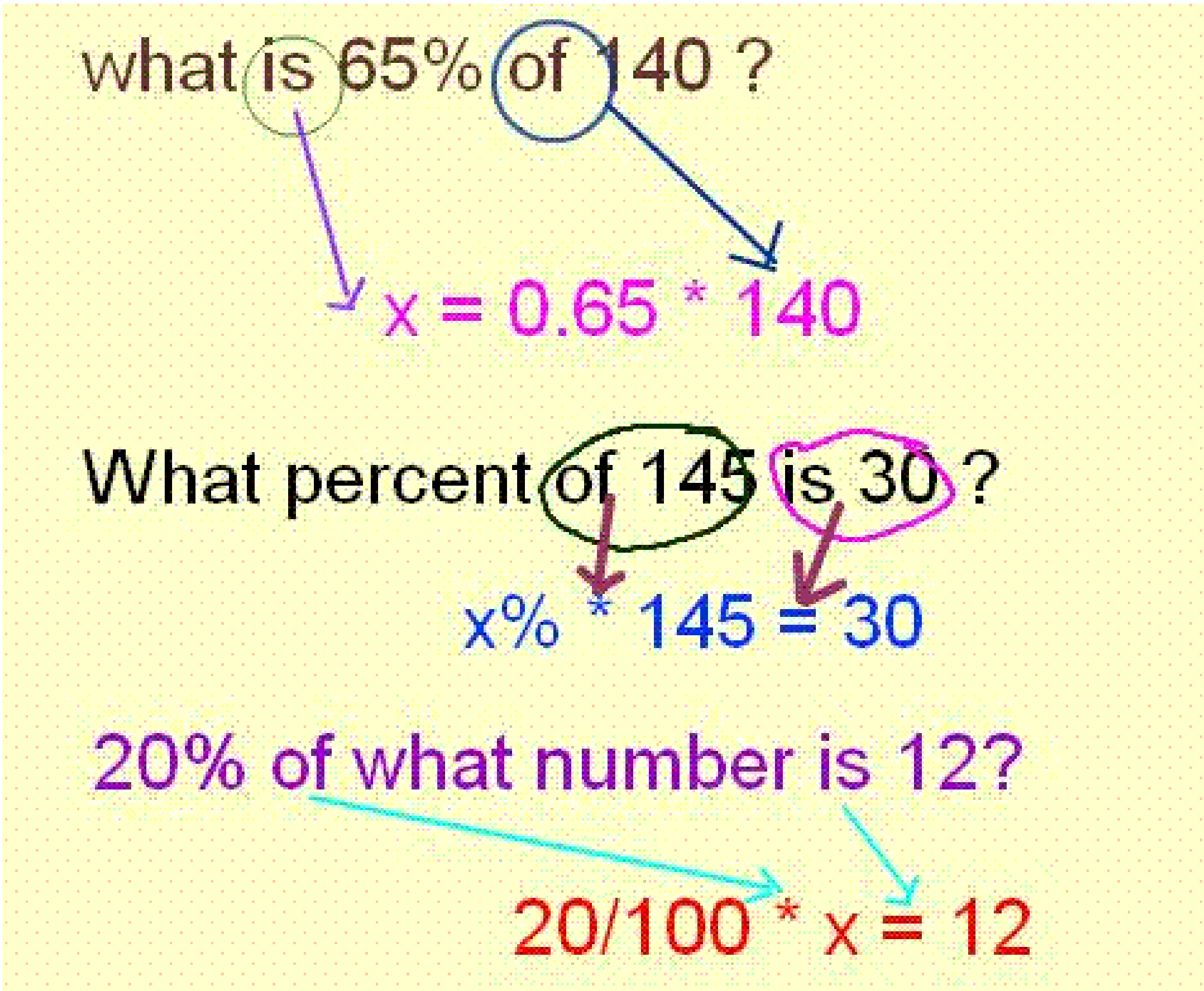
Solving Problems

- Solving Problems
  - Everyday and essential activity in Mathematics Classes
  - Features of existing classroom tasks



text

Solving Problems



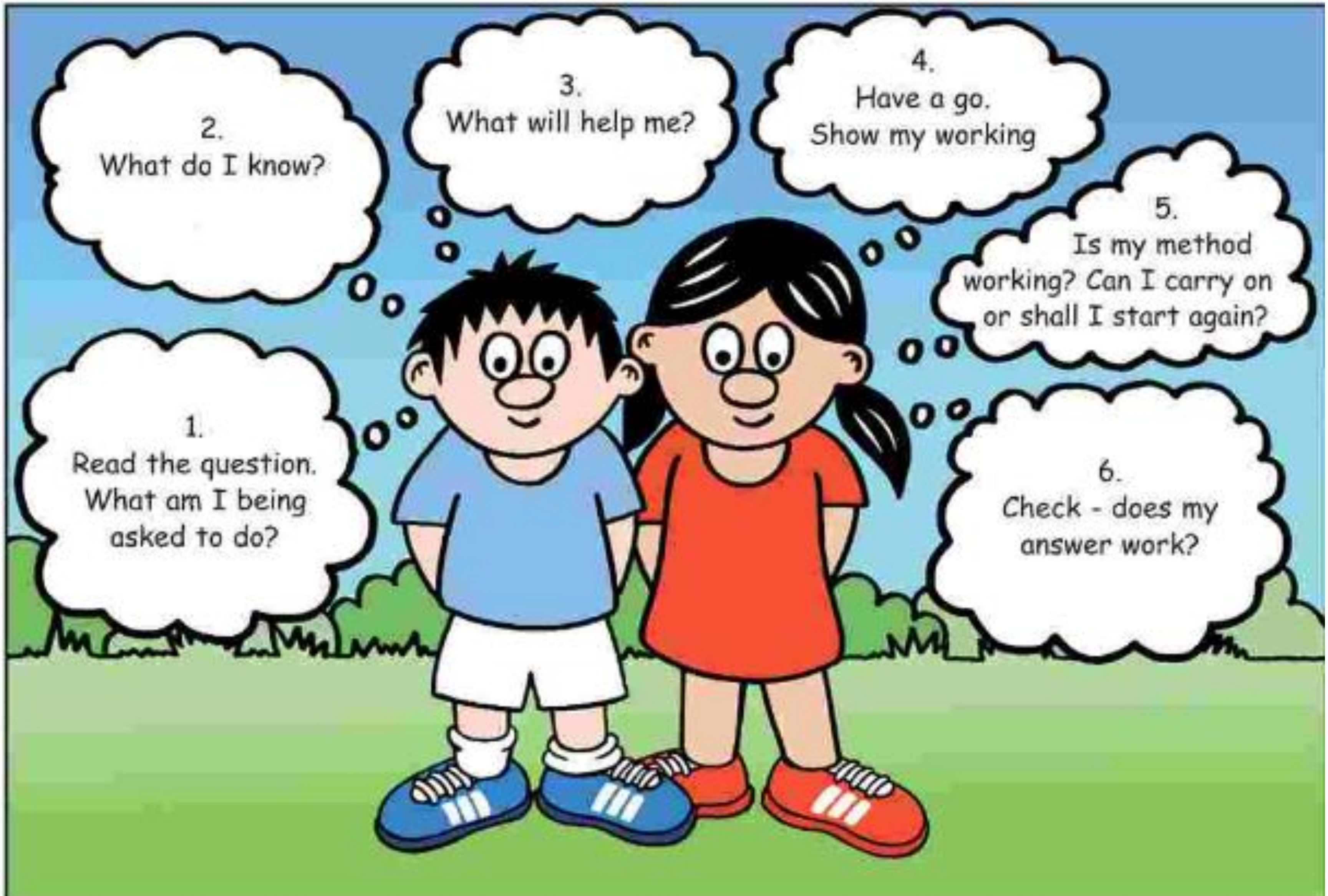
Problem Solving

Apollo 13: Fit a square peg into a round hole

CLICK TO PLAY VIDEO

Link: http://vimeo.com/61144423

Problem Solving



Mathematical Problem Solving

"People are generally better persuaded by the reasons which they have themselves discovered than by those which have come in to the mind of others."

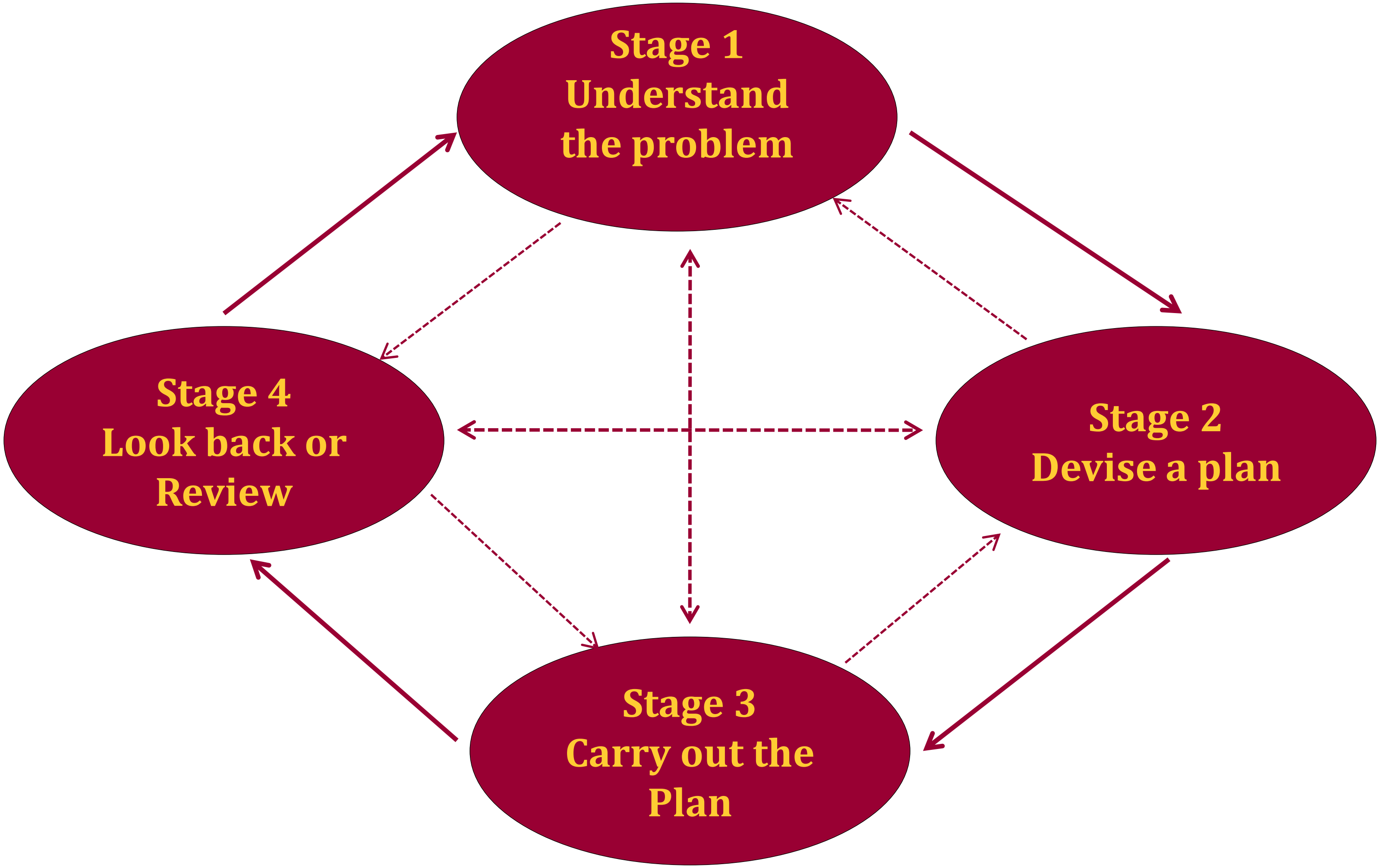
Blaise Pascal

Link: http://www.thocp.net/biographies/papers/pensees.htm

Is problem solving the same for us all?

What is a problem for one student may not be a problem for another!!

Four Stage Problem Solving Process



(Pólya, 1945)

Understanding the Problem

- Is the problem well defined?
- Do you understand all the words used in stating the problem?
- What are you asked to find or show?
- Can you restate the problem in your own words?

- Can you think of a picture or a diagram that might help you
- understand the problem?
- Is there enough information to enable you to find a solution?
- Do you need to ask a question to get the answer?

Devise a Plan

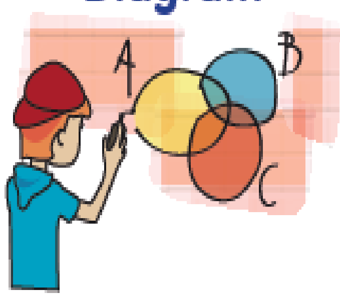
- Draw a picture
- Solve a simpler problem
- Use a model
- Work back-wards
- Use a formula
- Use your head
- Be creative
- Guess & Check
- Make a list
- Eliminate possibilities
- Consider special cases
- Use direct reasoning
- Solve an equation
- Look for a pattern

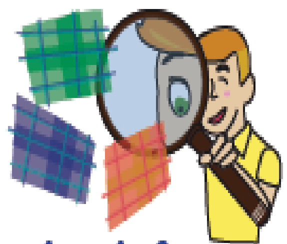
text

Problem Solving Strategies

Trial and Improvement

Draw a Diagram



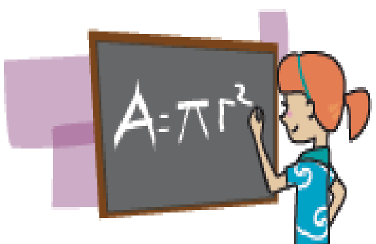


Look for a Pattern

Act It Out

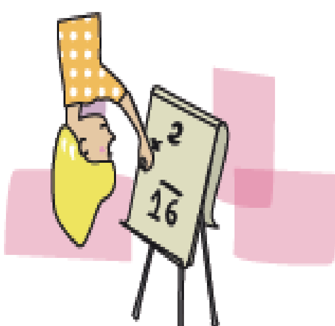
Draw a Table

Simplify the Problem



Use an Equation

Work Backwards



Eliminate Possibilities

Carry out the Plan

Monitor progress to make sure that things go according to plan.

Check each step

Record errors

Try a new plan if this plan fails

Review your Plan

- Can you check the result?
- Try to understand why you succeeded/failed?
- Reflect and look back at what you have done
- What worked and what didn't work.

Proof and Numbers

Proof

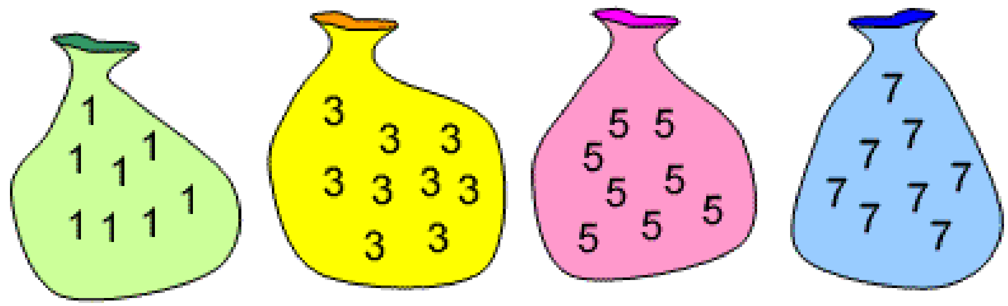
- Proof can be used to motivate or revise areas of the curriculum.
- Look to develop and apply proof in areas other than geometry.

Use of a counter example

Prove that the statement "a four sided figure with all sides equal in length must be a square" is untrue.

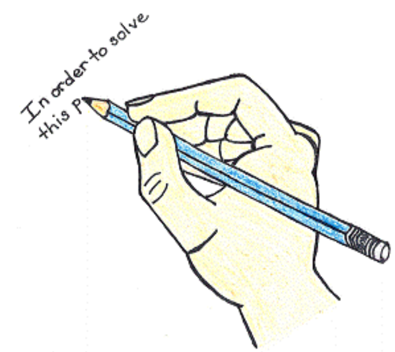
Activity 1 [Exploring Numbers]

Four bags contain a large number of 1's, 3's, 5's and 7's.



Pick any 10 numbers from the bag so that their sum equals 37.

Justify your solution.



Activity 1 [Exploring Numbers]

Four bags contain a large number of 1's, 3's, 5's and 7's.

Getting Started

What numbers *can* you make?

Do they have anything in common?

Have you made 37 with a different number (amount) of numbers? How many?

Do these numbers have anything in common?

What do you notice about the numbers in the bags?

Activity 1 [Exploring Numbers]

SOLUTION.

This problem is not possible because with an even number of odd numbers you cannot make an odd number. You can make 36 and 38 using 10 numbers but not 37. You can make 37, but by using 9 numbers. Here are some examples:

36 (10 numbers): 5 + 5 + 5 + 5 + 5 + 3 + 3 + 3 + 1 + 1

38 (10 numbers): 1 + 1 + 1 + 3 + 3 + 5 + 5 + 5 + 7 + 7

37 (9 numbers): 5 + 5 + 5 + 5 + 5 + 5 + 5 + 1 + 1

Exploring Numbers

- **Curricular Links**:

  1. Patterns in number
  2. Expanding algebraic expressions

- **Purpose**:

  Enhance the students appreciation of number and to motivate the expansion of algebraic expressions.

*

Activity 2 [Exploring Numbers]

Let's look at the properties of even and odd numbers...

Write down as many properties of even and odd numbers that you can!

Activity

Activity 2 [Exploring Numbers]

Try discover....

(a) A rule to represent every even number.

(b) A rule to represent every odd number.

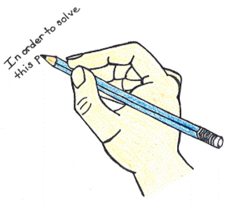
(c) The outcome when two even numbers are added.

(d) The outcome when two odd numbers are added.

(e) The outcome when two even numbers are multiplied.

(f) The outcome when two odd numbers are multiplied.

Prove:
The outcomes for **(c)** to **(f)** above.



Addition

(a) Rule for even numbers is:

$2n$

(b) Rule for odd numbers is:

$2n + 1$

(c) Adding two even numbers is:

$2n + 2k$

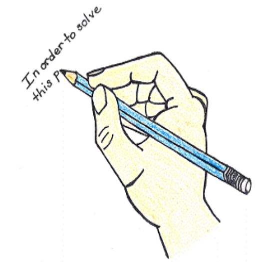
$= 2(n + k)$ an even number.

(d) Adding two odd numbers is:

$2k + 1 + 2n + 1$

$= 2k + 2n + 2$

$= 2(k + n + 1)$ an even number.



Multiplication

(e) Multiplying even numbers;

$(2k)(2n)$

$= 4nk$

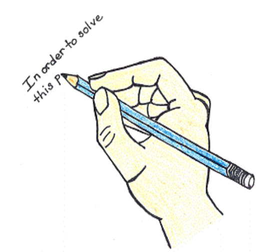
$= 2(2nk)$ an even number.

(f) Multiplying two odd numbers;

$(2k+1)(2n+1)$

$4nk + 2n + 2k + 1$

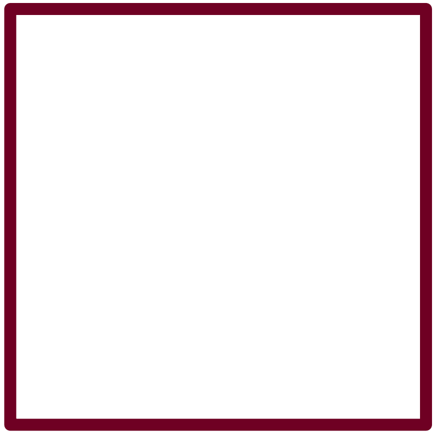
$= 2(2nk + n + k) + 1$ an odd number

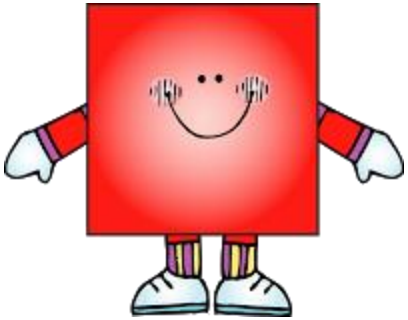


*

Activity 3

- Prove that a square with side an odd number in length, must have an odd area.



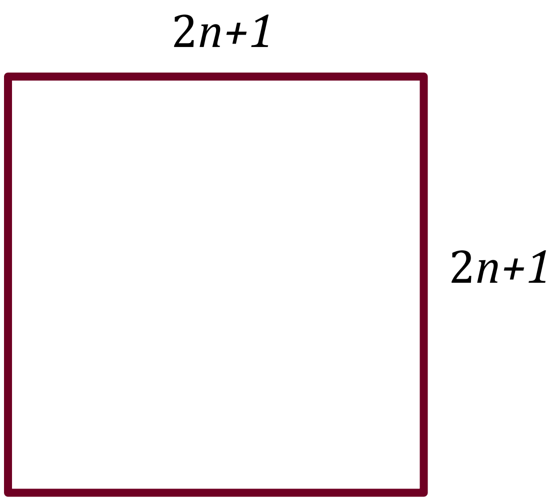


Activity

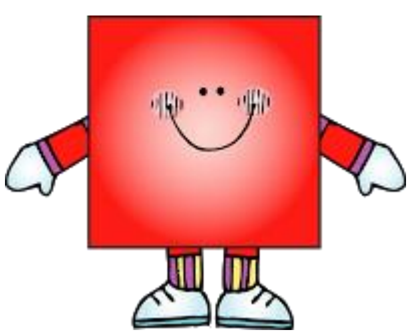
*

Activity 3

- Prove that a square with side an odd number in length, must have an odd area.



$$
\begin{aligned}
Area &= (2n+1)(2n+1) \\
&= 4n^2 + 4n + 1 \\
&= 2(2n^2 + 2n) + 1
\end{aligned}
$$



*

Activity

Proof by Contradiction - LCHL

Prove : $\sqrt{2}$ is irrational

Proof : Assume the contrary: $\sqrt{2}$ is rational

i.e. there exists integers p and q with no common factors such that:

$\frac{p}{q} = \sqrt{2}$ — Square both sides

$\Rightarrow \frac{p^2}{q^2} = 2$ — Multiply both sides by q^2

$\Rightarrow p^2 = 2q^2$ — ...it's a multiple of 2

$\Rightarrow p^2$ is even — $\text{even}^2 = \text{even}$

$\Rightarrow p$ is even

$\therefore p = 2k$ for some k

If $p = 2k$

$\Rightarrow p^2 = 2q^2$ becomes $(2k)^2 = 2q^2 \Rightarrow 4k^2 = 2q^2 \Rightarrow 2k^2 = q^2$

Then similarly $q = 2m$ from some m

$\Rightarrow \frac{p}{q} = \frac{2k}{2m} \Rightarrow \frac{p}{q}$ has a factor of 2 in common.

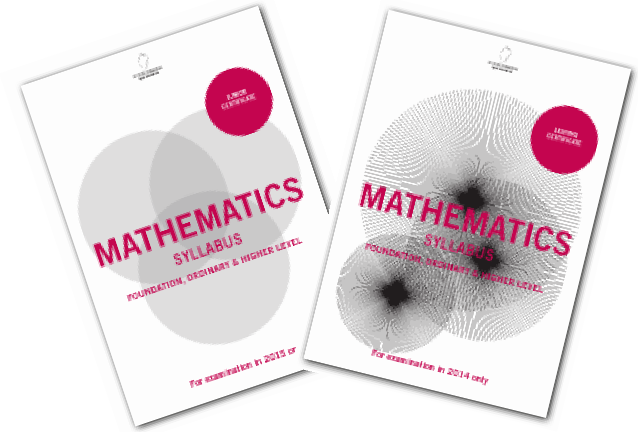
This contradicts the original assumption.

$\sqrt{2}$ is irrational Q.E.D.

Proof

Links to the syllabus

1. Variables and constants in expressions and equations
2. Whole Numbers/Integers
3. Integer Values



Consecutive Numbers

Try to discover a rule to represent the outcome when two consecutive numbers are multiplied:

First even and second odd:

$(2n)(2n+1) = (4n^2 + 2n)$ which is even.

First odd and second even?

Activity

Find all integer solutions of the equation:

$$x^2 + y^2 + x + y = 1997$$

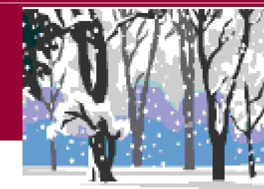
$x(x+1) + y(y+1) = 1997$

Here we have the product of two consecutive integers.

Since the product of two consecutive integers is even (or zero), we have the sum of two even integers.

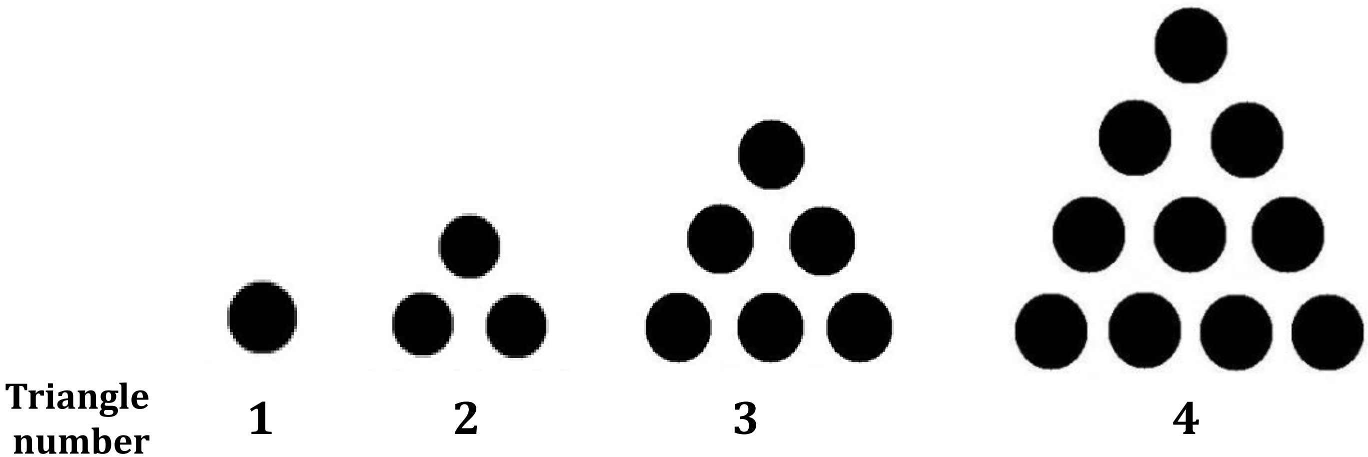
This can never be equal to the odd number, 1997, and thus the solution set is empty.

12 days of Christmas



Activity 4 [Triangular Numbers]

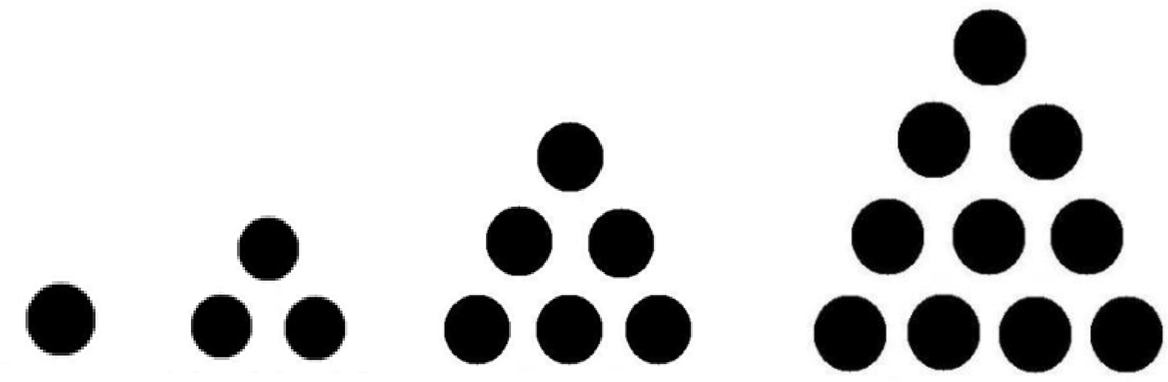
This pattern continues indefinitely.



Can you find a rule to define the number of circles in any given triangle?

Activity 4 [Triangular Numbers]

This pattern continues indefinitely.



For any triangular number eight times the number of circles when increased by 1 yields a perfect square.

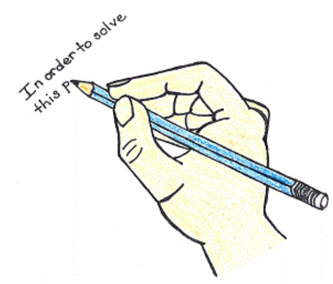
Can you prove this?

Activity 4 [Triangular Numbers]

(b) What seems to happen if any two consecutive triangle numbers are added? Can you prove this to be true?

$1, (1 + 2), (1 + 2 + 3), (1 + 2 + 3 + 4), \ldots$

Adding 2 consecutive triangle numbers yields a perfect square.

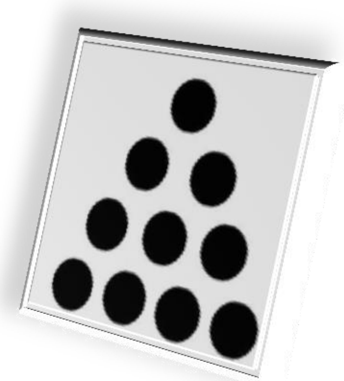


Triangular Numbers

Triangle	Pattern
1	1
2	3
3	6
4	10
5	15

General pattern is given by :

$$\frac{n^2 + n}{2}$$



Solving a Problem

Expand the following:

$x(x+2)$

$(x+1)^2$

$(x+1)(x-1)$

$(x+2)^2 - x(x+4)$

$(a+b)(a-b)$

$(p+q)^2 - (p+2q)p$

Problem Solving

Write down 3 consecutive numbers.
Square the middle number.
Multiply the other two numbers together.
What do you notice?

e.g. 81, 82, 83

$82 \times 82 =$

$81 \times 83 =$

Try other groups of consecutive numbers.

What happens if you use decimals?

e.g. 51.5, 52.5, 53.5

What happens if the numbers are not consecutive, but go up in "two's":

e.g. 412, 414, 416

Generalise your result

Syllabus

Synthesis and problem-solving skills

– explore patterns and formulate conjectures
– explain findings
– justify conclusions
– communicate mathematics verbally and in written form
– apply their knowledge and skills to solve problems in familiar and unfamiliar contexts
– analyse information presented verbally and translate it into mathematical form
– devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

Numbers and Indices

How many digits has the number, $8^{28}5^{80}$.

$$\begin{aligned} 8^{28}5^{80} &= (2^3)^{28}(5^{80}) \\ &= 2^{84}5^{80} \\ &= 2^4(2^{80}5^{80}) \\ &= 2^4(10^{80}) \\ &= 16\times 10^{80} \\ &= 1.6\times 10^{81} \end{aligned}$$

So, there are 82 digits

Numbers and Indices

LCOL (2011)

The number $2^{61} - 1$ is a prime number. Using your calculator, or otherwise, express its value, correct to two significant figures, in the form $a \times 10^{\boldsymbol{n}}$, where $1 \leq a < 10$ and $n \in N$.

How many digits are in $2^{61} - 1$?

LCFL (2012)

Let $a = 8640$.

Express a as a product of it's prime factors.

If $b = 2^{10} \times 3^5 \times 13^6$

Express ab as a product of prime factors.

Proof and the Curriculum

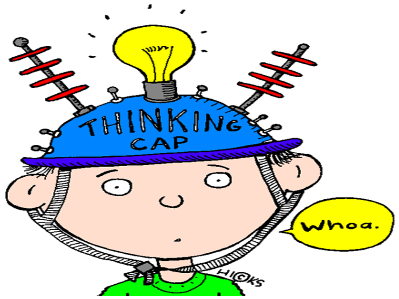
Curricular Links:

1. Patterns in number
2. Expanding algebraic expressions

Purpose:

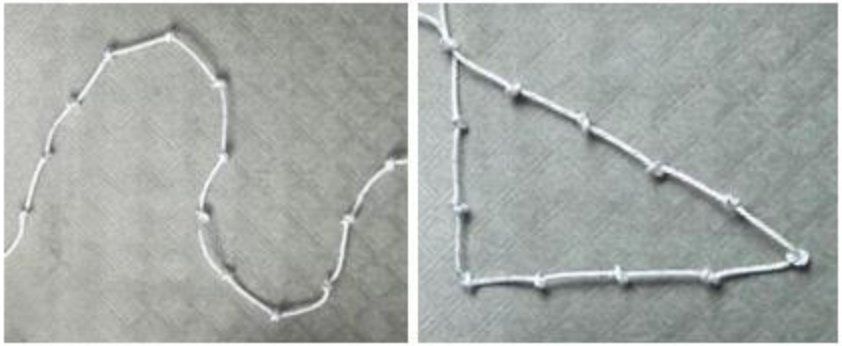
Enhance the students appreciation of number and to motivate the expansion of algebraic expressions.

Linking our Thinking



- You must have a reason for asking questions.
- Students think about how they thought about it.
- The student voice is LEAST clearly heard in maths than in any other subject.
- Use at start of lesson to motivate it.
- Some don’t have an obvious solution.
- The children you regard as best at maths aren’t always the best.
- Rigor is necessary.

Activity 5 [Linking Triangles]



(a) How many different triangles have a perimeter of 12 units?

(b) What kinds of triangles are they?

(c) Explain how you determined this.

Theorem 8

2 sides of a triangle are together greater than the third.

(d) Explain what you have discovered.

Activity 6 [Estimation of π]

Curricular Links

1. Geometry
2. Area
3. Algebraic manipulation
4. Inequalities

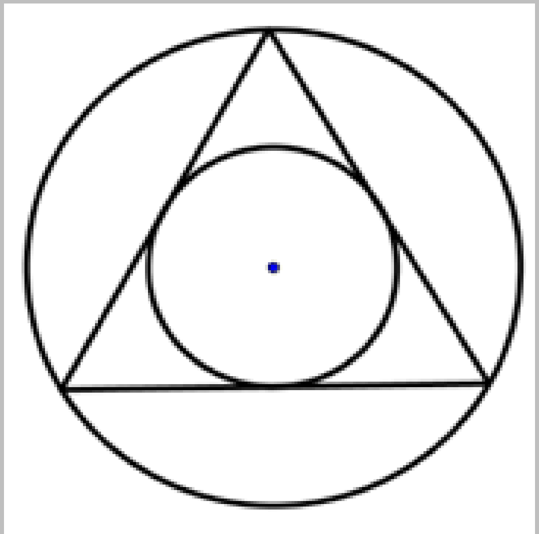
Purpose

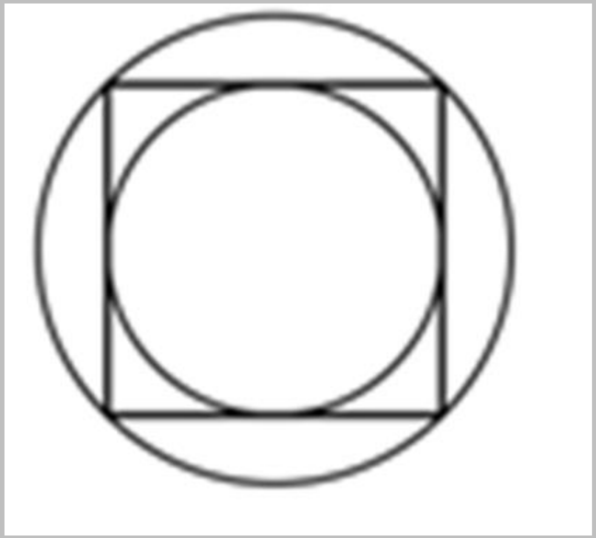
To consolidate and link certain concepts in Geometry and to see how these links can be used to estimate π.

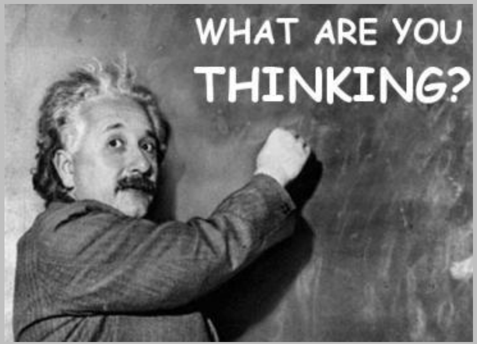
*

Activity 6 [Estimation of π]

Find the ratio of the areas of the circles below.







Can you get a better approximation for π?

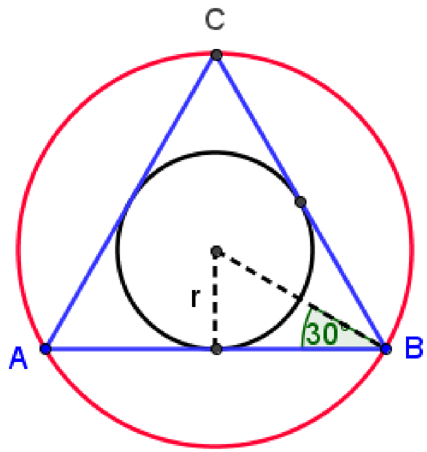
Area of Outer Circle> Area of Triangle> Area of Inner Circle

$$\tan 30^o = \frac{r}{x} \qquad \therefore \frac{1}{\sqrt{3}} = \frac{r}{x} \qquad \therefore x = r\sqrt{3}$$

$\therefore$ Area of Triangle in the original diagram $= x(height) = x(R+r)$

As $R = 2r$, area of the triangle $= r\sqrt{3}(3r) = 3\sqrt{3}r^2$

Area of outer circle > Area of Triangle > Area of inner Circle



$$\pi R^2 > 3\sqrt{3}r^2 > \pi r^2$$

$$\pi(2r)^2 > 3\sqrt{3}r^2 > \pi r^2$$

$$4\pi r^2 > 3\sqrt{3}r^2 > \pi r^2$$

$$4\pi > 3\sqrt{3} > \pi$$

$4\pi > 3\sqrt{3}$	$3\sqrt{3} > \pi$
$\pi > \frac{3\sqrt{3}}{4}$	$\pi < 3\sqrt{3}$
$\pi > 1.299$	$\pi < 5.196$

$$\therefore 1 < \pi < 6$$

Area of outer circle > Area of Square > Area of Inner Circle

$\pi R^2 > (2r)^2 > \pi r^2$

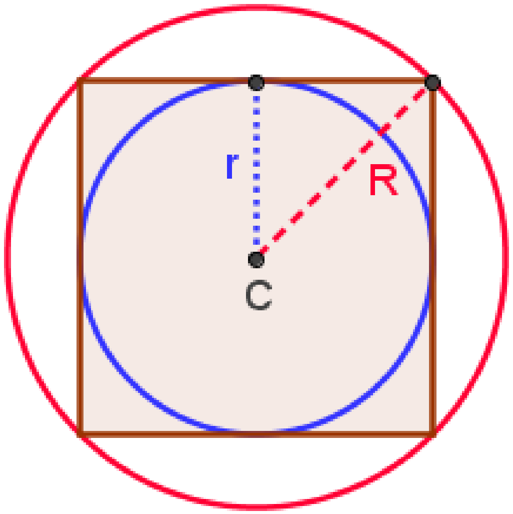
$2\pi > 4 > \pi$

$2\pi > 4$ or $4 > \pi$

$\pi > 2$ or $\pi < 4$

$\therefore 2 < \pi < 4$

Discussion: $R = \sqrt{2}r$, why?



Area of outer circle> Area of Regular Pentagon > Area of inner circle

$$\pi R^2 > 5\left[\frac{1}{2}R^2 \sin 72^o\right] > \pi r^2$$

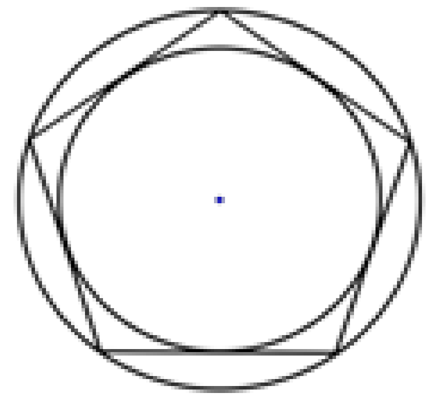
$$\pi\left(\frac{r}{\cos 36^o}\right)^2 > 5\left[\frac{1}{2}\left(\frac{r}{\cos 36^o}\right)^2 \sin 72^o\right] > \pi r^2$$

$$1.5279\pi r^2 > 3.6327 r^2 > \pi r^2$$

$$15279\pi > 3.6327 > \pi$$

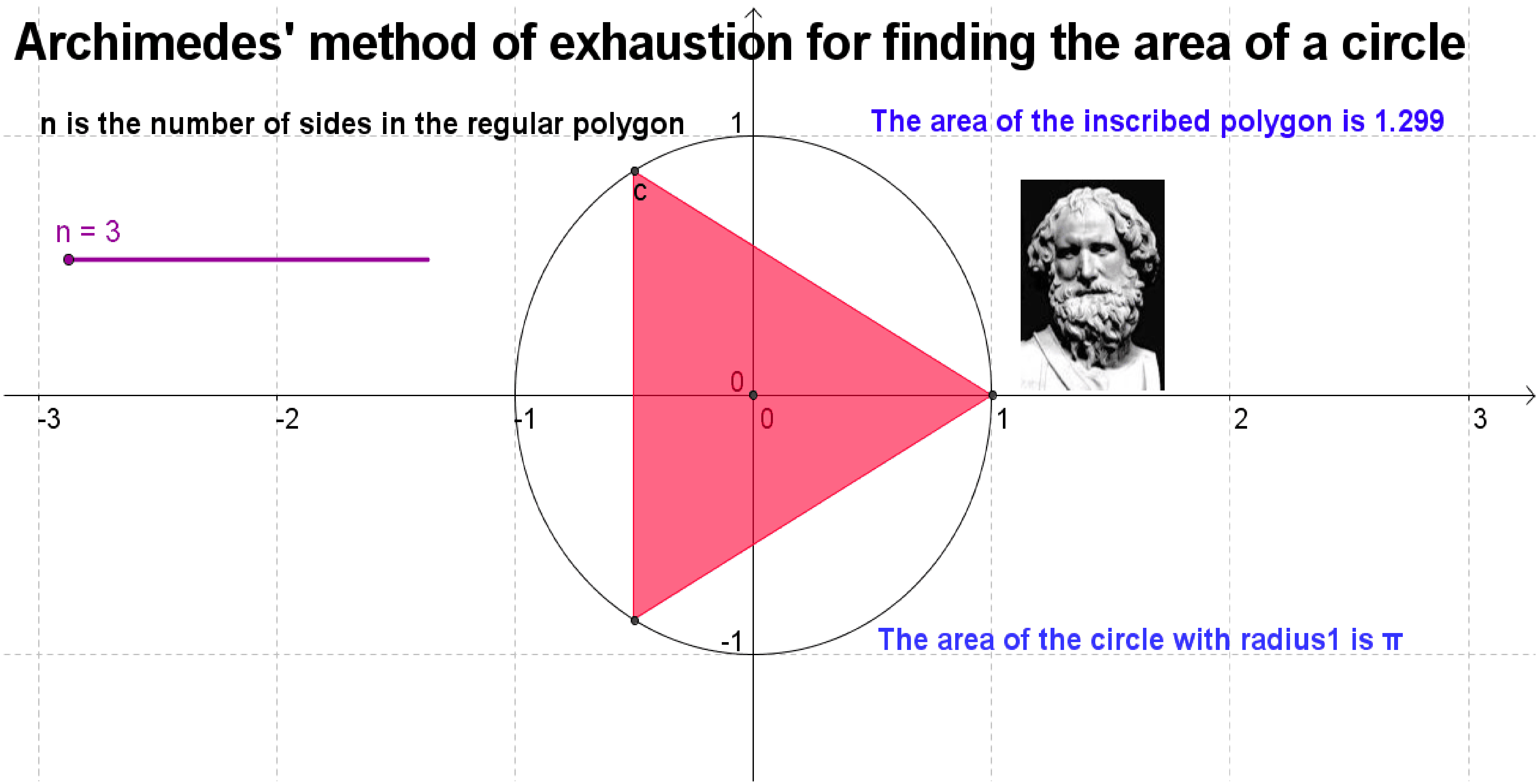
$$15279\pi > 3.6327 \text{ or } \pi < 3.6327$$

$$2.3776 < \pi < 3.6327$$



Discussion: where do the figures come from?

Early engagement with formulas for regions with curvilinear boundaries



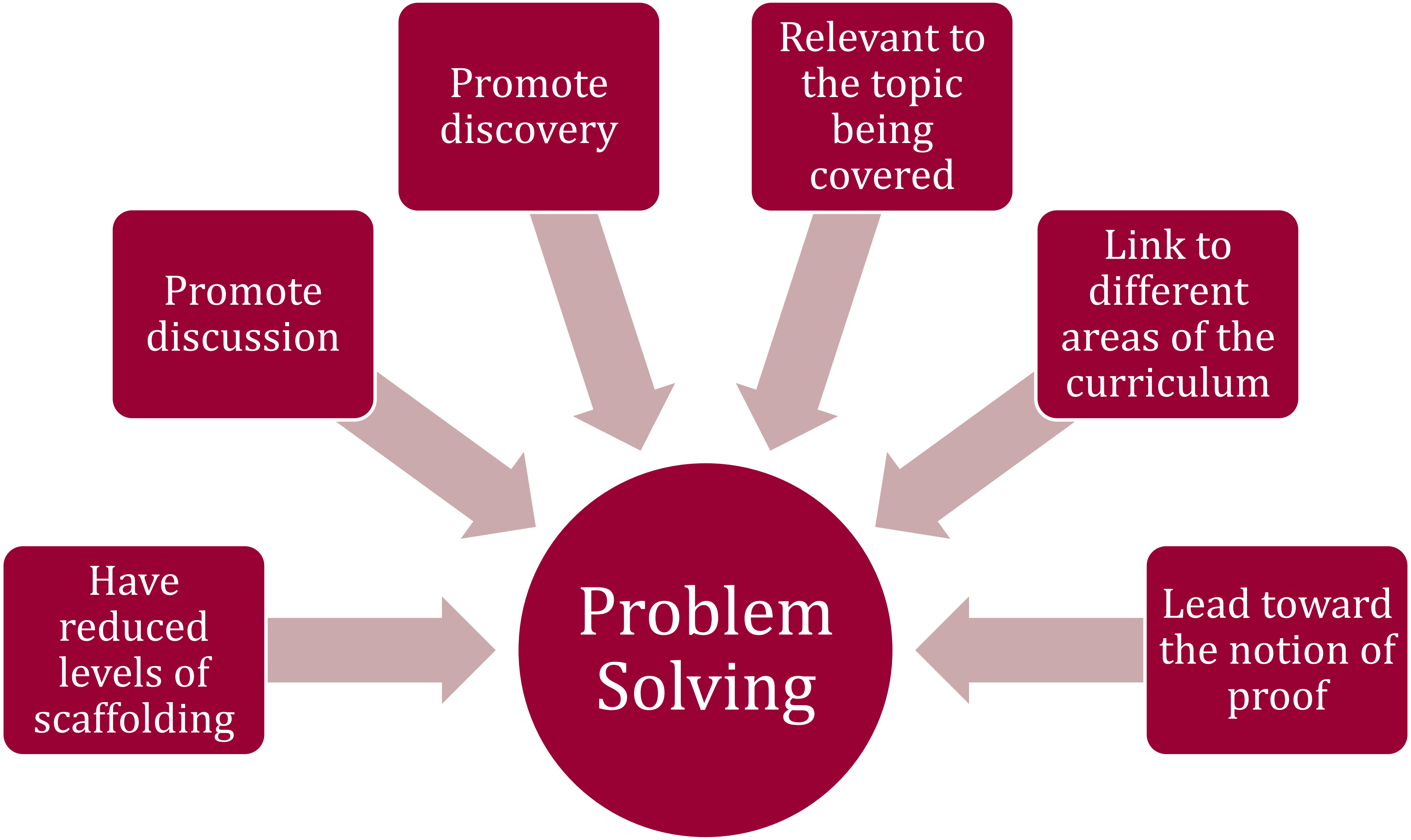
Syllabus

Synthesis and problem-solving skills

– explore patterns and formulate conjectures
– explain findings
– justify conclusions
– communicate mathematics verbally and in written form
– apply their knowledge and skills to solve problems in familiar and unfamiliar contexts
– analyse information presented verbally and translate it into mathematical form
– devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

Problem Solving

- Incorporating Problem Solving
  - Introduce questions that



Problem Solving

- Collaborative Problem solving
- Purpose
  - Engage students
  - Motivate curricular content
  - Place the student at the centre of the learning process
  - Encourage collaboration and discussion
  - Facilitate research, hypothesis development and testing
- Classroom organisation
- Minimal Instruction
- Group work
  - Unseen Problems
  - Collaboration
  - Discussion
  - Internet Access
  - Presentation of Solutions

Syllabus

Students learn about	Students should be able to
2.5 Synthesis and problem-solving skills	– explore patterns and formulate conjectures – explain findings – justify conclusions – communicate mathematics verbally and in written form – apply their knowledge and skills to solve problems in familiar and unfamiliar contexts – analyse information presented verbally and translate it into mathematical form – devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.