Project Maths Workshop 10

Choosing Learning & Teaching Approaches & Strategies

Name: _____

School: _____

Project Maths Tionscadal Mata Development Team

Contents

WS10.01 Lesson Study: A Form of Professional Development 1

Diagnostics Test on Simplifying Rational Algebraic Expressions 1

WS10.02 Cultivating Skills 2

The Concept and Notation of Number Systems 2

WS10.03 Using a Co – construction Approach 3

Developing Ideas 3

Analysing Distributions 4

Using a Sample to Make a Statement about the Population Mean, (μ) 8

Summary of Our Findings 9

WS10.04 Using an Enquiry Approach to Teaching 10

Fair Game – Student Activity 1 10

Expected Value – Student Activity 2 12

Expected Value – Student Activity 3 14

WS10.05 Problem – Solving Learning 15

Cabinet Problem – Student Activity 1 15

Tilted Squares – Student Activity 16

WS10.01 Lesson Study: A Form of Professional Development

Diagnostics Test on Simplifying Rational Algebraic Expressions

1. Simplify the following fractions where possible:

(i) $\dfrac{2+x}{2x}$

(ii) $\dfrac{a+ab}{a}$

(iii) $\dfrac{18x^2y}{27xy^3}$

(iv) $\dfrac{2(a-b)}{c+2}$

(v) $\dfrac{5-r}{5r}$

(vi) $\dfrac{y+yr}{r+1}$

2. Apply your knowledge of factorising to simplify the following fractions where possible:

(i) $\dfrac{b^2-9}{b+3}$

(ii) $\dfrac{b^2+9}{b+3}$

(iii) $\dfrac{16-4a^2}{4a+8}$

3. Apply your knowledge of factorising to simplify the following fractions where possible:

(i) $\dfrac{x^2+3x+2}{x+2}$

(ii) $\dfrac{x^2-3x+2}{x+1}$

(iii) $\dfrac{x^2(x-3)+4(3-x)}{x^2-x-6}$

4. Simplify:

(i) $\dfrac{2pq}{p+2q}\times\dfrac{2p^2-8q^2}{4p-8q}$

(ii) $\dfrac{x^2-4x+4}{3x+3}\times\dfrac{1-x^2}{4-2x}$

5. Simplify:

(i) $\dfrac{3y^2}{z^2}\div\dfrac{6z^3}{y^2}\times\dfrac{18z^4}{y^3}$

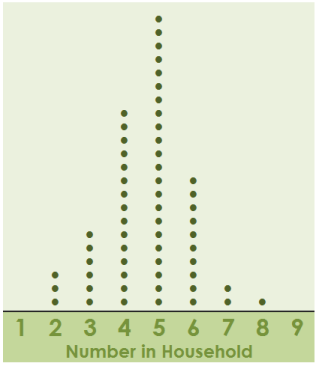
(ii) $\dfrac{p^2-4p-21}{p^2-9}\div\dfrac{p-7}{3-p}$

(iii) $\dfrac{r^2+5r+6}{3r+9}\div\dfrac{4-r^2}{4-2r}$

The Concept and Notation of Number Systems

WS10.03 Using a Co – construction Approach

Developing Ideas



The data for the line plot is from two classes in a school. The sample size is $n = 59$.

1. Describe the data in as many ways as you can using numerical and shape descriptions. Words, fractions, decimals and percentages can be used in your description.

2. One person was absent on the day of the survey. Someone said they would give you €100 if you could guess the number of people in that student's household. What number would you guess?

3. Explain why you chose this number.

4. Describe your chances of winning the €100 by picking this number. Words, fractions, decimals and percentages can be used in your description.

5. Describe how you might you increase your chances of winning the €100?

6. Describe your chances of winning the €100 now.

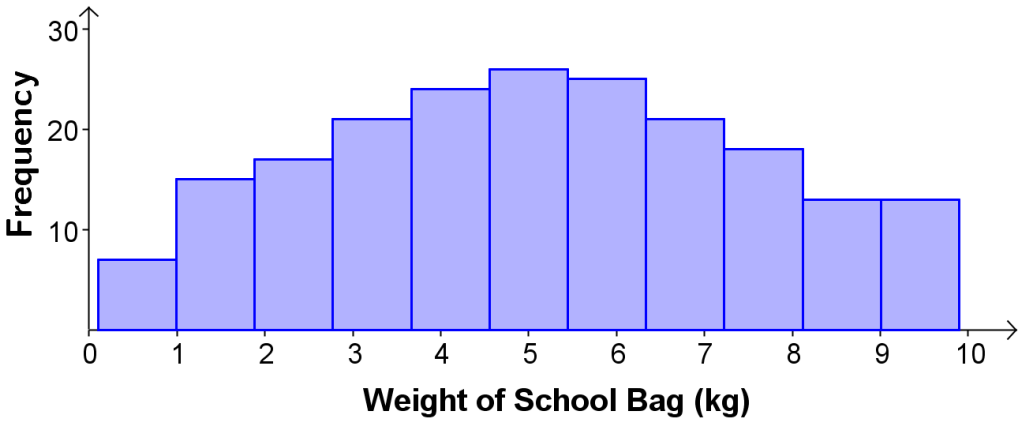
7. Explain how you could further increase your chances of winning.

8. Is it possible that the student who was absent could have 8 people in their household?

9. Is it probable that the student who was absent could have 8 people in their household? Explain.

Analysing Distributions

The distribution of weights of school bags of all 200 students in a school is shown in the diagram. We will call this our ***population distribution***.



1. Describe the population distribution by making one statement about each of the three characteristics indicated below:

(a) Shape of distribution:

(b) Location of data (central tendency):

(C) Spread of data (dispersion):

2. Before you take some samples from the distribution, you must make two predictions.

(a) If you took a sample of size 30 and found the mean of the sample describe the chances of getting a mean between 1 kg and 2 kg.

(b) If you took a sample of size 30 and found the mean of the sample what number would you expect to get?

3. How many different samples of size 30 can be taken from a population of size 200?

4. The table below shows the results of 19 samples each of size $n = 30$.
Create one more sample of size 30 and record the results in the table.

Sample Number	Data Sets										Mean	Standard Deviation
1	5.2	9.5	2	3.5	3.5	7.9	5	2.5	3	6	**4.9**	**2.4**
	5	7	2	5.5	5.1	1	5.6	1.7	1.5	2		
	8.1	7.1	4	8.8	6.8	6.5	4	4	5.5	8		
2	0.8	6	5	5	4	4	6.5	5.2	4	2	**4.8**	**2.5**
	5.6	7	7	1.5	9.9	5.7	4.6	3	6.8	9		
	2.5	3	2	8.9	6.5	3	1	3	8	2.5		
3	8	4	8	1.5	8.8	6	3.5	9.5	9	6.7	**4.6**	**2.6**
	7	4	4.1	2	4	5	7.9	2	4	1		
	2	4	3.5	4	3	7.1	1.1	5.1	0.5	3		
4	3	2	5	6.2	8	6.5	3	5.1	9	1	**4.7**	**2.5**
	2	6.8	2.5	3	5	7.5	6.8	2	2.5	2		
	1.7	3.5	9.9	8	5.8	8	1.5	3.5	4	5		
5	9.9	5	4	7	7.4	2.5	8	5	4	5.1	**5.3**	**2.4**
	9	6	5	6.8	5	1.7	5.4	9.7	3.5	4		
	2	6.6	7	3	5	1.6	5	8	1.1	5.6		
6	0.1	2	5	9.7	6.9	7.8	2	6.7	4	5	**5.4**	**2.7**
	9	8	6	9	9	3	3	2.5	5.6	8		
	7.4	1	4	9.9	2	5	6.5	4	4	5		
7	3	4.6	9.7	2.6	9	5	2	5.1	7	9	**5.6**	**2.9**
	4	9	4	7.8	5	8	0.5	8.9	3	7		
	9.9	3	2	8.5	4	1.3	3.5	9.5	8.1	3		
8	6	6.5	4	4	4	9	5.6	5	5	0.1	**5.8**	**2.6**
	5	9	6.8	9.7	0.8	8.9	6	5.9	4	9.9		
	8.8	3	2.5	6	9.9	8	8	3.5	3.5	5.5		
9	6	2	7.1	5.4	5	3	6.8	9	8	8	**5.4**	**2.5**
	7.8	8	9	1.2	3	5	3	5	5	9		
	5.7	0.1	2	2	5.1	7	3	8.1	5.6	5.9		
10	4	5	6	9.9	9	1	5	9.4	4	6.5	**4.9**	**2.9**
	4	4	4	0.1	6.5	2	7.5	1.5	4	3		
	4	0.5	5	9	2.5	6	3	9	9.5	1		
11	3	4	1	8	5	2	5	8	5	6.5	**5.0**	**2.8**
	9.4	8.4	4	5.5	5	1.5	1.2	6	8	4		
	3.5	6.7	0.5	2	4	8.8	9	1	6.2	9		
12	7	6.5	6	4	9	6.5	8	4	1.3	3	**5.0**	**2.8**
	1.9	0.5	3	1.1	6	5.3	9.7	5.9	9.5	4.1		
	8.1	5	4	7.9	1.1	4	8	0.5	3	7.1		
13	2.5	1	7.9	3	5	1.1	2.5	4	8.8	4	**4.8**	**2.6**
	1	6.9	9.1	5.2	5.8	8.4	7.4	2.5	4	6.5		
	7	5.3	6.8	1.7	1.5	2	3	5	6.8	8		
14	1.7	5	6	6	7.5	5	6	9	9.3	4	**5.2**	**2.6**
	8	4	4.8	4.1	7	5.9	6.2	1.1	8	7		
	2	0.1	9.5	4	3.5	3.5	8.8	1.5	6.7	1.5		
15	1	4	2	2.5	5.6	3	6	3.9	4	3.5	**4.4**	**2.6**
	4	8.8	9.7	2	1	9.9	5	5	8	3		
	4.5	4	3	9	5	1	5	1.1	1.7	5		
16	4	4	6	0.5	2.5	4	5.6	9.9	6.2	1.6	**4.9**	**2.4**
	6.2	4	4	7.8	5	8.4	3	6.5	3	5		
	4	3	4.9	1	7	8	7.9	8	2	5		
17	0.5	9	4	6	6.5	5.1	5	2	7	1.6	**4.7**	**2.5**
	2	2.5	4.7	3.5	1	6.5	2	6.7	3.5	6.6		
	5	6	9.7	6	5	0.5	3	8.1	8	3		
18	9.5	1.1	7	5	1	1.6	6	9.7	8	4	**5.5**	**2.5**
	6.7	6	6.8	7.5	4	8	4	6	7.1	4		
	5.2	7.8	6.8	4.8	8	4	0.5	1.3	5	8		
19	3.9	8.4	7	3	6	5.4	9.9	5	5	3	**4.9**	**2.7**
	2	8.8	4	9	5	7	1.2	1.1	7.8	5		
	3.5	3	5.5	2	3	9.9	6.8	0.5	4	2		
20												

For the next few questions use the STAT mode on your calculator.

5. Calculate, correct to one decimal place, the mean of your sample and write it in the **2nd last column**.

6. Calculate, correct to one decimal place, the standard deviation of your sample and write it in the **last column**.

7. **(a)** Calculate, correct to one decimal place, the mean of the **2nd last column**.

(b) What could we call this mean?

(c) Compare this mean to the mean of the population distribution (μ), which is 5.1 kg.

8. **(a)** Write down the range for the values in the **2nd last column**.

(b) What could we call this range?

(c) Compare this range to the range of the population distribution, which is 9.8 kg.

9. **(a)** Calculate, correct to one decimal place, the standard deviation of the **2nd last column.**

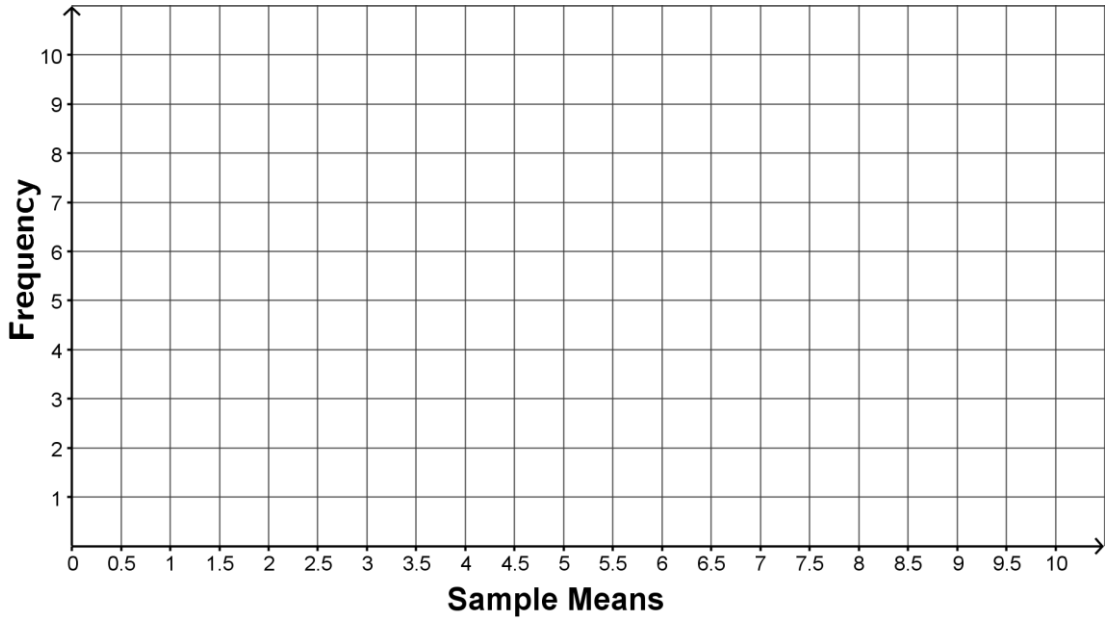
(b) What could we call this standard deviation?

(c) Compare this standard deviation to the standard deviation of the population distribution (σ), which is 2.6 kg.

10. Fill in the table below using your sample means. There is a row provided if you want to use tally marks. [Note that 5.0–5.5 means at least 5.0 kg but less than 5.5 kg etc.]

Sample Means	2.5 – 3.0	3.0 – 3.5	3.5 – 4.0	4.0 – 4.5	4.5 – 5.0	5.0 – 5.5	5.5 – 6.0	6.0 – 6.5	6.5 – 7.0	7.0 – 7.5	7.5 – 8.0
Tally											
Frequency											

11. Draw a histogram of the sample means.



12. Compare the distribution of the sample means to the population distribution by making one statement about each of the three characteristics indicated below:

(a) Shape of distribution:

(b) Location of data (central tendency):

(C) Spread of data (dispersion):

13. **(a)** In **Q2** above you described the chances of getting a mean between 1 kg and 2 kg from a sample of size 30. How good was your prediction?

(b) You also predicted what number you would expect to get for the mean of a sample of size 30. How good was your prediction?

Using a Sample to Make a Statement about the Population Mean, (μ)

N.B. Ignore any findings from the earlier investigation about the school bags.

We are going to see if we can use a sample to make a statement about the (unknown) population mean.

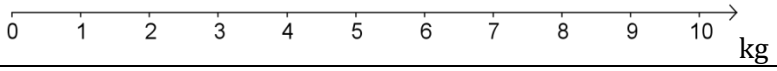
A sample of size 30 was taken of the weights of school bags of post-primary students. The mean of the sample ($\bar{x}$) is 4.5 kg. The standard deviation of the sample is 2.5 kg.

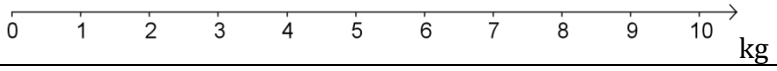
1. Make a statement about the value of the population mean.

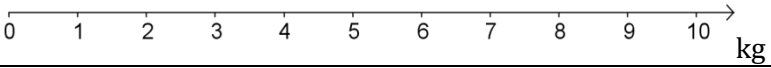
2. There are 3 spaces (Sample, Population, Sampling Distribution of the Means) provided below for any sketches you would like to draw.
(a) Write down and **(b)** sketch everything you know about each distribution where possible.

> **Hints**
> What do we know and what do we not know?
> What do we need to know to find what we do not know?
> What do I know that can be helpful?
> What's similar about this problem to what I already know, and what's different?

Sample







3. Write a statement about the population mean (stating the level of confidence being used).

Summary of Our Findings

Fill in what you know about the Shape, Centre and Spread of each of the distributions in the table below. Words, numbers and symbols can be used.

	Shape	**Centre**	**Spread**
Population		μ	σ
One Large Sample			
All Sample Means			

Fair Game – Student Activity 1

A funfair game called ***Numbers Up!*** involves rolling a single die.

These are the rules:

> ***You win the number that appears on the die in €uro***
>
>
>
> ***€4 for a 4***
>
> ***€6 for a 6 etc.***

Working in pairs, use a die or the random integer function on your calculator, to generate numbers from 1 to 6.

Play the game 20 times and record your results in the table below:

Game	Amount Won	Game	Amount Won
1		11	
2		12	
3		13	
4		14	
5		15	
6		16	
7		17	
8		18	
9		19	
10		20	

1. How much did you win?

2. Work out your average (mean) amount won per game having played the game 20 times.

3. When you have the value for the class mean, fill in the table below:

Your Average Amount Won	
The Class Average Amount Won	

4. Does your average differ from that of the class?

What could explain this?

5. What do you think the class average figure represents in the context of the game?

6. Would you pay €2 to play this game? Give a reason for your answer.

7. If you ran the ***Numbers Up!*** game at the funfair, how much would you charge people to play it? Explain your answer.

8. What do you think would be a fair price to pay to play this game? Why?

ROUND ONE

Number of opened box	Value of opened box	Boxes Remaining	
		21	
		20	
		19	
		18	
		17	
Prize Money Remaining		Banker's Offer	
Expected Value (EV)		Deal or No Deal?	

ROUND TWO

Number of opened box	Value of opened box	Boxes Remaining	
		16	
		15	
		14	
Prize Money Remaining		Banker's Offer	
Expected Value (EV)		Deal or No Deal?	

ROUND THREE

Number of opened box	Value of opened box	Boxes Remaining	
		13	
		12	
		11	
Prize Money Remaining		Banker's Offer	
Expected Value (EV)		Deal or No Deal?	

ROUND FOUR

Number of opened box	Value of opened box	Boxes Remaining	
		10	
		9	
		8	
Prize Money Remaining		Banker's Offer	
Expected Value (EV)		Deal or No Deal?	

ROUND FIVE

Number of opened box	Value of opened box	Boxes Remaining	
		7	
		6	
		5	
Prize Money Remaining		Banker's Offer	
Expected Value (EV)		Deal or No Deal?	

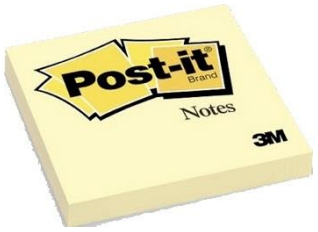
ROUND SIX

Number of opened box	Value of opened box	Boxes Remaining	
		4	
		3	
		2	
Prize Money Remaining		Banker's Offer	
Expected Value (EV)		Deal or No Deal?	

Expected Value – Student Activity 3

Fill in the missing values in the table below:

Amount	**Amount**	**Amount**	**Amount**	**Amount**	**Expected Value (EV)**	**Banker's Offer**	**Banker's Offer as % of EV**
€100	€10,000	€20,000	€35,000	€100,000		€30,000	
€1	€250	€3,000	€35,000	€75,000		€14,500	
50c	€750	€1,000	€3,000	€50,000			37.50%
€1,000	€10,000		€100,000	€250,000	€82,200	€67,000	
10c	€1	€100	€500	€20,000		€2,800	
€20,000		€50,000	€75,000	€100,000	€56,000	€41,000	
1c	€10	€5,000	€10,000	€75,000			69.44%

WS10.05 Problem – Solving Learning

Cabinet Problem – Student Activity 1