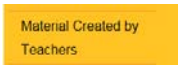
Syllabus and Resources DRAFT

Leaving Certificate Strands 1 - 4

The learning outcomes in the syllabus have been matched to resources which are all available on the Project Maths web site www.projectmaths.ie .

Most Teaching & Learning Plans are available by clicking on this icon on the home page.

The Teaching & Learning Plans denoted by * are available under "Material Created by Teachers".

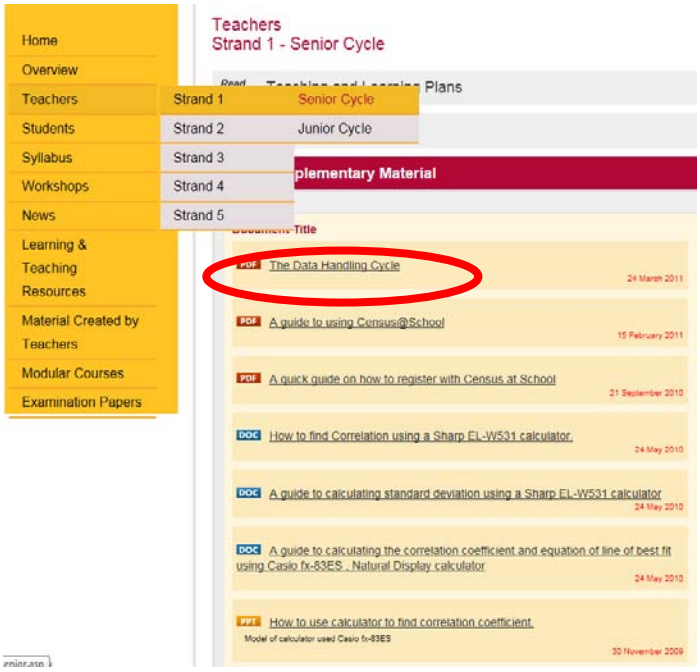


All Teaching & Learning Plans are also available under Teachers, Strand X, Senior Cycle

Patterns: A Relations Approach to Algebra is available by clicking on this icon on the home page.

Teacher Handbooks are available by clicking on this icon on the home page.

The Data Handling Cycle and other documents referred to in Strand 1 are available under Teachers, Strand 1, Senior Cycle, Supplementary Material.



There are activities on the Student's CD referring to the learning outcomes underlined or circled in blue. Some of these are in the Junior Certificate section.

There are hyperlinks to most of the resources referred to in the body of this document.

Strand 1: Statistics and Probability

Handbook

Students learn about	Students working at FL should be able to	In addition, students working at OL should be able to	In addition, students working at HL should be able to
1.1 Counting	– list outcomes of an experiment – apply the fundamental principle of counting	– count the arrangements of n distinct objects ($n!$) – count the number of ways of arranging r objects from n distinct objects	– count the number of ways of selecting r objects from n distinct objects
1.2 Concepts of probability	– decide whether an everyday event is likely or unlikely to occur – recognise that probability is a measure on a scale of 0-1 of how likely an event is to occur – use set theory; discuss experiments, outcomes, sample spaces – use the language of probability to discuss events, including those with equally likely outcomes – estimate probabilities from experimental data – recognise that, if an experiment is repeated, there will be different outcomes and that increasing the number of times an experiment is repeated generally leads to better estimates of probability – associate the probability of an event with its long run relative frequency	– discuss basic rules of probability (AND/OR, mutually exclusive) through the use of Venn diagrams – calculate expected value and understand that this does not need to be one of the outcomes – recognise the role of expected value in decision making and explore the issue of fair games	– extend their understanding of the basic rules of probability (AND/OR, mutually exclusive) through the use of formulae • Addition Rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ • Multiplication Rule (Independent Events): $P(A \cap B) = P(A) \times P(B)$ • Multiplication Rule (General Case): $P(A \cap B) = P(A) \times P(B \mid A)$ – solve problems involving conditional probability in a systematic way – appreciate that in general $P(A \mid B) \neq P(B \mid A)$ – examine the implications of $P(A \mid B) \neq P(B \mid A)$ in context
1.3 Outcomes of random processes	– construct sample spaces for two independent events – apply the principle that in the case of equally likely outcomes the probability is given by the number of outcomes of interest divided by the total number of outcomes (examples using coins, dice, spinners, urns with coloured objects, playing cards, etc.)	– find the probability that two independent events both occur – apply an understanding of Bernoulli trials* – solve problems involving up to 3 Bernoulli trials – calculate the probability that the 1st success occurs on the n^{th} Bernoulli trial where n is specified	– solve problems involving calculating the probability of k successes in n repeated Bernoulli trials (normal approximation not required) – calculate the probability that the k^{th} success occurs on the n^{th} Bernoulli trial – use simulations to explore the variability of sample statistics from a known population and to construct sampling distributions – solve problems involving reading probabilities from the normal distribution tables

*A Bernoulli trial is an experiment whose outcome is random and can be either of two possibilities: "success" or "failure".

T & L
Intro to Fundamental Principals of Counting *

2 T & Ls
Permutations, Combinations and Probability & Permutations leading to Combinations*

T & L
1, 2, 3, 4 & 5

Student's CD

T & L
Binomial Distribution *

2 T & Ls
Probability using Playing (Deck of) Cards *

Students learn about	Students working at FL should be able to	In addition, students working at OL should be able to	In addition, students working at HL should be able to
1.4 Statistical reasoning with an aim to becoming a statistically aware consumer	– engage in discussions about the purpose of statistics and recognise misconceptions and misuses of statistics – discuss populations and samples – decide to what extent conclusions can be generalised – work with different types of data: categorical: nominal or ordinal numerical: discrete or continuous in order to clarify the problem at hand	– work with different types of bivariate data	
1.5 Finding, collecting and organising data	– clarify the problem at hand – formulate one (or more) questions that can be answered with data – explore different ways of collecting data – generate data, or source data from other sources including the internet – select a sample (Simple Random Sample) – recognise the importance of representativeness so as to avoid biased samples – design a plan and collect data on the basis of above knowledge	– discuss different types of studies: sample surveys, observational studies and designed experiments – design a plan and collect data on the basis of above knowledge	– recognise the importance of randomisation and the role of the control group in studies – recognise biases, limitations and ethical issues of each type of study – select a sample (stratified, cluster, quota – no formulae required, just definitions of these) – design a plan and collect data on the basis of above knowledge

Data Handling Cycle

Students learn about	Students working at FL should be able to	In addition, students working at OL should be able to	In addition, students working at HL should be able to
1.6 Representing data graphically and numerically	**Graphical** – select appropriate graphical or numerical methods to describe the sample (univariate data only) – evaluate the effectiveness of different displays in representing the findings of a statistical investigation conducted by others – use stem and leaf plots and histograms (equal intervals) to display data **Numerical** – use a variety of summary statistics to describe the data • central tendency: mean, median, mode • variability: range	**Graphical** – describe the sample (both univariate and bivariate data) by selecting appropriate graphical or numerical methods – explore the distribution of data, including concepts of symmetry and skewness – compare data sets using appropriate displays, including back-to-back stem and leaf plots – determine the relationship between variables using scatterplots – recognise that correlation is a value from -1 to +1 and that it measures the extent of the linear relationship between two variables – match correlation coefficient values to appropriate scatter plots – understand that correlation does not imply causality **Numerical** – recognise standard deviation and interquartile range as measures of variability – use a calculator to calculate standard deviation – find quartiles and the interquartile range – use the interquartile range appropriately when analysing data – recognise the existence of outliers	**Graphical** – analyse plots of the data to explain differences in measures of centre and spread – draw the line of best fit by eye – make predictions based on the line of best fit – calculate the correlation coefficient by calculator **Numerical** – recognise the effect of outliers – use percentiles to assign relative standing

Data Handling Cycle

T & L
Correlation Coefficient

Document
(Supplementary Material)

Students learn about	Students working at FL should be able to	In addition, students working at OL should be able to	In addition, students working at HL should be able to
1.7 Analysing, interpreting and drawing inferences from data*	– recognise how sampling variability influences the use of sample information to make statements about the population – use appropriate tools to describe variability, drawing inferences about the population from the sample – interpret the analysis – relate the interpretation to the original question	– interpret a histogram in terms of distribution of data – make decisions based on the empirical rule	– recognise the concept of a hypothesis test – calculate the margin of error ($\frac{1}{\sqrt{n}}$) for a population proportion – conduct a hypothesis test on a population proportion using the margin of error
Students learn about	**Students should be able to**		
1.8 Synthesis and problem-solving skills	– explore patterns and formulate conjectures – explain findings – justify conclusions – communicate mathematics verbally and in written form – apply their knowledge and skills to solve problems in familiar and unfamiliar contexts – analyse information presented verbally and translate it into mathematical form – devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.		

Problem Solving Questions

* The final syllabus will contain additional material in this section, which has been deferred for an interim peroiod until students coming through to senior cycle have completed the relevant revised syllabus material in the junior cycle.

Strand 2: Geometry and Trigonometry

Students learn about	Students working at FL should be able to	In addition, students working at OL should be able to	In addition, students working at HL should be able to
2.1 Synthetic geometry *	– perform constructions **18, 19, 20** (see *Geometry for Post-primary School Mathematics*)	– perform constructions **16, 17, 21** (see *Geometry for Post-primary School Mathematics*) – use the following terms related to logic and deductive reasoning: theorem, proof, axiom, corollary, converse, implies – investigate theorems **7, 8, 11, 12, 13, 16, 17, 18, 20, 21** and **corollary 6** (see *Geometry for Post-primary School Mathematics*) and use them to solve problems	– perform constructions **1-15** and **22** (see *Geometry for Post-primary School Mathematics*) – use the following terms related to logic and deductive reasoning: is equivalent to, if and only if, proof by contradiction – prove theorems **11, 12, 13,** concerning ratios (see *Geometry for Post-primary School Mathematics*), which lay the proper foundation for the proof of the theorem of Pythagoras studied at junior cycle
2.2 Co-ordinate geometry	– use slopes to show that two lines are • parallel • perpendicular – recognise the fact that the relationship $ax + by + c = 0$ is linear – solve problems involving slopes of lines	– calculate the area of a triangle – recognise that $(x-h)^2 + (y-k)^2 = r^2$ represents the relationship between the x and y co-ordinates of points on a circle centre (h, k) and radius r – solve problems involving a line and a circle with centre (0, 0)	– solve problems involving • the perpendicular distance from a point to a line • the angle between two lines – divide a line segment internally in a given ratio m:n – recognise that $x^2+y^2+2gx+2fy+c = 0$ represents the relationship between the x and y co-ordinates of points on a circle centre $(-g,-f)$ and radius r where $r = \sqrt{(g^2+f^2 - c)}$ – solve problems involving a line and a circle

Student's CD

* In the examination, candidates will have the option of answering a question on the synthetic geometry set out here, or answering a problem-solving question based on the geometrical results from the corresponding syllabus level at Junior Certificate. This option will apply for a three year period only, for candidates sitting the Leaving Certificate examination in 2012, 2013 and 2014. There will be no choice after that stage.

Students learn about	Students working at FL should be able to	In addition, students working at OL should be able to	In addition, students working at HL should be able to
2.3 Trigonometry	– use the theorem of Pythagoras to solve problems (2D only)	– use trigonometry to calculate the area of a triangle – solve problems using the sine and cosine rules (2D)	– use trigonometry to solve problems in 3D – graph the trigonometric functions sine, cosine, tangent – graph trigonometric functions of type $a\text{Sin } n\theta$, $a\text{Cos } n\theta$ for $a, n \in \mathbb{N}$
		– define sin θ and cos θ for all values of θ – define tan θ	– solve trigonometric equations such as $\text{Sin } n\theta = 0$ and $\text{Cos } n\theta = \frac{1}{2}$ giving all solutions
		– solve problems involving area of a sector of a circle and the length of an arc – work with trigonometric ratios in surd form	– use the radian measure of angles – derive the trigonometric formulae 1, 2, 3, 4, 5, 6, 7, 9 (see appendix) – apply the trigonometric formulae 1-24 (see appendix)
2.4 Transformation geometry, enlargements	– investigate enlargements paying attention to • centre of enlargement • scale factor k, where $0<k<1$, $k>1$ $k \in \mathbf{Q}$ • area – solve problems involving enlargements		

Students learn about	Students should be able to
2.5 Synthesis and problem-solving skills	– explore patterns and formulate conjectures – explain findings – justify conclusions – communicate mathematics verbally and in written form – apply their knowledge and skills to solve problems in familiar and unfamiliar contexts – analyse information presented verbally and translate it into mathematical form – devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

T & L 8

T & L 9

T & L 10

End of T & L 9 is the start of this section

T & L Radians*

T & L Enlargements*

Student's CD

Problem Solving Questions

Strand 3: Number

Students learn about	Students working at FL should be able to	In addition, students working at OL should be able to	In addition, students working at HL should be able to
3.1 Number systems	– recognise irrational numbers and appreciate that **R**≠**Q** – revisit the operations of addition, multiplication, subtraction and division in the following domains: • **N** of natural numbers • **Z** of integers • **Q** of rational numbers • **R** of real numbers and represent these numbers on a number line – appreciate that processes can generate sequences of numbers or objects – investigate patterns among these sequences – use patterns to continue the sequence – generate rules/formulae from those patterns – develop decimals as special equivalent fractions strengthening the connection between these numbers and fraction and place value understanding – consolidate their understanding of factors, multiples, prime numbers in **N** – express numbers in terms of their prime factors – appreciate the order of operations, including brackets – express non-zero positive rational numbers in the form $a \times 10^n$, where $n \in \mathbf{N}$ and $1 \leq a < 10$ and perform arithmetic operations on numbers in this form	– work with irrational numbers – investigate the operations of addition, multiplication, subtraction and division with complex numbers **C** in the form $a+ib$ – illustrate complex numbers on an Argand diagram – interpret the modulus as distance from the origin on an Argand diagram and calculate the complex conjugate – generalise and explain patterns and relationships in algebraic form – recognise whether a sequence is arithmetic, geometric or neither – find the sum to n terms of an arithmetic series – express non-zero positive rational numbers in the form $a \times 10^n$, where $n \in \mathbf{Z}$ and $1 \leq a < 10$ and perform arithmetic operations on numbers in this form	– geometrically construct $\sqrt{2}$ and $\sqrt{3}$ – calculate conjugates of sums and products of complex numbers – verify and justify formulae from number patterns – investigate geometric sequences and series – prove by induction • simple identities such as the sum of the first n natural numbers and the sum of a finite geometric series • simple inequalities such as $n! > 2^n$ $2^n > n^2 \ (n \geq 4)$ $(1+x)^n \geq 1+nx \ (x > -1)$ • factorisation results such as 3 is a factor of 4^n-1 – apply the rules for sums, products, quotients of limits – find by inspection the limits of sequences such as $\lim_{n\to\infty} \frac{n}{n+1}$; $\lim_{n\to\infty} r^n \ |r|<1$ – solve problems involving finite and infinite geometric series including applications such as recurring decimals and financial applications, e.g. deriving the formula for a mortgage repayment – derive the formula for the sum to infinity of geometric series by considering the limit of a sequence of partial sums

2 T & Ls
Arithmetic Sequences
Arithmetic Series

T & L
Complex Numbers

Student's CD

T & L
Patterns

Patterns
A Relations Approach to Algebra

3 T & Ls
Intro to Decimals & Place Value, Decimal Operations, Percentages*

Students learn about	Students working at FL should be able to	In addition, students working at OL should be able to	In addition, students working at HL should be able to
3.2 Indices	– solve problems using the rules for indices (where $a \in \mathbf{Q}$, $p, q \in \mathbf{N}$; $a \neq 0$): • $a^p a^q = a^{p+q}$ • $\frac{a^p}{a^q} = a^{p-q}$ $p > q$ • $(a^p)^q = a^{pq}$ • $a^0 = 1$	– solve problems using the rules for indices (where $a, b \in \mathbf{R}$; $p, q \in \mathbf{Q}$; $a^p, a^q \in \mathbf{Q}$; $a, b \neq 0$): • $a^p a^q = a^{p+q}$ • $\frac{a^p}{a^q} = a^{p-q}$ • $a^0 = 1$ • $(a^p)^q = a^{pq}$ • $a^{\frac{1}{q}} = \sqrt[q]{a}$ $q \in \mathbf{Z}$, $q \neq 0, a > 0$ • $a^{\frac{p}{q}} = \sqrt[q]{a^p} = (\sqrt[q]{a})^p$ $p, q \in \mathbf{Z}$, $q \neq 0, a > 0$ • $a^{-p} = \frac{1}{a^p}$ • $(ab)^p = a^p b^p$ • $(\frac{a}{b})^p = \frac{a^p}{b^p}$	– solve problems using the rules of logarithms • $\log_a(xy) = \log_a x + \log_a y$ • $\log_a(\frac{x}{y}) = \log_a x - \log_a y$ • $\log_a x^q = q \log_a x$ • $\log_a a = 1$ and $\log_a 1 = 0$ • $\log_a x = \frac{\log_b x}{\log_b a}$
3.3 Arithmetic	– check a result by considering whether it is of the right order of magnitude and by working the problem backwards; round off a result – make and justify estimates and approximations of calculations; calculate percentage error and tolerance – calculate average rates of change (with respect to time) – solve problems involving • finding depreciation (reducing balance method) • costing: materials, labour and wastage • metric system; change of units; everyday imperial units (conversion factors provided for imperial units) – estimate of the world around them, e.g. how many books in a library	– accumulate error (by addition or subtraction only) – solve problems that involve calculating cost price, selling price, loss, discount, mark up (profit as a % of cost price), margin (profit as a % of selling price), compound interest, depreciation (reducing balance method), income tax and net pay (including other deductions)	– use *present value* when solving problems involving loan repayments and investments

Student's CD

Students learn about	Students working at FL should be able to	In addition, students working at OL should be able to	In addition, students working at HL should be able to
3.4 Length, area and volume	– select and use suitable strategies to find • length of the perimeter and the area of the following plane figures: parallelogram, trapezium, and figures made from combinations of these • surface area and volume of the following solid figures: cylinder, right cone, right prism and sphere – use the Trapezoidal Rule to approximate area – investigate the nets of prisms (polygonal bases), cylinders and cones	– solve problems involving the length of the perimeter and the area of plane figures: disc, triangle, rectangle, square, parallelogram, trapezium, sectors of discs, and figures made from combinations of these – solve problems involving surface area and volume of the following solid figures: rectangular block, cylinder, right cone, triangular-based prism (right angle, isosceles and equilateral), sphere, hemisphere, and solids made from combinations of these	

Student's CD

Students learn about	Students will be able to
3.5 Synthesis and problem-solving skills	– explore patterns and formulate conjectures – explain findings – justify conclusions – communicate mathematics verbally and in written form – apply their knowledge and skills to solve problems in familiar and unfamiliar contexts – analyse information presented verbally and translate it into mathematical form – devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

Problem Solving Questions

Students learn about	Students working at FL should be able to	In addition, students working at OL should be able to	In addition, students working at HL should
4.1 Expressions	– evaluate expressions given the value of the variables – expand and simplify expressions	– factorise expressions of order 2 – add and subtract expressions of the form • $(ax+by+c)\pm\ldots\pm(dx+ey+f)$ • $(ax^2+bx+c)\pm\ldots\pm(dx^2+ex+f)$ where $a,b,c,d,e,f\in\mathbf{Z}$ • $\frac{a}{bx+c}\pm\frac{q}{px+r}$ where $a,b,c,p,q,r\in\mathbf{Z}$ – use the associative and distributive properties to simplify expressions of the form • $(bx+cy+d)+\ldots+e(fx+gy+h)$ • $(x\pm y)(w\pm z)$ – rearrange formulae	– perform the arithmetic operations of addition, subtraction, multiplication and division on polynomials and rational algebraic expressions paying attention to the use of brackets and surds
4.2 Solving equations	– select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to equations of the form: • $f(x)=g(x)$ with $f(x)=ax+b$, $g(x)=cx+d$ where $a, b, c, d\in\mathbf{Z}$ • $f(x)=0$ with $f(x)=ax^2+bx+c$ where $b^2\geq 4ac$; $a, b, c\in\mathbf{Z}$ and interpret the results	– select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to equations of the form: • $f(x)=g(x)$ with $f(x)=ax+b$, $g(x)=cx+d$ where $a, b, c, d\in\mathbf{Q}$ • $f(x)=g(x)$ with $f(x)=\frac{a}{bx+c}\pm\frac{q}{px+r}$; $g(x)=\frac{e}{f}$ where $a, b, c, d, e, f, p, q, r\in\mathbf{Z}$ • $f(x)=k$ with $f(x)=ax^2+bx+c$ (and not necessarily factorisable) where $a, b, c\in\mathbf{Q}$ and interpret the results – select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to • simultaneous linear equations with two unknowns and interpret the results • one linear equation and one equation of order 2 with two unknowns (restricted to the case where either the coefficient of x or the coefficient of y is ± 1 in the linear equation) and interpret the results – form quadratic equations given whole number roots	– select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to equations of the form: • $f(x)=g(x)$ with $f(x)=\frac{ax+b}{ex+f}\pm\frac{cx+b}{px+q}$; $g(x)=k$ where $a, b, c, d, e, f, p, q\in\mathbf{Z}$ – use the *Factor Theorem* for polynomials – select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to • cubic equations with at least one integer root • simultaneous linear equations with three unknowns • one linear equation and one equation of order 2 with two unknowns and interpret the results

Student's CD

Students learn about	Students working at FL should be able to	In addition, students working at OL should be able to	In addition, students working at HL should
4.3 Inequalities	– select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to inequalities of the form: • $g(x) \leq k$, $g(x) \geq k$, • $g(x) < k$, $g(x) > k$, where $g(x) = ax + b$ and $a, b, k \in \mathbf{Z}$	– select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to inequalities of the form: • $g(x) \leq k$, $g(x) \geq k$, $g(x) < k$, $g(x) > k$, where $g(x) = ax + b$ and $a, b, k \in \mathbf{Q}$	– select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to inequalities of the form: • $g(x) \leq k$, $g(x) \geq k$, $g(x) < k$, $g(x) > k$, where $g(x) = ax^2 + bx + c$ or $g(x) = \frac{ax+b}{cx+d}$ and $a, b, c, d, k \in \mathbf{Q}$, $x \in \mathbf{R}$ – use notation $\lvert x \rvert$ – select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to inequalities of the form: $\lvert x - a \rvert < b$, $\lvert x - a \rvert > b$ and combinations of these, where $a, b \in \mathbf{Q}$, $x \in \mathbf{R}$
4.4 Complex Numbers		See strand 3, section 3.1	– use the *Conjugate Root Theorem* to find the roots of polynomials – work with complex numbers in rectangular and polar form to solve quadratic and other equations including those in the form $z^n = a$, where $n \in \mathbf{Z}$ and $z = r \operatorname{Cos} \theta + i \operatorname{Sin} \theta$ – use De Moivre's Theorem – prove De Moivre's Theorem by induction for $n \in \mathbf{N}$ – use applications such as n^{th} roots of unity, $n \in \mathbf{N}$ and identities such as $\operatorname{Cos} 3\theta = 4 \operatorname{Cos}^3 \theta - 3 \operatorname{Cos} \theta$

Student's CD

Students learn about	Students should be able to
4.5 Synthesis and problem-solving skills	– explore patterns and formulate conjectures – explain findings – justify conclusions – communicate mathematics verbally and in written form – apply their knowledge and skills to solve problems in familiar and unfamiliar contexts – analyse information presented verbally and translate it into mathematical form – devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

Problem Solving Questions

APPENDIX

Other Resources not mentioned already: All available on www.projectmaths.ie

Booklet to accompany Student's CD. Click on this icon. 

Notes from Modular Courses on Strand 1 Content, Strand 2 Content and ICT

Teachers

Strand 1

Senior Cycle

Supplementary Material

A Guide to using Census@School

Teachers

Strand 2

Senior Cycle

Supplementary Material

Theorems in your own words

Student Activities on Theorems 7, 8, 13, 15 & 21

Materials Created by Teachers

Strand 1

Senior Cycle

An Introduction to Statistics & Data Handling
Tossing 2 Dice
Paired Data & Correlation Coefficient
Scatter Plots

Strand 2

Senior Cycle

The Theorem of Pythagoras
Exploring the Distance Formula

Other useful websites

NCCA Student Resources including Revision Material www.ncca.ie/projectmaths

Census@School www.censusatschool.ie

National Centre for Excellence in Mathematics and Science Teaching & Learning
www.nce-mstl.ie