Leaving Certificate Sample and Examination Papers Phase 1

For the 2012 Examination

HIGHER LEVEL

Includes:

2011 Leaving Certificate Examination Paper 2
2011 Sample Examination Paper 2
2010 Leaving Certificate Examination Paper 2
2010 Sample Examination Paper 2

Leaving Certificate 2012 Examination Guidelines

HIGHER LEVEL

Some Points to Note:

- **Paper one** will be unchanged from previous years; same structure, same choice.

- The full **paper two** will be changed from previous years. There will be two sections on the paper. Section A (worth 150 marks), on concepts and skills, will contain six questions. Section B of the paper (also worth 150 marks), on contexts and applications, will contain three questions.

- Paper two will examine **Statistics and Probability** (*Strand 1)* and **Geometry and Trigonometry** (*Strand 2).*

- Learning outcomes from more than one strand can be examined in a single question.

- All questions to be attempted, there is **no*** choice.

  **Section 2.1 of the syllabus indicates that candidates will have the option of answering a question on the synthetic geometry set out in the syllabus, or answering a problem-solving question based on the geometrical results from the corresponding syllabus level at Junior Certificate. This option will apply for a three year period only that is for candidates sitting the Leaving Certificate examination in 2012, 2013 and 2014. There will be no choice after that stage.*

- The old option questions are no longer on the syllabus (i.e. Further calculus and Series, Further Probability and Statistics, Groups or Further Geometry)

- There is a new type of marking scheme in place for examining the new syllabus; more information and these marking schemes can be found on: *http://www.examinations.ie*

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination, 2011

Mathematics
(Project Maths – Phase 2)

Paper 2

Higher Level

Monday 13 June Morning 9:30 – 12:00

300 marks

Examination number

Centre stamp

Running total	

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Grade

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	2 questions

Answer **all eight** questions, as follows:

In Section A, answer:

Questions 1 to 5 and

either Question 6A **or** Question 6B.

In Section B, answer Question 7 and Question 8.

Write your answers in the spaces provided in this booklet. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the booklet of *Formulae and Tables*. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Section A — Concepts and Skills — 150 marks

Answer **all six** questions from this section.

Question 1 **(25 marks)**

(a) A random variable X follows a normal distribution with mean 20 and standard deviation 5. Find $P(14 \leq X \leq 26)$.

(b) There are 16 girls and 8 boys in a class. Half of these 24 students study French. The probability that a randomly selected girl studies French is 1·5 times the probability that a randomly selected boy studies French. How many of the boys in the class study French?

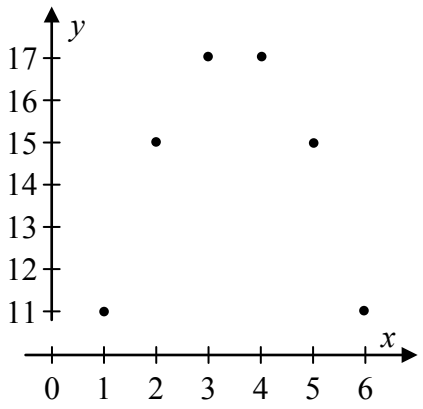
Question 2 **(25 marks)**

(a) Explain, with the aid of an example, what is meant by the statement:

"Correlation does not imply causality."

(b) The data given in the table below and represented in the scatter diagram are pairs of observations of the variables x and y.

x	1	2	3	4	5	6
y	11	15	17	17	15	11



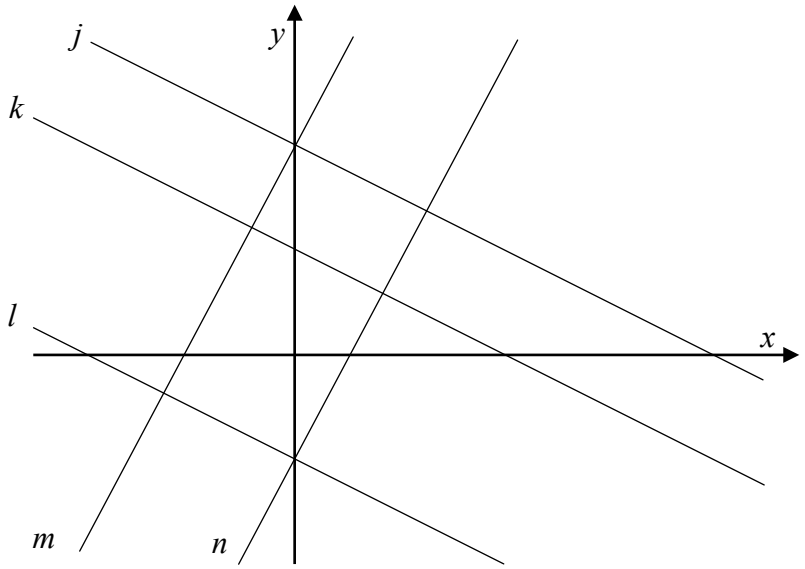
(i) Calculate the correlation coefficient.

Answer: _____

(ii) What kind of relationship, if any, do the observed data suggest exists between x and y?

Question 3 **(25 marks)**

In the co-ordinate diagram shown, the lines j, k, and l are parallel, and so are the lines m and n. The equations of four of the five lines are given in the table below.



Equation	Line
$x+2y=-4$	
$2x-y=-4$	
$x+2y=8$	
$2x-y=2$	

(a) Complete the table, by matching four of the lines to their equations.

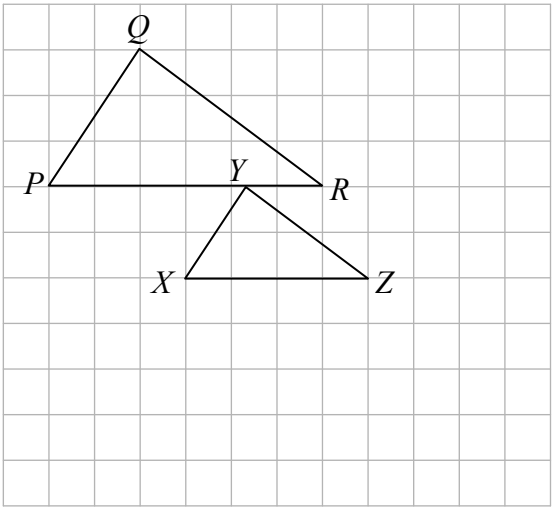
(b) Hence, insert scales on the x-axis and y-axis.

(c) Hence, find the equation of the remaining line, given that its x-intercept and y-intercept are both integers.

page	running

Question 4 **(25 marks)**

Two triangles are drawn on a square grid as shown. The points P, Q, R, X, and Z are on vertices of the grid, and the point Y lies on $[PR]$. The triangle PQR is an enlargement of the triangle XYZ.



(a) Calculate the scale factor of the enlargement, showing your work.

(b) By construction or otherwise, locate the centre of enlargement on the diagram above.

(c) Calculate $|YR|$ in grid units.

Question 5 **(25 marks)**

The line $x+3y=20$ intersects the circle $x^2+y^2-6x-8y=0$ at the points P and Q.
Find the equation of the circle that has $[PQ]$ as diameter.

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Question 6 **(25 marks)**

Answer **either** 6A **or** 6B.

Question 6A

Prove that if three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal line.

Diagram:

Given:

To prove:

Construction:

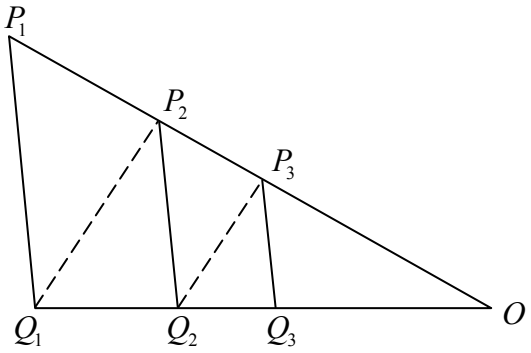
Proof:

OR

Question 6B

In the diagram, P_1Q_1, P_2Q_2, and P_3Q_3 are parallel and so also are Q_1P_2 and Q_2P_3.

Prove that $|P_1Q_1| \times |P_3Q_3| = |P_2Q_2|^2$.



Section B — Contexts and Applications — 150 marks

Answer Question 7 and Question 8.

Question 7 (75 marks)

(a) Some students are using a database of earthquakes to investigate the times between the occurrences of serious earthquakes around the world. They extract information about all of the earthquakes in the 20th century that caused at least 1000 deaths. There are 115 of these.

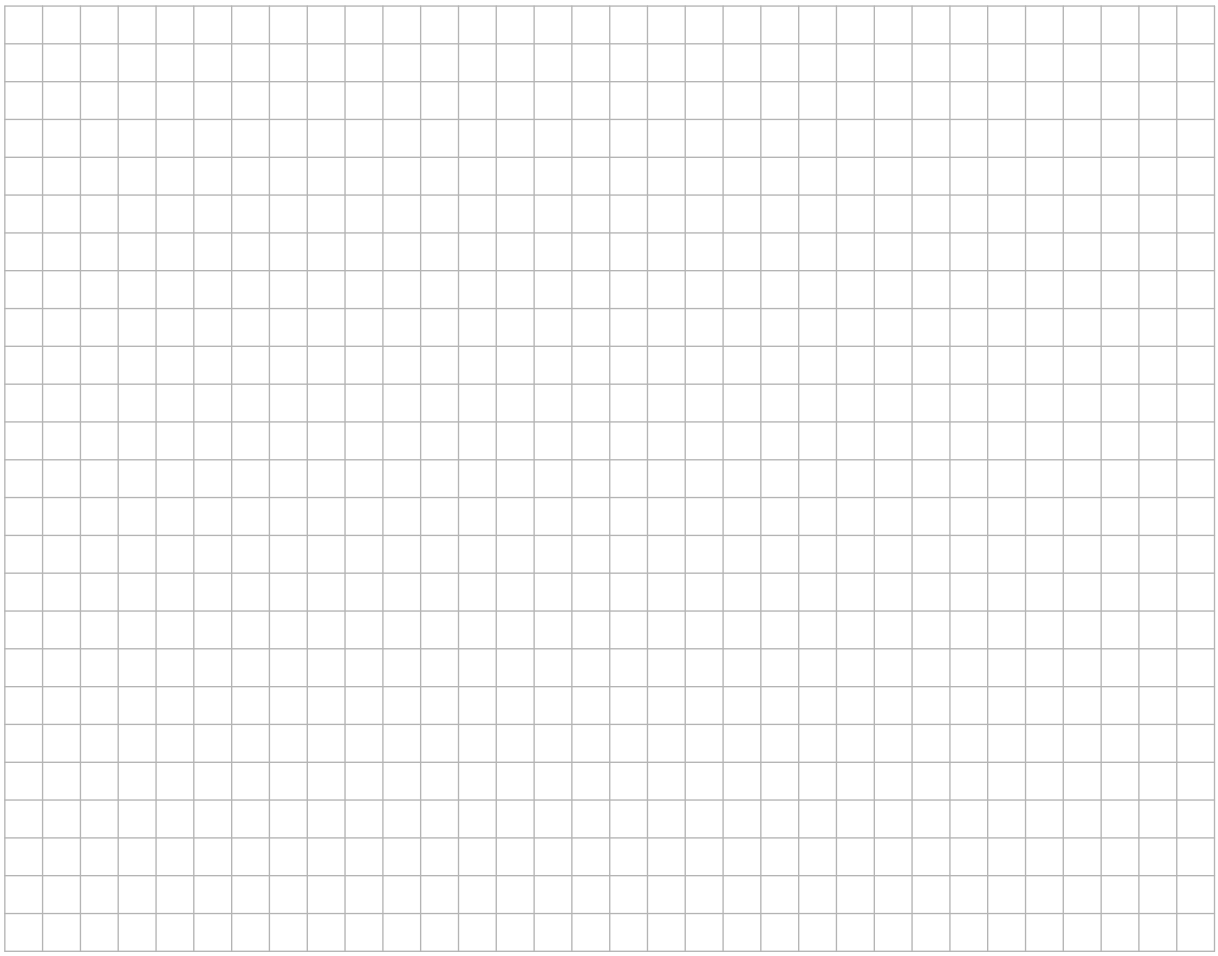
The students wonder whether there are patterns in the timing of these earthquakes, so they look at the number of days between each successive pair of these earthquakes.

They make the following table, showing the number of earthquakes for which the time interval from the previous earthquake is as shown.

Time in days from previous earthquake	0 – 100	100 – 200	200 – 300	300 – 400	400 – 500	500 – 600	600 – 700	700 – 800	800 – 1000	1000 – 1300
Number of earthquakes	31	24	12	14	8	7	5	6	5	3

[Source: National geophysical data center, significant earthquake database: www.ngdc.noaa.gov]

(i) Create a suitable graphical representation of the distribution.



(ii) Describe the distribution. Your description should refer to the shape of the distribution and should include an estimate of the median.

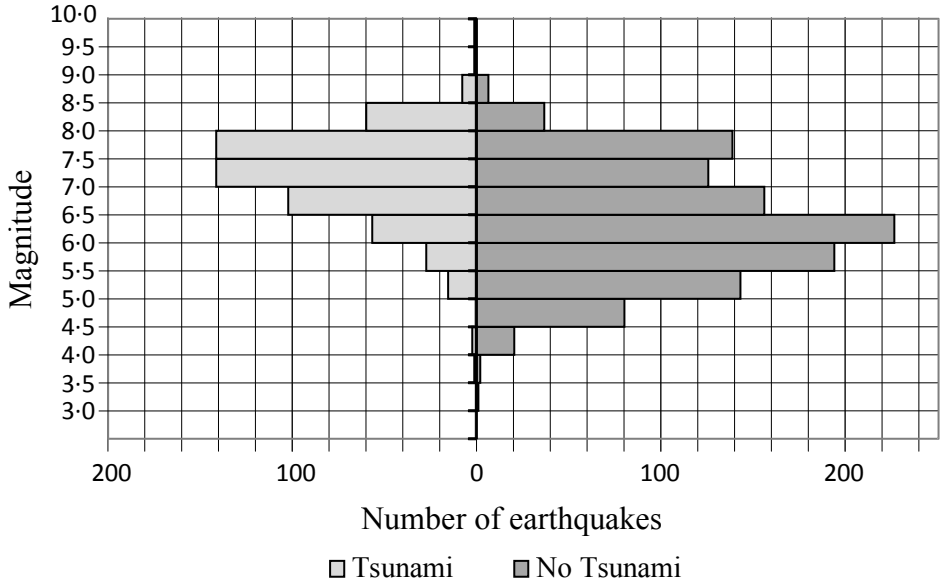
(iii) The mean time between these earthquakes is 309 days and the standard deviation is 277 days. Suppose that such an earthquake has just occurred and that we want to find the probability that the time to the next one will be between 100 and 200 days. Explain why it would **not** be correct to use standard normal distribution tables (z-tables) to do this.

(iv) Based on the information presented in this question so far, what is the best estimate for the probability described in part **(iii)** above? Explain your reasoning.

(v) As stated at the beginning, the students chose to analyse earthquake timings by looking at the time intervals between the occurrences of a particular type of earthquake. Suggest a different way that they could have looked at the data in the database in order to try to find patterns in the timing of earthquakes.

page	running

(b) The students heard a reporter saying that "strong earthquakes will cause large destructive ocean waves called tsunamis, while weaker ones will not". They decide to check this. They draw two histograms back to back, one showing the magnitudes of the earthquakes that caused tsunamis, and the other showing the magnitudes of those that did not. They use all of the suitable data from the 20th century that were recorded in this particular database.



(i) Comment on the reporter's statement, using information from the diagram to support your answer, and suggest a more accurate statement.

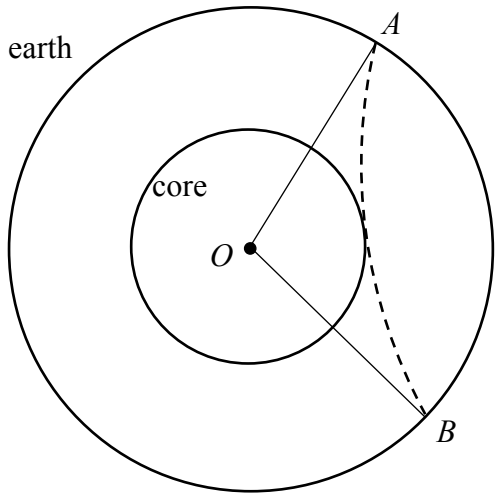
(ii) By taking suitable readings from the diagram, estimate the probability that an earthquake of magnitude between 6·5 and 7·0 will cause a tsunami.

(iii) Consider the next six earthquakes of magnitude at least 7·5. Find an estimate for the probability that at least four of them will cause a tsunami, assuming that these six events are independent of each other.

(c) Scientists use information about seismic waves from earthquakes to find out about the internal structure of the earth.

The diagram below represents a circular cross-section of the earth. The dashed curve represents the path of a seismic wave travelling through the earth from an earthquake near the surface at A to a monitoring station at B. The radius of the earth is 6·4 units and the path of this wave is a circular arc of radius 29·1 units, where 1 unit = 1000 km. Based on information from other stations, it is known that this particular path just touches the earth's core. The angle AOB measures 104°, where O is the centre of the earth.

Find the radius of the earth's core.
(There is space for work on the next page.)

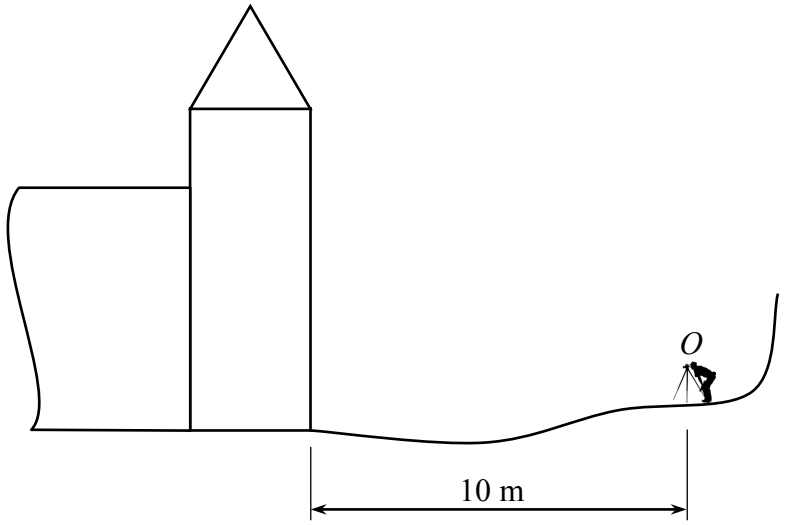


Question 8 **(75 marks)**

(a) A tower that is part of a hotel has a square base of side 4 metres and a roof in the form of a pyramid. The owners plan to cover the roof with copper. To find the amount of copper needed, they need to know the total area of the roof.

A surveyor stands 10 metres from the tower, measured horizontally, and makes observations of angles of elevation from the point *O* as follows:

The angle of elevation of the top of the roof is 46°.
The angle of elevation of the closest point at the bottom of the roof is 42°.
The angle of depression of the closest point at the bottom of the tower is 9°.



(i) Find the vertical height of the roof.

(ii) Find the total area of the roof.

(iii) If all of the angles observed are subject to a possible error of ±1°, find the range of possible areas for the roof.

(b) Twenty five students each measure and record a particular angle of elevation, in degrees, each using his or her own home-made clinometer. The results are as follows:

24	20	22	15	70
15	16	15	16	15
18	16	21	21	73
16	20	12	18	20
18	18	14	22	18

(i) Find what you consider to be the best estimate of the true value of the angle, explaining your reasoning.

(ii) Based on previous experience, a teacher has claimed that, in these circumstances, half of all students will measure the angle correctly to within two degrees. Taking these students to be a simple random sample, and assuming the true value of the angle is the one you calculated in part **(i)**, is there sufficient evidence to reject the teacher's claim at the 5% level of significance?

You may use this page for extra work.

You may use this page for extra work.

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Leaving Certificate 2011 – Higher Level

Mathematics (Project Maths – Phase 2) – Paper 2

Monday 13 June
Morning 9:30 – 12:00

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination, 2011
Sample Paper

Mathematics
(Project Maths – Phase 2)

Paper 2

Higher Level

Time: 2 hours, 30 minutes

300 marks

Examination number

Centre stamp

Running total	

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Grade

Instructions

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Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	2 questions

Answer **all eight** questions, as follows:

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Questions 1 to 5 and

either Question 6A **or** Question 6B.

In Section B, answer Question 7 and Question 8.

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Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Section A — Concepts and Skills — 150 marks

Answer **all six** questions from this section.

Question 1 (25 marks)

The random variable X has a discrete distribution. The probability that it takes a value other than 13, 14, 15 or 16 is negligible.

(a) Complete the probability distribution table below and hence calculate $E(X)$, the expected value of X.

x	13	14	15	16
$P(X = x)$	0·383	0·575		0·004

(b) If X is the age, in complete years, on 1 January 2010 of a student selected at random from among all second-year students in Irish schools, explain what $E(X)$ represents.

(c) If ten students are selected at random from this population, find the probability that exactly six of them were 14 years old on 1 January 2010. Give your answer correct to three significant figures.

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Question 2 **(25 marks)**

(a) Explain what is meant by *stratified sampling* and *cluster sampling*. Your explanation should include:

- a clear indication of the difference between the two methods
- one reason why each method might be chosen instead of simple random sampling.

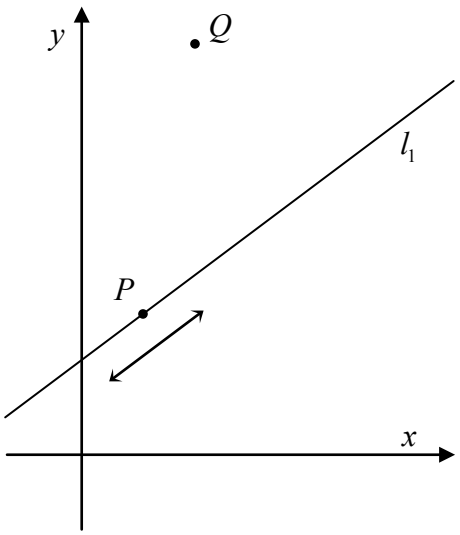
(b) A survey is being conducted of voters' opinions on several different issues.

(i) What is the overall margin of error of the survey, at 95% confidence, if it is based on a simple random sample of 1111 voters?

(ii) A political party had claimed that it has the support of 23% of the electorate. Of the voters in the sample above, 234 stated that they support the party. Is this sufficient evidence to reject the party's claim, at the 5% level of significance?

Question 3 **(25 marks)**

(a) Show that, for all $k \in \mathbb{R}$, the point $P(4k-2, 3k+1)$ lies on the line $l_1 : 3x - 4y + 10 = 0$.



(b) The line l_2 passes through P and is perpendicular to l_1. Find the equation of l_2, in terms of k.

(c) Find the value of k for which l_2 passes through the point $Q(3, 11)$.

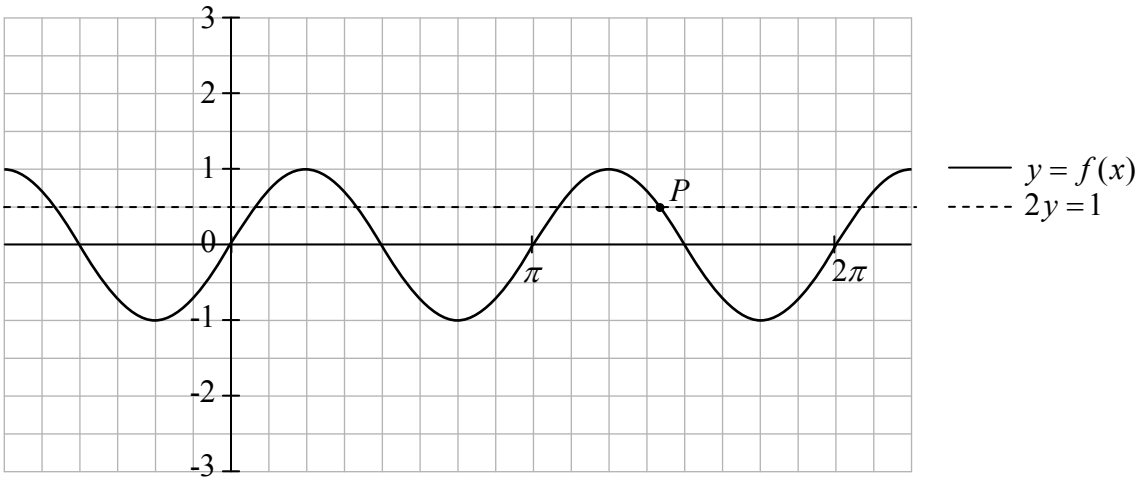
(d) Hence, or otherwise, find the co-ordinates of the foot of the perpendicular from Q to l_1.

Question 4 **(25 marks)**

The centre of a circle lies on the line $x + 2y - 6 = 0$. The x-axis and the y-axis are tangents to the circle. There are two circles that satisfy these conditions. Find their equations.

Question 5 **(25 marks)**

The diagram below shows the graph of the function $f : x \mapsto \sin 2x$. The line $2y = 1$ is also shown.



(a) On the same diagram above, sketch the graphs of $g : x \mapsto \sin x$ and $h : x \mapsto 3\sin 2x$. Indicate clearly which is *g* and which is *h*.

(b) Find the co-ordinates of the point *P* in the diagram.

Question 6 **(25 marks)**

Answer **either** 6A **or** 6B.

Question 6A

Explain, with the aid of an example, what is meant by *proof by contradiction*.

Note: you do not need to provide the full proof involved in your example. Give sufficient outline to illustrate how contradiction is used.

Explanation:

Example:

OR

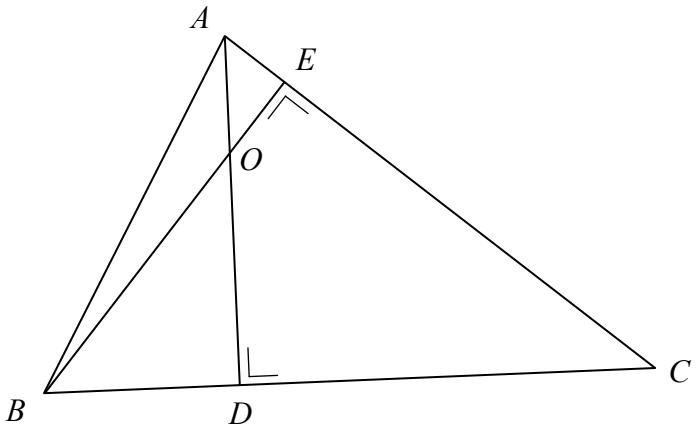
Question 6B

ABC is a triangle.

D is the point on BC such that $AD \perp BC$.
E is the point on AC such that $BE \perp AC$.

AD and BE intersect at O.

Prove that $|\angle DOC| = |\angle DEC|$.



Section B — Contexts and Applications — 150 marks

Answer Question 7 and Question 8.

Question 7 (75 marks)

The *King of the Hill* triathlon race in Kinsale consists of a 750 metre swim, followed by a 20 kilometre cycle, followed by a 5 kilometre run.

The questions below are based on data from 224 athletes who completed this triathlon in 2010.

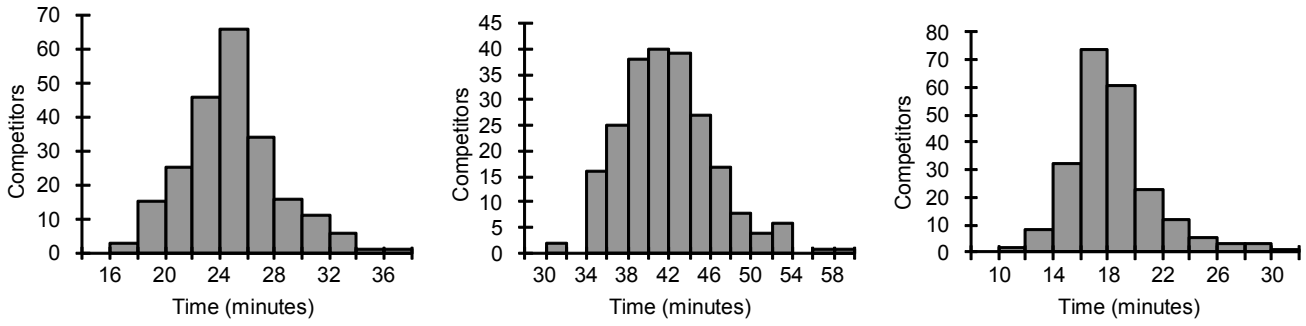
Máire is analysing data from the race, using statistical software. She has a data file with each competitor's time for each part of the race, along with various other details of the competitors.

Lizzie Lee, winner of the women's event

Máire gets the software to produce some *summary statistics* and it produces the following table. Three of the entries in the table have been removed and replaced with question marks (?).

	Swim	*Cycle*	*Run*
Mean	18.329	41.927	**?**
Median	17.900	41.306	**?**
Mode	#N/A	#N/A	#N/A
Standard Deviation	**?**	4.553	3.409
Sample Variance	10.017	20.729	11.622
Skewness	1.094	0.717	0.463
Range	19.226	27.282	20.870
Minimum	11.350	31.566	16.466
Maximum	30.576	58.847	37.336
Count	224	224	224

Máire produces histograms of the times for the three events. Here are the three histograms, without their titles.



(a) **(i)** Use the summary statistics in the table to decide which histogram corresponds to each event. Write the answers above the histograms.

(ii) The mean and the median time for the run are approximately equal. Estimate this value from the corresponding histogram.

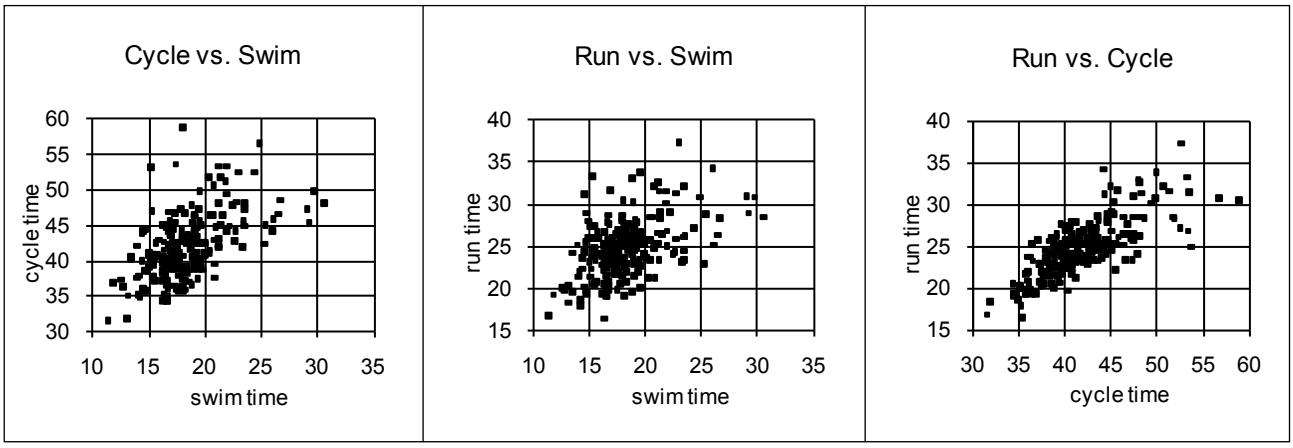
mean ≈ median ≈ _____

(iii) Estimate from the relevant histogram the standard deviation of the times for the swim.

standard deviation ≈ _____

(iv) When calculating the summary statistics, the software failed to find a *mode* for the data sets. Why do you think this is?

Máire is interested in the relationships between the athletes' performance in the three different events. She produces the following three scatter diagrams



(b) Give a brief summary of the relationship between performance in the different events, based on the scatter diagrams.

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(c) The best-fit line for run-time based on swim-time is $y = 0{\cdot}53x + 15{\cdot}2$. The best-fit line for run-time based on cycle-time is $y = 0{\cdot}58x + 0{\cdot}71$. Brian did the swim in 17·6 minutes and the cycle in 35·7 minutes. Give your best estimate of Brian's time for the run, and justify your answer.

The mean finishing time for the overall event was 88·1 minutes and the standard deviation was 10·3 minutes.

(d) Based on an assumption that the distribution of overall finishing times is approximately normal, use the *empirical rule* to complete the following sentence:

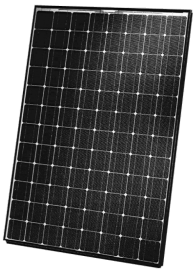
"95% of the athletes took between _____ and _____ minutes to complete the race."

(e) Using normal distribution tables, estimate the number of athletes who completed the race in less than 100 minutes.

(f) After the event, a reporter wants to interview two people who took more than 100 minutes to complete the race. She approaches athletes at random and asks them their finishing time. She keeps asking until she finds someone who took more than 100 minutes, interviews that person, and continues until she finds a second such person. Assuming the athletes are cooperative and truthful, what is the probability that the second person she interviews will be the sixth person she approaches?

Question 8 **(75 marks)**

(a) A stand is being used to prop up a portable solar panel. It consists of a support that is hinged to the panel near the top, and an adjustable strap joining the panel to the support near the bottom.



By adjusting the length of the strap, the angle between the panel and the ground can be changed.

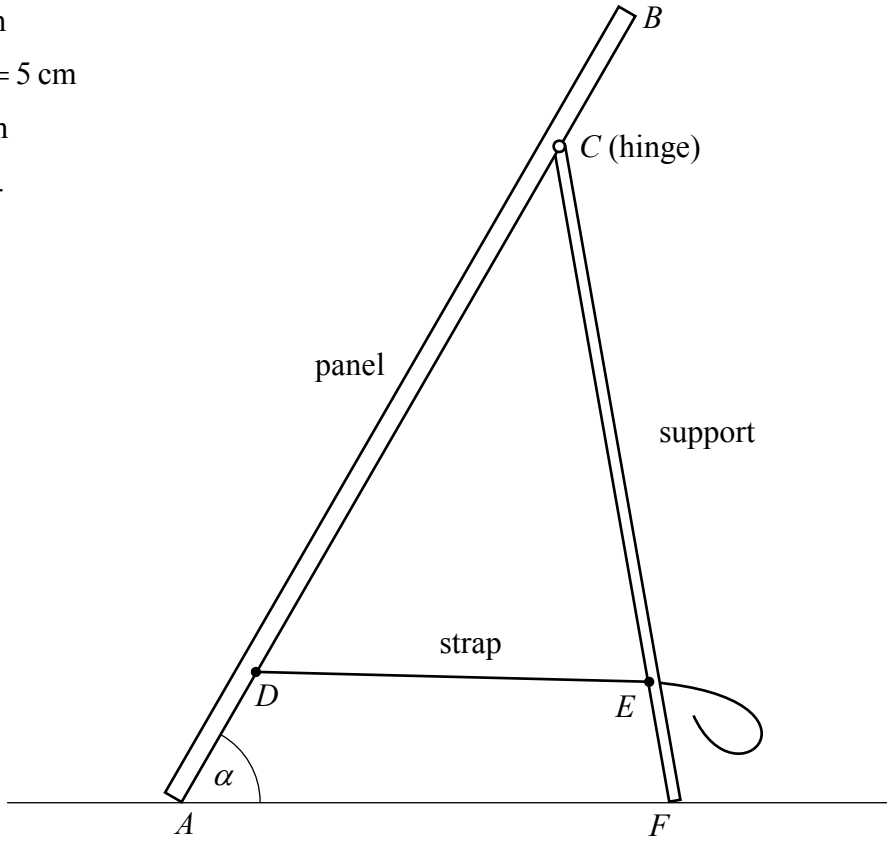
The dimensions are as follows:

$|AB| = 30$ cm

$|AD| = |CB| = 5$ cm

$|CF| = 22$ cm

$|EF| = 4$ cm.



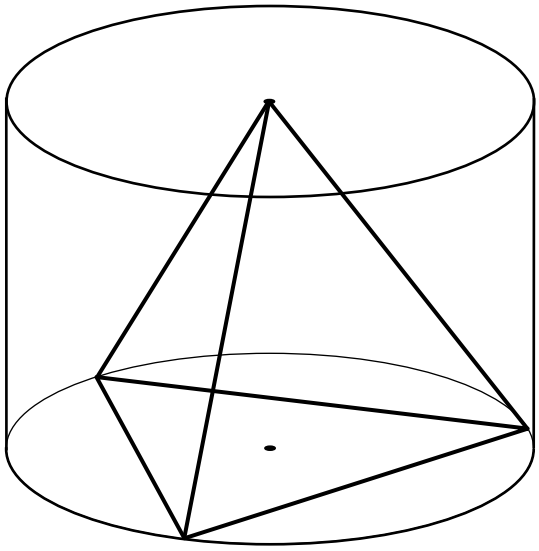
(i) Find the length of the strap [DE] such that the angle α between the panel and the ground is 60°.

(ii) Find the maximum possible value of α, correct to the nearest degree.

(b) A regular tetrahedron has four faces, each of which is an equilateral triangle.

A wooden puzzle consists of several pieces that can be assembled to make a regular tetrahedron. The manufacturer wants to package the assembled tetrahedron in a clear cylindrical container, with one face flat against the bottom.

If the length of one edge of the tetrahedron is $2a$, show that the volume of the smallest possible cylindrical container is $\left(\frac{8\sqrt{6}}{9}\right)\pi a^3$.



You may use this page for extra work

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Note to readers of this document:

This sample paper is intended to help teachers and candidates prepare for the June 2011 examination in the *Project Maths* initial schools. The content and structure do not necessarily reflect the 2012 or subsequent examinations in the initial schools or in all other schools.

Leaving Certificate 2011 – Higher Level

Mathematics (Project Maths – Phase 2) – Paper 2

Sample Paper
Time: 2 hours 30 minutes

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination

Mathematics (Project Maths)

Paper 2

Higher Level

Monday 14 June Morning 9:30 – 12:00

300 marks

Examination number

Centre stamp

Running total	

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Grade

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer **all nine** questions, as follows:

In Section A, answer all six questions.

In Section B, answer:

Question 7

Question 8

either Question 9A **or** Question 9B.

Write your answers in the spaces provided in this booklet. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

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Answers should include the appropriate units of measurement, where relevant.

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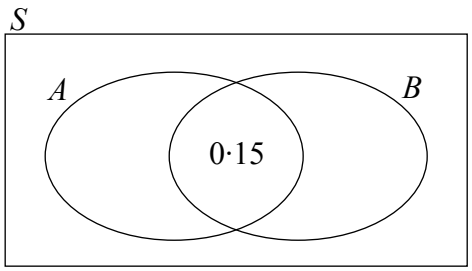
Section A — Concepts and Skills — 150 marks

Answer **all six** questions from this section.

Question 1 (25 marks)

Two events A and B are such that $P(A) = 0{\cdot}2$, $P(A \cap B) = 0{\cdot}15$ and $P(A' \cap B) = 0{\cdot}6$.

(a) Complete this Venn diagram.



(b) Find the probability that neither A nor B happens.

(c) Find the conditional probability $P(A \mid B)$.

(d) State whether A and B are independent events and justify your answer.

Question 2 **(25 marks)**

(a) ~~The back-to-back stem-and-leaf diagram below shows data from two samples. The corresponding populations are assumed to be identical in shape and spread. Use the *Tukey quick test* to test, at the 5% significance level, the hypothesis that the populations have the same average~~.

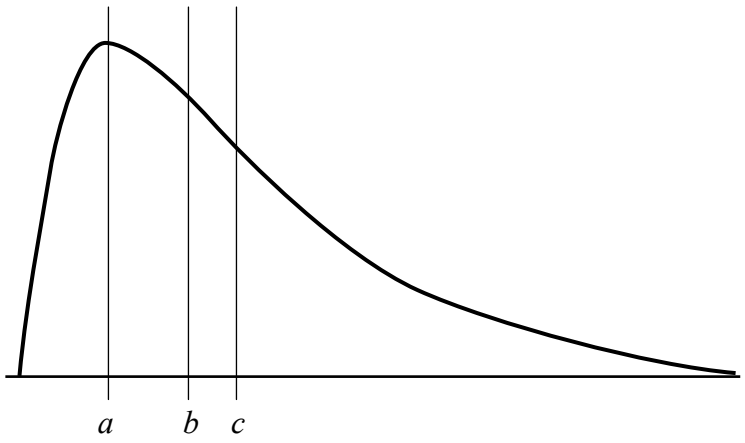
				6	3						
8	6	6	3	0	4	4					
	5	3	2	0	5	1	5	5	7	7	8	9
		2	0	0	6	1	4	9				
					7	2	4					
					8	1						

key: 6 | 3 | means 36 key: | 8 | 1 means 81

This topic is no longer on the syllabus.

Conclusion:

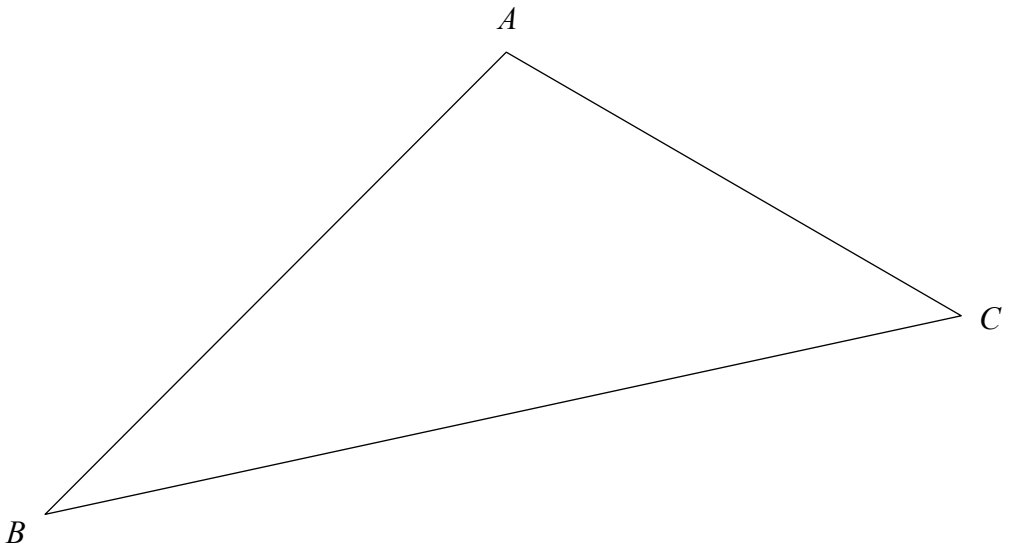
(b) The diagram below shows a skewed frequency distribution. Vertical lines have been drawn through the mean, mode and median. Identify which is which by inserting the relevant letters in the spaces below.



mean = _____ mode = _____ median = _____

Question 3 **(25 marks)**

(a) Construct the incircle of the triangle *ABC* below using only a compass and straight edge. Show all construction lines clearly.



(b) An equilateral triangle has sides of length 2 units.
Find the area of its incircle.

page	running

Question 4 **(25 marks)**

(a) The centre of a circle lies on the line $x-2y-1=0$. The *x*-axis and the line $y=6$ are tangents to the circle. Find the equation of this circle.

(b) A different circle has equation $x^2+y^2-6x-12y+41=0$. Show that this circle and the circle in part **(a)** touch externally.

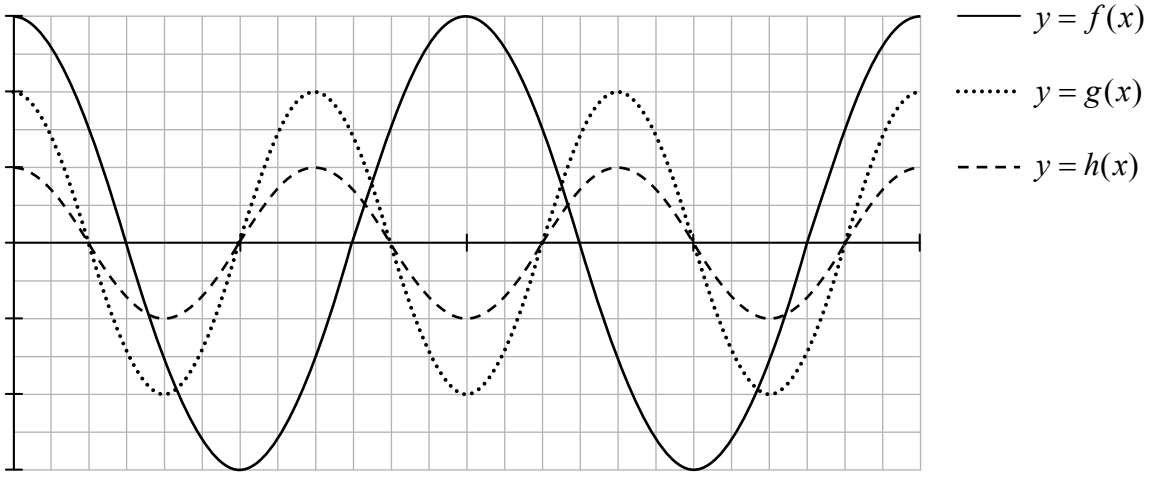
Question 5 **(25 marks)**

(a) Solve the equation $\cos 3\theta = \frac{1}{2}$, for $\theta \in \mathbb{R}$, (where θ is in radians).

(b) The graphs of three functions are shown on the diagram below. The scales on the axes are not labelled. The three functions are:

$$x \to \cos 3x$$
$$x \to 2\cos 3x$$
$$x \to 3\cos 2x$$

Identify which function is which, and write your answers in the spaces below the diagram.



$f : x \to$ _____ $g : x \to$ _____ $h : x \to$ _____

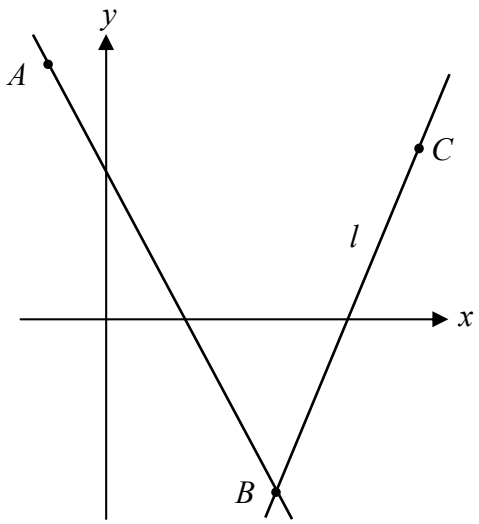
(c) Label the scales on the axes in the diagram in part **(b)**.

page	running

Question 6 **(25 marks)**

Three points A, B and C have co-ordinates:
$A(-2,9)$, $B(6,-6)$ and $C(11,6)$.

The line l passes through B and has equation $12x-5y-102=0$.



(a) Verify that C lies on l.

(b) Find the slope of AB, and hence find $\tan(\angle ABC)$, as a fraction.

(c) ~~Find the vectors $\overrightarrow{BC}$ and $\overrightarrow{BA}$ in terms of $\vec{i}$ and $\vec{j}$.~~

This topic is no longer on the syllabus.

(d) ~~Use the dot product to find~~ $\cos(\angle ABC)$ ~~and show that the answer is consistent with the answer to part **(b)**.~~

This topic is no longer on the syllabus.

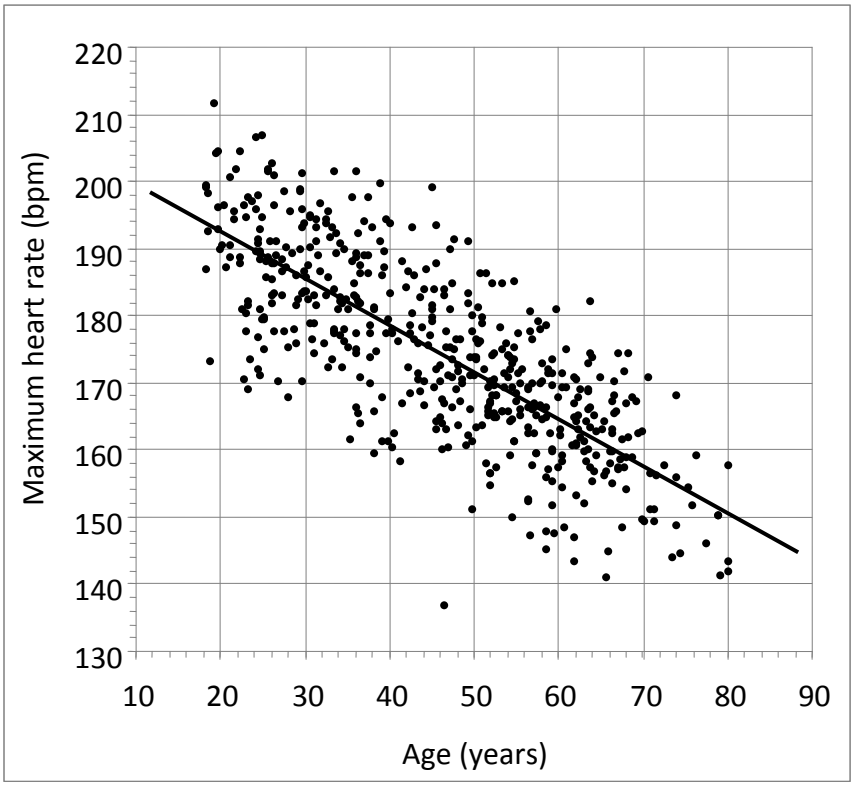
You may use this space for extra work.

Section B	Contexts and Applications	150 marks

Answer Question 7, Question 8, and **either** Question 9A **or** Question 9B.

Question 7 **Probability and Statistics** **(50 marks)**

A person's *maximum heart rate* is the highest rate at which their heart beats during certain extreme kinds of exercise. It is measured in beats per minute (bpm). It can be measured under controlled conditions. As part of a study in 2001, researchers measured the maximum heart rate of 514 adults and compared it to each person's age. The results were like those shown in the scatter plot below.



Source: Simulated data based on:Tanaka H, Monaghan KD, and Seals DR. *Age-predicted maximal heart rate revisited*, J. Am. Coll. Cardiol. 2001;37;153-156.

(a) From the diagram, estimate the correlation coefficient. Answer: ____

(b) Circle the *outlier* on the diagram and write down the person's age and maximum heart rate.

Age = ____ Max. heart rate = ____

(c) The line of best fit is shown on the diagram. Use the line of best fit to estimate the maximum heart rate of a 44-year-old person.

Answer: ____

(d) By taking suitable readings from the diagram, calculate the slope of the line of best fit.

(e) Find the equation of the line of best fit and write it in the form: $MHR = a - b \times (\text{age})$, where *MHR* is the maximum heart rate.

(f) The researchers compared their new rule for estimating maximum heart rate to an older rule. The older rule is: $MHR = 220 - \text{age}$. The two rules can give different estimates of a person's maximum heart rate. Describe how the level of agreement between the two rules varies according to the age of the person. Illustrate your answer with two examples.

(g) A particular exercise programme is based on the idea that a person will get most benefit by exercising at 75% of their estimated *MHR*. A 65-year-old man has been following this programme, using the old rule for estimating *MHR*. If he learns about the researchers' new rule for estimating *MHR*, how should he change what he is doing?

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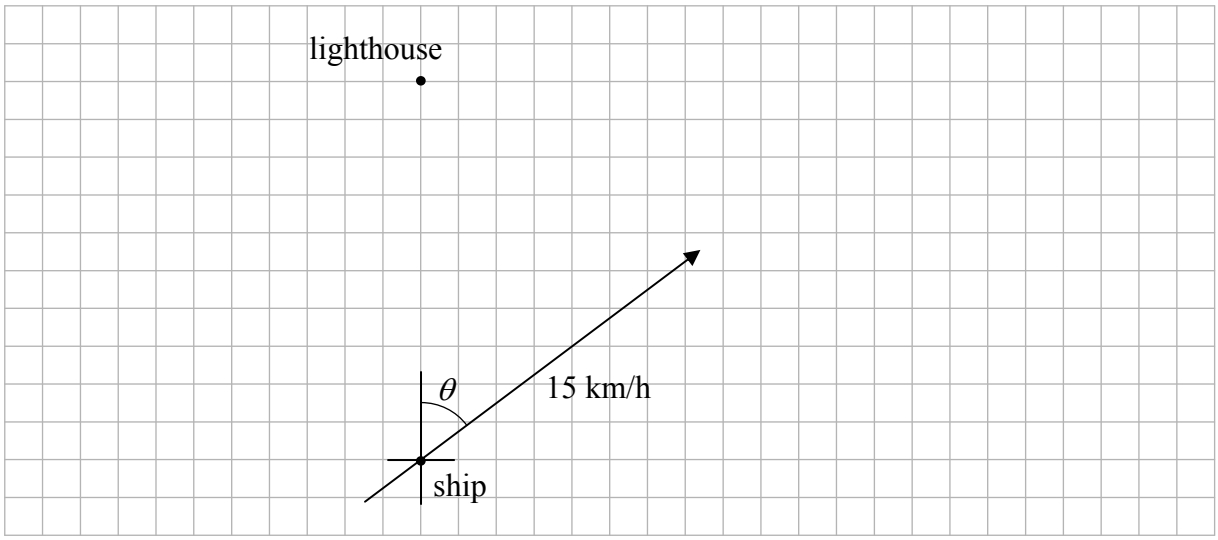
Question 8 **Geometry and Trigonometry** **(50 marks)**

A ship is 10 km due South of a lighthouse at noon.

The ship is travelling at 15 km/h on a bearing of θ, as shown below, where $\theta = \tan^{-1}\left(\frac{4}{3}\right)$.



(a) On the diagram above, draw a set of co-ordinate axes that takes the lighthouse as the origin, the line East-West through the lighthouse as the *x*-axis, and kilometres as units.

(b) Find the equation of the line along which the ship is moving.

(c) Find the shortest distance between the ship and the lighthouse during the journey.

(d) At what time is the ship closest to the lighthouse?

(e) Visibility is limited to 9 km. For how many minutes in total is the ship visible from the lighthouse?

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Question 9A **Probability and Statistics** **(50 marks)**

A factory manufactures aluminium rods. One of its machines can be set to produce rods of a specified length. The lengths of these rods are normally distributed with mean equal to the specified length and standard deviation equal to 0·2 mm.

The machine has been set to produce rods of length 40 mm.

(a) What is the probability that a randomly selected rod will be less than 39·7 mm in length?

(b) Five rods are selected at random. What is the probability that at least two of them are less than 39·7 mm in length?

(c) The operators want to check whether the setting on the machine is still accurate. They take a random sample of ten rods and measure their lengths. The lengths in millimetres are:

39·5	40·0	39·7	40·2	39·8
39·7	40·2	39·9	40·1	39·6

Conduct a hypothesis test at the 5% level of significance to decide whether the machine's setting has become inaccurate. You should start by clearly stating the null hypothesis and the alternative hypothesis, and finish by clearly stating what you conclude about the machine.

This has been moved to deferred material and hence will not be examined in 2012

Question 9B **Geometry and Trigonometry** **(50 marks)**

(a) Prove that if three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal line.

Diagram:

Given:

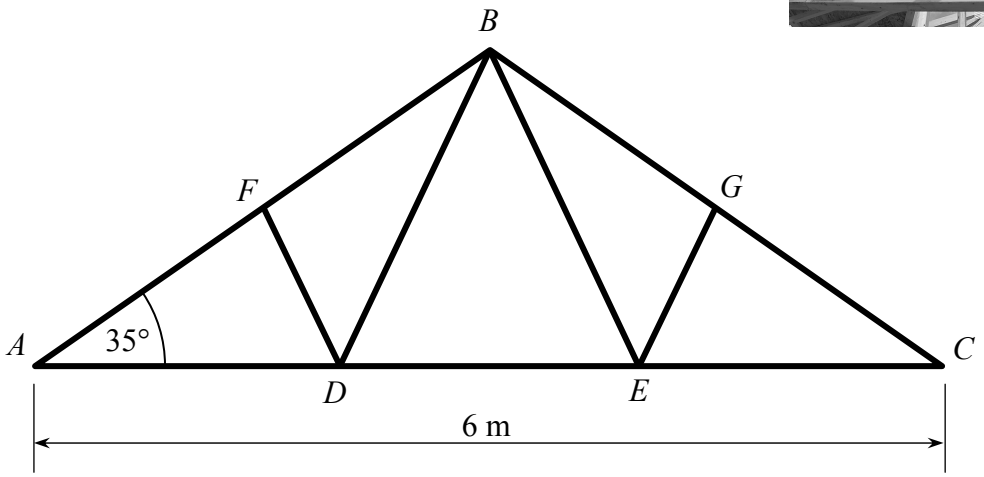
To prove:

Construction:

Proof:

(b) Roofs of buildings are often supported by frameworks of timber called *roof trusses*.

A quantity surveyor needs to find the total length of timber needed in order to make the triangular truss shown below.



The length of $[AC]$ is 6 metres, and the pitch of the roof is 35°, as shown.
$|AD| = |DE| = |EC|$ and $|AF| = |FB| = |BG| = |GC|$.

(i) Calculate the length of $[AB]$, in metres, correct to two decimal places.

(ii) Calculate the total length of timber required to make the truss.

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You may use this page for extra work

You may use this page for extra work

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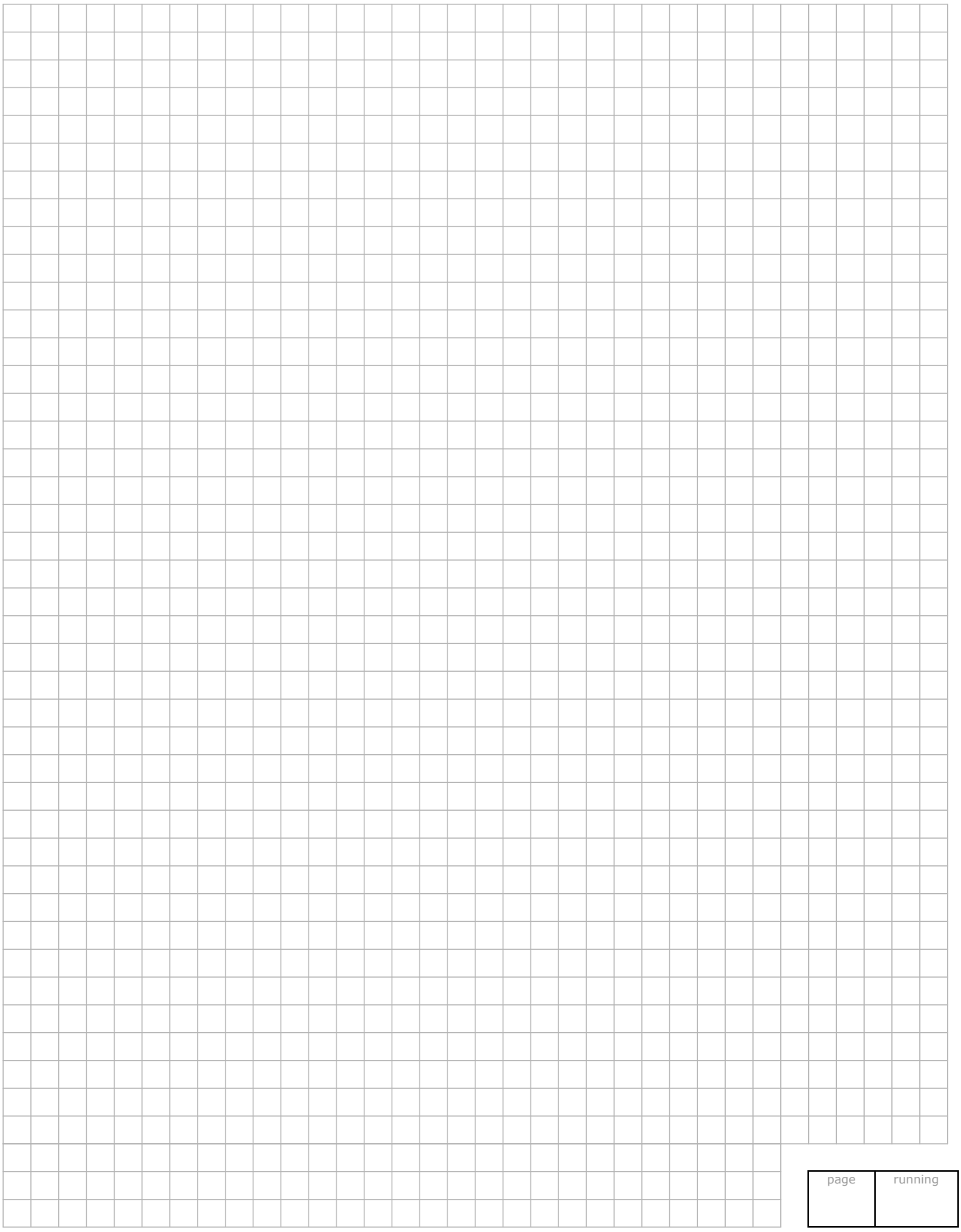

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Leaving Certificate – Higher Level

Mathematics (Project Maths) – Paper 2

Monday 14 June
Morning 9:30 – 12:00

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination
Sample Paper

Mathematics (Project Maths)

Paper 2

Higher Level

Time: 2 hours, 30 minutes

300 marks

Examination number

Centre stamp

Running total	

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Grade

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer **all nine** questions, as follows:

In Section A, answer all six questions

In Section B, answer:

Question 7

Question 8

either Question 9A **or** Question 9B.

Write your answers in the spaces provided in this booklet. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the booklet of *Formulae and Tables*. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Section A — Concepts and Skills — 150 marks

Answer **all six** questions from this section.

Question 1 (25 marks)

The events A and B are such that $P(A) = 0{\cdot}7$, $P(B) = 0{\cdot}5$ and $P(A \cap B) = 0{\cdot}3$.

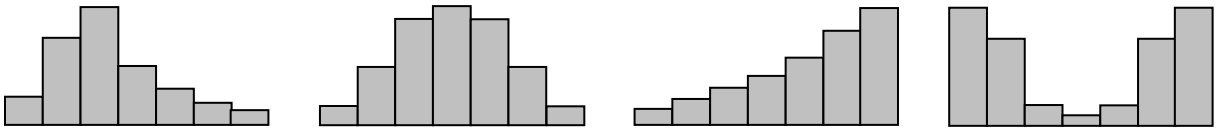
(a) Find $P(A \cup B)$

(b) Find $P(A \mid B)$

(c) State whether A and B are independent events, and justify your answer.

Question 2 **(25 marks)**

The shapes of the histograms of four different sets of data are shown below.



A B C D

(a) Complete the table below, indicating whether the statement is correct (✓) or incorrect (✗) with respect to each data set.

	A	B	C	D
The data are skewed to the left				
The data are skewed to the right				
The mean is equal to the median				
The mean is greater than the median				
There is a single mode				

(b) Assume that the four histograms are drawn on the same scale.
State which of them has the largest standard deviation, and justify your answer.

Answer: _____

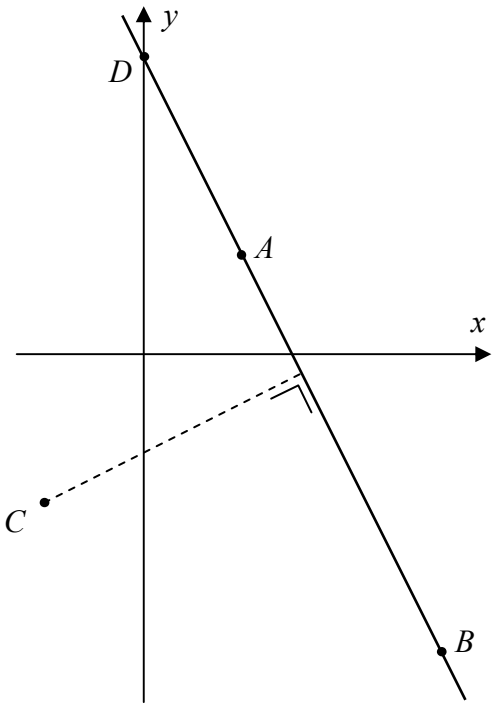
Justification:

Question 3 **(25 marks)**

The co-ordinates of three points A, B, and C are: $A(2, 2)$, $B(6, -6)$, $C(-2, -3)$.
(See diagram on facing page.)

(a) Find the equation of AB.

(b) The line AB intersects the y-axis at D.
Find the coordinates of D.



(iii) Find the perpendicular distance from C to AB.

(iv) Hence, find the area of the triangle ADC.

Question 4 **(25 marks)**

(a) Write down the equation of the circle with centre (–3, 2) and radius 4.

(b) A circle has equation $x^2 + y^2 - 2x + 4y - 15 = 0$.
Find the values of m for which the line $mx + 2y - 7 = 0$ is a tangent to this circle.

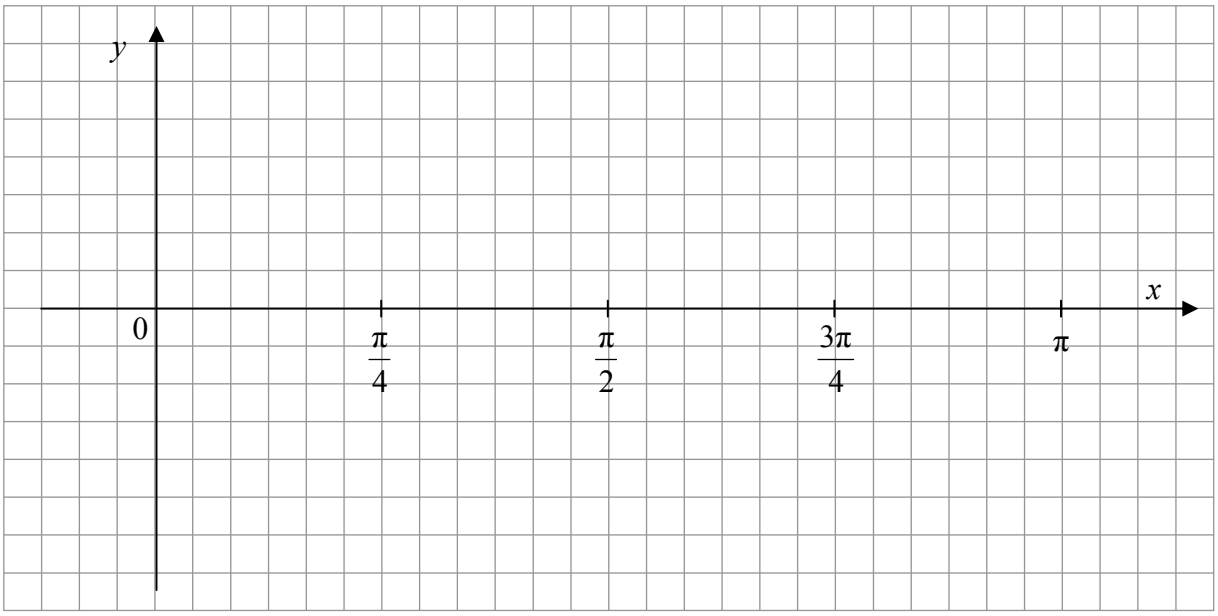
Question 5 **(25 marks)**

The function $f(x) = 3\sin(2x)$ is defined for $x \in \mathbb{R}$.

(i) Complete the table below

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$2x$					
$\sin(2x)$					
$3\sin(2x)$					

(ii) Draw the graph of $y = f(x)$ in the domain $0 \le x \le \pi,\ x \in \mathbb{R}$.



(iii) Write down the range and the period of f.

Range = _____ Period = _____

Question 6 **(25 marks)**

$ABCD$ is a parallelogram in which $[AC]$ is a diagonal.
$\vec{a} = 2\vec{i} - \vec{j}$, $\vec{b} = 5\vec{i} + 3\vec{j}$, and $\vec{c} = -\vec{i} - \vec{j}$.

(i) Express $\vec{d}$ in terms of $\vec{i}$ and $\vec{j}$.

This topic is no longer on the syllabus.

(ii) Find $\overrightarrow{AC}$ and $\overrightarrow{AB}$.

This topic is no longer on the syllabus.

(iii) Hence, find $|\angle CAB|$, correct to the nearest degree.

This topic is no longer on the syllabus.

You may use this page for extra work

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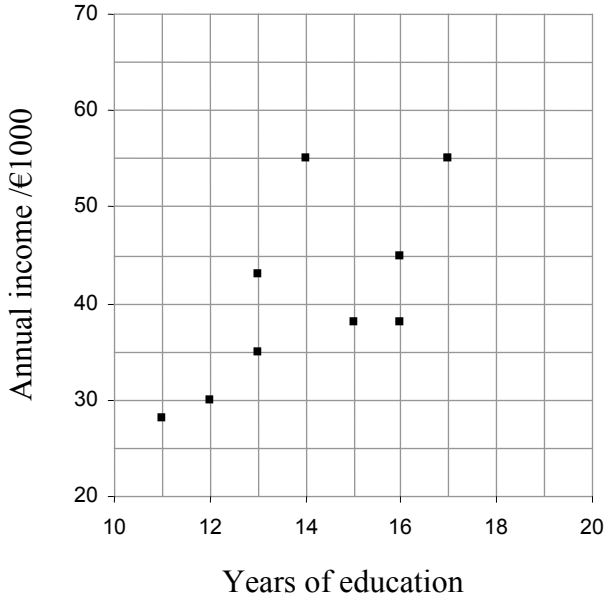
Section B — Contexts and Applications — 150 marks

Answer Question 7, Question 8, and **either** Question 9A **or** Question 9B.

Question 7 — Probability and Statistics — (50 marks)

An economics student is interested in finding out whether the length of time people spend in education affects the income they earn. The student carries out a small study. Twelve adults are asked to state their annual income and the number of years they spent in full-time education. The data are given in the table below, and a partially completed scatter plot is given.

Years of education	Income /€1,000
11	28
12	30
13	35
13	43
14	55
15	38
16	45
16	38
17	55
17	60
17	30
19	58



(i) The last three rows of data have not been included on the scatter plot. Insert them now.

(ii) Calculate the correlation coefficient.

Answer:

(iii) What can you conclude from the scatter plot and the correlation coefficient?

(iv) Add the line of best fit to the completed scatter plot above.

(v) Use the line of best fit to estimate the annual income of somebody who has spent 14 years in education.

Answer:

(vi) By taking suitable readings from your diagram, or otherwise, calculate the slope of the line of best fit.

(vii) Explain how to interpret this slope in this context?

(viii) The student collected the data using a telephone survey. Numbers were randomly chosen from the Dublin area telephone directory. The calls were made in the evenings, between 7 and 9 pm. If there was no answer, or if the person who answered did not agree to participate, then another number was chosen at random.

List **three** possible problems regarding the sample and how it was collected that might make the results of the investigation unreliable. In each case, state clearly why the issue you mention could cause a problem.

Problem 1:

Problem 2:

Problem 3:

page	running

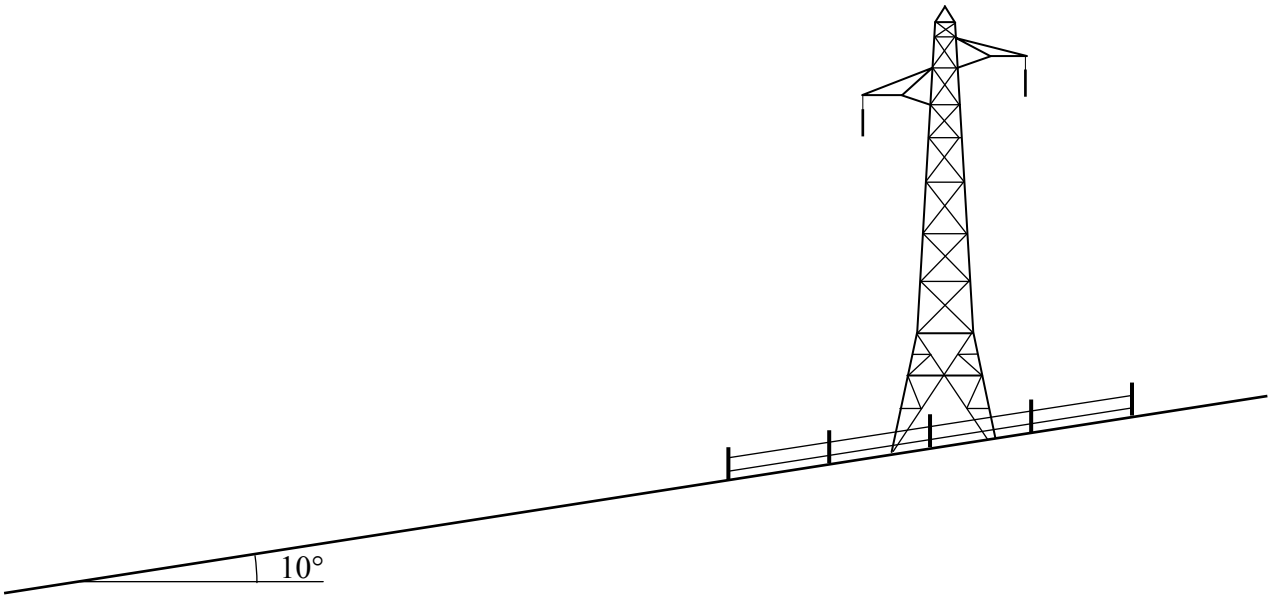
Question 8 **Geometry and Trigonometry** **(50 marks)**

Two surveyors want to find the height of an electricity pylon. There is a fence around the pylon that they cannot cross for safety reasons. The ground is inclined at an angle. They have a clinometer (for measuring angles of elevation) and a 100 metre tape measure. They have already used the clinometer to determine that the ground is inclined at 10° to the horizontal.

(a) Explain how they could find the height of the pylon.

Your answer should be illustrated on the diagram below. Show the points where you think they should take measurements, write down clearly what measurements they should take, and outline briefly how these can be used to find the height of the pylon.

Diagram:



Measurements to be taken:

Procedure used to find the height:

(b) Write down possible values for the measurements taken, and use them to show how to find the height of the pylon. (That is, find the height of the pylon using your measurements, and showing your work.)

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Question 9A **Probability and Statistics** **(50 marks)**

A car rental company has been using *Evertread* tyres on their fleet of economy cars. All cars in this fleet are identical. The company manages the tyres on each car in such a way that the four tyres all wear out at the same time. The company keeps a record of the lifespan of each set of tyres. The records show that the lifespan of these sets of tyres is normally distributed with mean 45 000 km and standard deviation 8000 km.

(i) A car from the economy fleet is chosen at random. Find the probability that the tyres on this car will last for at least 40 000 km.

(ii) Twenty cars from the economy fleet are chosen at random. Find the probability that the tyres on at least eighteen of these cars will last for more than 40 000 km.

(iii) The company is considering switching brands from *Evertread* tyres to *SafeRun* tyres, because they are cheaper. The distributors of *SafeRun* tyres claim that these tyres have the same mean lifespan as *Evertread* tyres. The car rental company wants to check this claim before they switch brands. They have enough data on *Evertread* tyres to regard these as a known population. They want to test a sample of *SafeRun* tyres against it.

The company selects 25 economy cars at random from the fleet and fits them with the new tyres. For these cars, it is found that the mean life span of the tyres is 43 850 km.

Test, at the 5% level of significance, the hypothesis that the mean lifespan of *SafeRun* tyres is the same as the known mean of *Evertread* tyres. State clearly what the company can conclude about the tyres.

This has been moved to deferred material and hence will not be examined in 2012

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Question 9B **Geometry and Trigonometry** **(50 marks)**

(a) Prove that, if two triangles ΔABC and $\Delta A'B'C'$ are similar, then their sides are proportional, in order:

$$\frac{|AB|}{|A'B'|} = \frac{|BC|}{|B'C'|} = \frac{|CA|}{|C'A'|}.$$

Diagram:

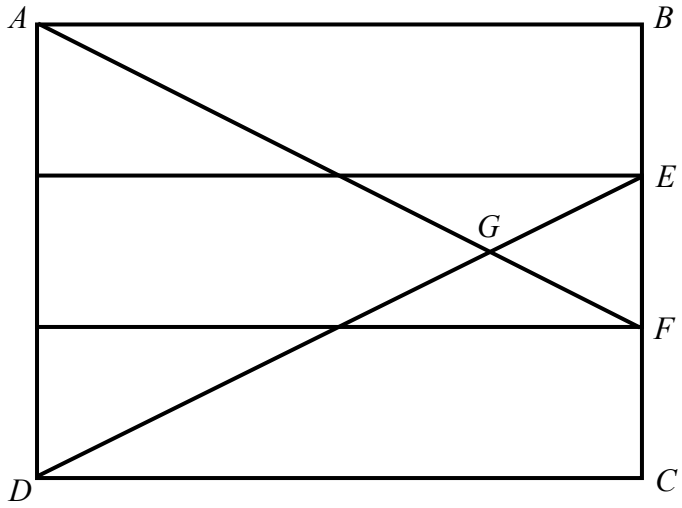
Given:

To prove:

Construction:

Proof:

(b) Anne is having a new front gate made and has decided on the design below.



The gate is 2 metres wide and 1·5 metres high. The horizontal bars are 0·5 metres apart.

(i) Calculate the common length of the bars [AF] and [DE], in metres, correct to three decimal places.

(ii) In order to secure the bar [AF] to [DE], the manufacturer needs to know:

- the measure of the angle EGF, and
- the common distance $|AG| = |DG|$.

Find these measures. Give the angle correct to the nearest degree and the length correct to three decimal places.

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You may use this page for extra work

You may use this page for extra work

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Note to readers of this document:

This sample paper is intended to help teachers and candidates prepare for the June 2010 examination in the *Project Maths* initial schools. The content and structure do not necessarily reflect the 2011 or subsequent examinations in the initial schools or in all other schools.

Leaving Certificate – Higher Level

Mathematics (Project Maths) – Paper 2

Sample Paper
Time: 2 hours 30 minutes