Senior cycle

TEACHER HANDBOOK

Senior cycle Ordinary Level

5th & 6th year

DRAFT

Project Maths Tionscadal Mata Development Team

Project Maths Tionscadal Mata Development Team

Contents

Introduction

Student Learning

While this is a handbook for teachers, it must be emphasised that student learning and the process of mathematical thinking and building understanding are the main focus of this document.

Information and Communications Technologies are used whenever and wherever appropriate to help to support student learning. In this document the  symbol appears at the corresponding position of the content to indicate that an interactive IT module is available on the Project Maths Student's CD.

Students with mild general learning disabilities

Teachers are reminded that the NCCA Guidelines on mathematics for students with mild general learning disabilities can be accessed at

http://www.ncca.ie/uploadedfiles/PP_Maths.pdf

This document includes

- Approaches and Methodologies (from Page 4)
- Exemplars (from page 20).

Note on Strand 2 Synthetic Geometry (see syllabus)

"In the examination, candidates will have the option of answering a question on the synthetic geometry set out here, or answering a problem- solving question based on the geometrical results from the corresponding syllabus level at Junior Certificate. This option will apply for a three year period only, for candidates sitting the Leaving Certificate in 2012, 2013 and 2014. There will be no choice after that period."

Note on timeline for examinations for national roll out

Strands 1 and 2 will be examined the first time in the Junior Certificate Mathematics Examination of 2013 and the Leaving Certificate Mathematics Examination of 2012

Strands 1, 2, 3 and 4 will be examined in the Junior Certificate Mathematics Examination in 2014 and the Leaving Certificate Mathematics Examination in 2013

Strands 1, 2, 3, 4 and 5 will be examined in the Junior Certificate Mathematics Examination in 2015 and Leaving Certificate Mathematics Examination in 2014

Note: Synthesis and problem solving listed below must be incorporated into all of the Strands. The list of skills below, is taken from Strand 1of the syllabus but, an identical list is given at the end of each Strand, in the syllabus.

Students learn about	Students should be able to
1.8 Synthesis and problem-solving skills	– explore patterns and formulate conjectures – explain findings – justify conclusions – communicate mathematics verbally and in written form – apply their knowledge and skills to solve problems in familiar and unfamiliar contexts – analyse information presented verbally and translate it into mathematical form – devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

Colour coding used in the suggested sequence below:

Strand 1 **Statistics and probability**	**Strand 2** **Synthetic, coordinate& transformation geometry & trigonometry**	**Strand 3** **Number**	**Strand 4** **Algebra**	**Strand 5** **Functions**

Suggested sequence of topics

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Section number	Strand	Lesson Idea	Title of lesson idea	Suggested number of class periods	Cumulative totals
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Section number	Strand	Lesson Idea	Title of lesson idea	Suggested number of class periods	Cumulative totals
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	2.1	LCOL.33	Revision of JC synthetic geometry: More on quadrilaterals	2	107
	2.1	LCOL.34	Theorem 11	1	108
	2.1	LCOL.35	Theorem 12 and revision of Theorem 13	3	111
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Section number	Strand	Lesson Idea	Title of lesson idea	Suggested number of class periods	Cumulative totals
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Proposed beginning of 6th year programme					
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Section number	Strand	Lesson Idea	Title of lesson idea	Suggested number of class periods	Cumulative totals
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Section 1 Strand 3 Number

Lesson Idea LCOL.1

Six class periods

Title
Number systems

Resources

Teaching and Learning Plans:

Integers

Fractions Diagnostic Test

Partitioning
Equivalent fractions
Addition and subtraction of fractions
Multiplication of fractions
Division of fractions
Decimals

Student's CD 

Content
These lessons will involve the students in investigating and understanding:

- The operations of addition, multiplication, subtraction and division (emphasising the order of operations, including brackets) in the following domains:
  $\mathbb{N}, \mathbb{Z}, \mathbb{Q}$ and representing these numbers on a number line
- Decimals as special equivalent fractions and strengthening the connection between these numbers and fractions and place value understanding
- Rounding of decimals
- Terminating and non-terminating decimals
- Irrational numbers $\mathbb{R} \setminus \mathbb{Q}$
- Number system $\mathbb{R}$, and appreciating that $\mathbb{R} \neq \mathbb{Q}$ and representing $\mathbb{R}$ on a number line

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.2

Three class periods

Title
Rules for indices

Resources

Student's CD 

Content
These lessons will involve the students in investigating and understanding:

- The rules for indices (where $a, b \in \mathbb{R},\ p, q \in \mathbb{Q};\ a^p, a^q \in \mathbb{Q}$)

$$a^p a^q = a^{p+q}$$

$$\frac{a^p}{a^q} = a^{p-q}$$

$$a^0 = 1$$

$$\left(a^p\right)^q = a^{pq}$$

$$a^{-p} = \frac{1}{a^p}$$

$$(ab)^p = a^p b^p$$

$$\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$$

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.3 (Possibly for 6th year depending on the class)

Four class periods

Title
Adding, subtracting, multiplying and dividing real numbers

Resources

Student's CD 

Content
These lessons will involve the students in investigating and understanding:

- Irrational numbers, including surds (Roots that are irrational are called ***surds***.)
- How to apply the following rules:

  $a^{\frac{1}{n}} = \sqrt[n]{a}, \quad n \in \mathbb{Z}, n \neq 0, a > 0$

- $a^{\frac{m}{n}} = \sqrt[n]{a^m}, \quad m, n \in \mathbb{Z}, n \neq 0, a > 0$

  $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m, \quad m, n \in \mathbb{Z}, n \neq 0, a > 0$

- Adding, subtracting, multiplying and dividing surds

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.4

Two class periods

Title
Factors, multiples and primes

Resources

Teaching and Learning Plans:

Factors, multiples and primes

Student's CD 

Content
These lessons will involve the students in investigating and understanding:

- Factors, multiples and prime numbers in $\mathbb{N}$
- Expressing numbers in terms of their prime factors
- Highest Common Factor and Lowest Common Multiple

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.5

Four class periods

Title
Percentages, estimating using real world contexts, % error

Resources

Teaching and Learning Plans:

Percentages

Student's CD 

Content
These lessons will involve the students in investigating and understanding:

- How to make and justify estimates and approximations of calculations
- How to check a result by considering whether it is of the right order of magnitude and by working the problem backwards
- How to make estimates of the world around them e.g. how many books in a library
- How to calculate percentage error and tolerance
- How to calculate accumulated error (due to addition or subtraction only)

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.6

Six class periods

Title
Financial Maths

Resources

Teaching and Learning Plans:

Percentages

Student's CD 

Content
These lessons will involve the students in investigating and understanding:

- How to solve problems involving
  - Cost price, selling price, loss, discount
  - Markup (profit as a % of cost price)
  - Margin (profit as a % of selling price)
  - Income tax and net pay including other deductions
  - Compound interest including terms such as AER, APR, CAR (refer to later when dealing with exponential functions and geometric sequences)
  - Depreciation (reducing balance method)
  - Compound interest and depreciation (reducing balance method)
  - Currency transactions

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.7

Five class periods

Title
Ratio and proportion

Resources

Teaching and Learning Plans:

Percentages

Ratio and proportion

Student's CD 

Content
These lessons will involve the students in investigating and understanding:

- Ratios as comparing two quantities of the same kind by division (no units)
- Contexts involving ratio and proportion
  - Metric system; change of units; everyday imperial units (conversion factors provided for imperial units)
  - Rates as the comparison of two quantities by division but with different units
  - Average rates of change with respect to time and units
  - Diagrams drawn to scale

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.8

Four class periods

Title
Scientific notation involving real world contexts

Resources

Student's CD 

Content
These lessons will involve the students in investigating and understanding:

- How to express non -zero positive rational numbers in the form $a \times 10^n$, where $n \in \mathbb{Z}$ and $1 \leq a < 10$ involving real world contexts
- How to enter very large numbers on the calculator
- How to enter very small numbers on the calculator
- How to perform arithmetic operations on numbers in scientific notation
- How to solve problems involving numbers in scientific notation

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Section 2 Strands 3, 4, 5 patterns, functions and algeb

Lesson Idea LCOL.9

Eight class periods

Title
Relations approach to algebra- revision and extension of Junior Cycle material

Resources

Teaching and Learning Plan:

Introduction to Patterns

Patterns: A Relations Approach to Algebra

Workshop 4 booklet

Dynamic software package

Student's CD 

Content
These lessons will involve the students in investigating and understanding:

- That processes can generate sequences of numbers or objects
- How to investigate and discover patterns among these sequences
- How to use patterns to continue the sequence
- How to develop generalising strategies and ideas, present and interpret solutions, in the following:
    - The use of tables, diagrams, graphs and formulae as tools for representing and analysing **linear** patterns and relations
        - Discuss rate of change and the y - intercept. Consider how these relate to the context from which the relationship is derived and identify how they can appear in a table, in a graph and in a formula
        - Decide if two linear relations have a common value (decide if two lines intersect and where the intersection occurs).
        - Recognise that the distinguishing feature of a linear relationship is a constant rate of change
        - Recognise linear relationships as arithmetic sequences
    - The use of tables, diagrams, graphs and formulae as tools for representing and analysing **quadratic** patterns and relations

- Recognise that a distinguishing feature of quadratic relations is that the rate of change of the rate of change is constant

- The concept of a function as a relationship between a set of inputs and a set of outputs where each input is related uniquely to only one output
- The use of tables, diagrams, graphs and formulae as tools for representing and analysing **exponential** patterns and relations
  - Recognise that a distinguishing feature of exponential relations is a constant ratio between successive outputs
  - Recognise exponential relationships as geometric sequences
- Recognise whether a sequence is arithmetic, geometric or neither

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.10

Four class periods

Title
Arithmetic sequences and series

Resources

Teaching and Learning Plan:

Introduction to Patterns

Arithmetic sequences and series

Patterns: A Relations Approach to Algebra

Workshop 4 booklet

Dynamic software package

Student's CD 

Content
These lessons will involve the students in investigating and understanding:

- The link between linear relations and the formula for the general term (T_n)of an arithmetic sequence
- How to find the sum (S_n)of *n* terms of an arithmetic series
- How to apply the formula for the *n*th term of an arithmetic sequence and the formula for the sum to *n* terms of an arithmetic series to different contexts.

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.11

Six class periods

Title
Functions -interpreting and representing linear, quadratic and exponential functions in graphical form

Resources
Patterns: A Relations Approach to Algebra

Student's CD 
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- That a function assigns a unique output to a given input
- Domain, co -domain and range of a function
- The use of function notation $f(x) =$, $f : x \to$, and $y =$
- How to graph functions of the form:
  - ax where $a \in \mathbb{Q}, x \in \mathbb{R}$
  - $ax + b$ where $a, b \in \mathbb{Q}, x \in \mathbb{R}$
  - $ax^2 + bx + c$, where $a, b, c \in \mathbb{Z}, x \in \mathbb{R}$
  - $ax^3 + bx^2 + cx + d$, where $a, b, c, d \in \mathbb{Z}, x \in \mathbb{R}$
  - ab^x where $a \in \mathbb{N}, b, x \in \mathbb{R}$
- How to interpret equations of the form $f(x) = g(x)$ as a comparison of the above functions
- How to use graphical methods to find approximate solutions to

- o $f(x) = 0$
- o $f(x) = k$
- o $f(x) = g(x)$ where $f(x)$ and $g(x)$ are of the above form

- How to find local maximum and minimum values of quadratic functions from a graph
- The relationship between the graphs of $f(x) = x^2$, $f(x) = ax^2$ and $f(x) = x^2 + c$

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.12

Three class periods

Title
Composition of functions

Resources

Student's CD 
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- The composition of functions including the notation used

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.13

Four class periods

Title
Revision of basic JC algebra - evaluating and expanding expressions

Resources

Student's CD 

Content
These lessons will involve the students in investigating and understanding:

- The use of letters to express quantities that are variable
- Indices in algebra (exponents $\in \mathbb{N}$)
- Terms, coefficients and expressions
- How to generate algebraic expressions from simple contexts
- How to carry out operations of the form:

$$(ax+by+c)\pm(dx+ey+f)$$
$$(ax^2+bx+c)\pm(dx^2+ex+f) \text{ where } a,b,c,d,e,f \in \mathbb{Z}$$

- How to evaluate expressions such as:

$$ax+by$$
$$a(x+y)$$
$$x^2+bx+c$$
$$\frac{ax+by}{cx+dy}$$
$$axy$$
where $a,b,c,d,x,y \in \mathbb{Z}$ (JC ordinary level)

- How to carry out operations of the form:
  - $(ax+by+c)\pm.....(dx+ey+f)$
  - $(ax^2+bx+c).....\pm(dx^2+ex+f)$ where $a,b,c,d,e,f \in \mathbb{Z}$

- How carry out operations of the type:
  $a(bx+cy+d)+.....+e(fx+gy+h)$
  $a(bx^2+cx+d)$
  $ax(bx^2+c)$ where $a,b,c,d,e,f,g,h \in \mathbb{Z}$

  $(x \pm y)(x \pm y)$
- $(x \pm y)(w \pm z)$
  $(ax+b)(cx+d)$

(These include the use of brackets and the associative and distributive laws.)

- How carry out operations of the form :
  $ax^2+bx+c \div dx+e,$ where $a,b,c,d,e \in \mathbb{Z}$

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.14

Four class periods

Title
Factorising in Algebra - revision and extension of JC material

Resources

Student's CD 

Content
These lessons will involve the students in investigating and understanding:

- Factors and multiples
- Factorising expressions of the form:

ax, axy where $a \in \mathbb{Z}$

$abxy + ay$ where $a, b \in \mathbb{Z}$

x^2

$ax^2 + bx$ where $a, b \in \mathbb{Z}$

$sx - ty + tx - sy$ where s, t, x, y are variable

$x^2 + bx + c$, where $b, c \in \mathbb{Z}$

$x^2 - y^2$

$ax^2 + bx + c, a \in \mathbb{N}, b, c \in \mathbb{Z}$

$a^2x^2 - b^2y^2$, where $a, b \in \mathbb{N}$

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Idea LCOL.15

Two class periods

Title
Rearrangement of formulae

Resources

Student's CD 

Content
These lessons will involve the students in investigating and understanding:

- How to rearrange formulae using familiar context

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.16

Three class periods

Title
Adding algebraic fractions - revision and extension of JC material

Resources

Student's CD 

Content
These lessons will involve the students in investigating and understanding:

- Operations of the form:
  $\frac{ax+b}{c} \pm \frac{dx+e}{f}$ where $a, b, c, d, e, f \in \mathbb{Z}$
  $\frac{a}{bx+c} \pm \frac{q}{px+r}$ where $a, b, c, p, q, r \in \mathbb{Z}$

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.17

Five class periods

Title
Solving linear and quadratic equations

Resources
Teaching and Learning Plan:

Equations

Student's CD 
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Selecting and using of suitable strategies (graphic, numeric and algebraic) for finding solution to equations of the form:
  - $f(x) = g(x)$ with $f(x) = ax + b, \quad g(x) = cx + d$ and where $a, b, c, d \in \mathbb{Q}$
  - $f(x) = g(x)$ with $f(x) = \frac{a}{bx + c} \pm \frac{p}{qx + r}$ $g(x) = \frac{e}{f}$ where $a, b, c, e, f, p, q, r \in \mathbb{Z}$
  - $f(x) = k$ with $f(x) = ax^2 + bx + c$ (and not necessarily factorisable), $a, b, c \in \mathbb{Q}$ and interpret the result
  - Linear equations with two unknowns and interpret the results
  - Simultaneous equations where one is linear and the other is of order two with two unknowns (restricted to the case where either the coefficient of x or the coefficient of y is ± 1 in the linear equation) and interpret the results.
- How to form quadratic equations given whole number roots

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.18

Three class periods

Title

Inequalities

Resources

Teaching and Learning Plan:

Equations

Student's CD
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

Selecting and using suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to inequalities of the form:

- $g(x) \leq k,\ g(x) \geq k,$

  $g(x) < k,\ g(x) > k$ where $g(x) = ax + b, a, b, k \in \mathbb{Q}$

- Graph solution sets on the number line for linear inequalities in one variable

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.19
Three class periods

Title
Relations without formulae (See 4.5 JCOL)

Resources

Student's CD
Dynamic software package

Motion sensor
Workshop 4 booklet (graph matching)

Content
These lessons will involve the students in investigating and understanding:

- Graphs of motion
- Quantitative graphs and drawing conclusions from them
- The connections between the shape of a graph and the story of a phenomenon

- Quantity and change of quantity on a graph

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Section 3 Strand 2 Coordinate Geometry of the line

Lesson Idea LCOL.20

Two class periods

Title

Review of Junior Cycle ordinary-level coordinate geometry - coordinating the plane, distance and midpoint formula

Resources

Graph paper, mathematical set
Teaching and Learning Plans:

- *Coordinating the Plane*
- *Distance Formula*

Student's CD 

Content

These lessons will involve the students in investigating and understanding:

- The coordination of the plane
- The distance formula
- The midpoint formula

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.21

Two class periods

Title

Review of slope including application to parallel and perpendicular lines

Resources

Graph paper, mathematical set, dynamic geometry software package, set of board drawing instruments

Student's CD

Content
These lessons will involve the students in investigating and understanding:

- The idea of slope as $\frac{Rise}{Run}$
- The slope formula
- The meaning of *positive, negative, zero and undefined slope.*
- Use of slopes to investigate if two lines are parallel
- The use of slopes to investigate if two lines are perpendicular or not
- That 3 points on the coordinate plane but not all on the same vertical line are collinear if and only if the slope between any two of them is the same
  (This approach can be used to find the equation of a line between two points)

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL. 22
Two class periods

Title
Area of a triangle

Resources
Graph paper, mathematical set, dynamic geometry software package, set of board drawing instruments

Student's CD

Content
This lesson will involve the students in investigating and understanding:

- How to calculate the area of a triangle using coordinates

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.23

Three class periods

Title

Equation of Line

Resources

Graph paper, mathematical set, dynamic geometry software package, set of board drawing instruments

Student's CD 

Content

These lessons will involve the students in investigating and understanding:

- The equation of a line in the forms :

$$y - y_1 = m(x - x_1)$$

$$y = mx + c$$

$$ax + by + c = 0$$

- The significance of the variables m and c
- Whether or not a point is in a line
- Where a line intersects the axes and why these points might be of interest to someone trying to interpret or plot a graph
- The interpretation of the intercepts in context
- How to find the slope of a line given its equation
- How to solve problems involving slopes of lines
- The link between coordinate geometry of the line and functions

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.24

Two class periods

Title

Intersection of two lines

Resources

Graph paper, mathematical set, dynamic geometry software package, set of board drawing instruments

Student's CD 

Content

These lessons will involve the students in investigating and understanding:

- A graphical approach to the intersection of two lines
- An algebraic approach to the intersection of two lines, using simultaneous equations
- The comparison and verification of both of these methods above

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Section 4 Strand 2 Synthetic Geometry 1

Prior Knowledge:

Students who have taken **Ordinary Level** in Junior Certificate (2007 - 2013) will have studied the following (refer to section 2 of this document):

Axioms: 1 - 5

Theorems: 1 - 10

14 - 18

Corollaries 3, 4 and 5 of Theorem 19

Constructions: 1, 2, 6, 10 -14

Students who have taken **Higher Level** in Junior Certificate (2007 - 2013) will have studied the following (refer to section 2 of this document):

Axioms: 1 - 5

Theorems: 1 – 10, 12 – 15, 17 – 20

(Proofs of 4, 6, 9, 14 and 19)

Constructions: 1, 2, 6, 7, 10 - 14 and 16, 17

Concepts:
Set, plane, point, line, ray, angle, real number, length, degree, triangle, right-angle, congruent triangles, similar triangles, parallel lines, parallelogram, area, tangent to a circle, subset, segment, collinear points, distance, midpoint of a line segment, reflex angle, ordinary angle, straight angle, null angle, full angle, supplementary angles, vertically-opposite angles , acute angle, obtuse angle, angle bisector, perpendicular lines, perpendicular bisector of a line segment, ratio, isosceles triangle, equilateral triangle, scalene triangle, right-angled triangle, exterior angles of a triangle, interior opposite angles, hypotenuse, alternate angles, corresponding angles, polygon, quadrilateral, convex quadrilateral, rectangle, square, rhombus, base and corresponding apex and height of triangle or parallelogram, transversal line, circle, radius, diameter, chord, arc, sector, circumference of a circle, disc, area of a disc, point of contact of a tangent, concurrent lines. Vertex, vertices (of angle, triangle, triangle, polygon), endpoints of segment, arms of an angle, equal segments, equal angles, adjacent sides, angles, or vertices of triangles or quadrilaterals, the side opposite an angle of a triangle, opposite sides or angles of a quadrilateral, centre of a circle.

The following is a suggested sequence for teaching the Leaving Certificate Course. In teaching these lessons, teachers and students can draw from the Teaching and Learning Plans and student activities on the website at www.projectmaths.ie

As outlined at the workshops, the use of learning materials such as "geostrips", "anglegs", geo-boards etc. can make the learning so much more enjoyable for students of all perceived abilities.

While proofs are not the issue as regards informal introduction, it is important that students are kept aware that the theorems build logically.

The lesson divisions which follow are for guidance only. The initial lesson ideas give the students a chance to revisit the material they met in the Junior Cycle. This can be done at a pace that is appropriate to the student's needs. It is recommended that new activities and challenges be introduced during this revision so that students do not see it as too much repetition and that they can see new ways of investigating familiar situations.

Note on experimentation and experimental results:

With experimentation, involving measurement, the results are only approximations and won't agree exactly. It is important for students to report faithfully what they find e.g. for a triangle they could find the sum of the angles to be 179^0 or 181 etc. The conclusion is that the angles appear to add up to 180^0 .This is a plausible working assumption. There is a distinction between what you can discover and what you can prove.
See Section 8.2 (From Discovery to Proof) of *Geometry for Post-primary School Mathematics*"

Strand 2: Geometry and trigonometry LC syllabus

Students learn about	**Students working at FL should be able to**	**In addition, students working at OL should be able to**	**In addition, students working at HL should be able to**
2.1 Synthetic geometry *	– perform constructions **18,19,20** (see *Geometry for Post-primary School Mathematics*)	– perform constructions **16,17,21** (see *Geometry for Post-primary School Mathematics*) – use the following terms related to logic and deductive reasoning: theorem, proof, axiom, corollary, converse, implies – investigate theorems **7, 8, 11, 12, 13, 16, 17, 18, 20, 21** and **corollary 6** (see *Geometry for Post-primary School Mathematics*) and use them to solve problems	– perform constructions **1-15** and **22** (see *Geometry for Post-primary School Mathematics*) – use the following terms related to logic and deductive reasoning: is equivalent to, if and only if, proof by contradiction – prove theorems **11**,**12**,**13**, concerning ratios (see *Geometry for Post-primary School Mathematics*), which lay the proper foundation for the proof of the theorem of Pythagoras studied at junior cycle

*In the examination, candidates will have the option of answering a question on the synthetic geometry set out here, or answering a problem-solving question based on the geometrical results from the corresponding syllabus level at Junior Certificate. This option will apply for a three year period only, for candidates sitting the Leaving Certificate examination in 2012, 2013 and 2014.There will be no choice after that stage.

Lesson Idea LCOL.25

One class period

Title

Revision of preliminary concepts - Plane and points

Resources

Geometry Course for Post-Primary School Mathematics

Teaching and Learning Plans:

Plan 6: Planes and Points

Plan7: Introduction to Angles

A mathematical instruments set

Dynamic software package

Student's CD 

Content

These lessons will involve the students in investigating and understanding:

- Plane, points, lines, line segments, rays, collinear points, length of a line segment
- What is meant by the terms "axiom" and "implies"
- **Axiom 1:** There is exactly one line through any two given points.
- **Axiom 2:** [Ruler Axiom]: The properties of the distance between points
- Angles as a rotation, angles in different orientations
- How to estimate angles in degrees, naming angles
- Terms: Perpendicular, parallel, vertical, horizontal

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.26

One class period

Title

Revision - Angles, Axiom 3, Theorem 1, Constructions 8 & 9

Resources

Geometry Course for Post-Primary School Mathematics

Teaching and Learning Plan:

Plan7: Introduction to Angles

NCCA Student Resources

A mathematical instruments set, an angle estimator

Dynamic software package

Student's CD 

Content
These lessons will involve the students in investigating and understanding:

- Measurement of angles using a protractor
- Possible misconceptions: Students thinking that size of an angle varies with arm or arc-length; failure to recognise equal angles in different orientations
  Common error: Reading from the incorrect scale on the protractor
- The addition of angles (**Axiom 3**)
- That a straight angle measures 180^0 (supplementary angles)
- Vertically opposite angles
- What is meant by the term "theorem"
- **Theorem 1**: Vertically opposite angles are equal in measure.
- What is meant by "proof"
- The use of the compass
- **Construction 8**: Line segment of a given length on a given ray
- **Construction 9**: Angle of a given number of degrees with a given ray as one arm

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.27

Two class periods

Title

Revision of JC synthetic geometry

Resources

Geometry Course for Post-Primary School Mathematics
NCCA Student Resources

A mathematical instruments set

Dynamic software package

Student's CD 

Content

These lessons will involve the students in investigating and understanding:

- **Construction 5**: Line parallel to a given line, through a given point;
- **Axiom 5:** Given any line l and a point P, there is exactly one line through P parallel to l.
- **Construction 6**: Division of a line segment into 2 or 3 equal segments without measuring it
- Triangles and congruent triangles
    - Triangles: scalene, isosceles, equilateral, right-angled
    - **Construction 10**: Triangle given SSS – Congruent triangles (Axiom 4)
    - **Construction 11**: Triangle given SAS - Congruent triangles (Axiom 4)
    - **Construction 12**: Triangle given ASA - Congruent triangles (Axiom 4)
    - By construction, show that AAA and AAS are not sufficient conditions for congruence.
    - What is meant by the term "converse"
    - **Theorem 2**: (i) In an isosceles triangle the angles opposite the equal sides are equal.

      (ii) Conversely, if two angles are equal, then the triangle is isosceles

Suggested class activities

Students might engage in activities in relation to Scalene, equilateral, isosceles, right-angled and obtuse-angled triangles

- Describe each in words
- Draw three examples of each in different orientations
- Determine the sum of the angles in each
- Determine which triangles, if any, contain two or more equal angles

- Establish if they can say for certainty the size of the angles in each
- Establish if any of the triangles can belong to more than one category

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.28

One class period

Title

Revision of JC synthetic geometry

Resources

Geometry Course for Post-Primary School Mathematics

NCCA Student Resources

A mathematical instruments set

Dynamic software package

Student's CD 

Content

These lessons will involve the students in investigating and understanding:

- Alternate angles by examples and measuring
- **Theorem 3**: (i) If a transversal makes equal alternate angles on two lines, then the lines are parallel.

  (ii) Conversely, if two lines are parallel, then any transversal will make equal alternate angles with them.
- **Theorem 4**: The angles in any triangle add to 180^0.
- Corresponding angles explained by examples and measuring
- **Theorem 5**: Two lines are parallel if and only if for any transversal, corresponding angles are equal.

- **Theorem 6**: Each exterior angle of a triangle is equal to the sum of the interior opposite angles.

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.29

Two class periods

Title

Revision of JC synthetic geometry

Resources

Geometry Course for Post-Primary School Mathematics
NCCA Student Resources

A mathematical instruments set

Dynamic software package

Student's CD 

Content

These lessons will involve the students in investigating and understanding:

- **Theorem 7**: The angle opposite the greater of two sides is greater than the angle opposite the lesser. Conversely, the side opposite the greater of two angles is greater than the side opposite the lesser angle.

- **Theorem 8**: Two sides of a triangle are together greater than the third.

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.30

Two class periods

Title

Revision - Constructions 1,2,& 4

Resources

Geometry Course for Post-Primary School Mathematics
NCCA Student Resources

A mathematical instruments set

Dynamic software package

Student's CD 

Content

These lessons will involve the students in investigating and understanding:

- **Construction 1**: The use of a compass and straight angle to bisect an angle
- **Construction 2**: The use of a compass and straight edge to draw the perpendicular bisector of a line segment
- **Construction 4**: Line perpendicular to a given line l, passing through a given point on l

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.31

One class period

Title

Revision of translations, central symmetries and axial symmetries

Resources

Geometry Course for Post-Primary School Mathematics
NCCA Student Resources

A mathematical instruments set

Dynamic software package

Student's CD

Content
These lessons will involve the students in investigating and understanding:

- Translations: an intuitive approach using drawings
- Axial symmetry: an intuitive approach using drawings
- Axis of symmetry
- Central symmetry- an intuitive approach using drawings
- Centre of symmetry

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.32

Five class periods

Title
Revision of quadrilaterals, parallelograms, Theorems 9 & 10, Construction 20

Resources

Geometry Course for Post-Primary School Mathematics
NCCA Student Resources

A mathematical instruments set

Dynamic software package

Student's CD

Content
These lessons will involve the students in investigating and understanding:

- **Theorem 9**: In a parallelogram, opposite sides are equal, and opposite angles are equal Conversely, (1) if the opposite angles of a convex quadrilateral are equal, then it is a parallelogram; (2) if the opposite sides of a convex quadrilateral are equal, then it is a parallelogram.

Remark 1: Sometimes it happens that the converse of a true statement is false. For example, it is true that if a quadrilateral is a rhombus, then its diagonals are perpendicular. But it is not true that a quadrilateral whose diagonals are perpendicular is always a rhombus.
Remark 2: The converse of Corollary 1 is false: it may happen that a diagonal divides a convex quadrilateral into two congruent triangles, even though the quadrilateral is not a parallelogram.

- **Theorem 10:** The diagonals of a parallelogram bisect each other.

  Conversely, if the diagonals of a quadrilateral bisect one another, then the quadrilateral is a parallelogram.
- **Construction 20**: Parallelogram, given the length of the sides and the measure of the angles

- The properties of different quadrilaterals

Suggested class activities
Students might engage in the following activities which lead to an informal proof of theorem 9:
Draw a parallelogram ABCD which is not a rectangle or a rhombus
Draw in one diagonal BD
Mark in all the alternate angles – they should find 2 pairs
Establish that triangles ABD and BCD are congruent and explain their reasoning
Establish what this means about the opposite sides of parallelogram ABCD?
Make a deduction about the opposite angles of parallelogram ABCD?

The students might determine:

If the diagonal bisects the angles at the vertex

The sum of the four angles of parallelogram ABCD
The result if two adjacent angles of the parallelogram are added together

Students might engage in the following activities which lead to an informal proof of theorem 10: (In all instances they should be encouraged to explain their reasoning.)
Draw a parallelogram ABCD which is not a rectangle or a rhombus
Draw in the two diagonals AC and BD intersecting at E
Determine if the two diagonals equal in length. (Measure)
Mark in all the equal sides and angles in the triangles AED and BEC
Explain why triangles ADE and BEC are congruent (Give a reason.)

Possible further investigations:

The students might determine:

If the triangles AEB and DEC congruent
If the diagonals perpendicular
If the parallelogram contains 4 two pairs of congruent triangles
If the diagonals bisect the vertex angles of the parallelogram

The number of axes of symmetry the parallelogram has
If the parallelogram has a centre of symmetry and its location if it does exist

Students might engage in the following activities about a square, rhombus, parallelogram and rectangle: (In all instances they should be encouraged to explain their reasoning.)

Describe each of them in words.
Draw three examples of each in different orientations.
Determine which sides are equal in length
Determine the sum of the angles in each case
Determine which angles are equal
Determine the sum of two adjacent angles in each case
Establish if a diagonal bisect the angles it passes through
Establish if the diagonals are perpendicular
Determine if a diagonal divide it into two congruent triangles
Calculate the length of a diagonal given the length of its sides, where possible
Establish if the two diagonals equal in length
Determine if the diagonals divide the different shapes into 4 congruent triangles?
Establish if the diagonals bisect each other?

The students should determine the number of axes of symmetry each of the shapes has and which ones have a centre of symmetry

An interesting option would be to conduct the activities above on a KITE.

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.33

Two class periods

Title

Revision: More Quadrilaterals – Investigating a Square

Resources

Geometry Course for Post-Primary School Mathematics
NCCA Student Resources

A mathematical instruments set

Dynamic software package

Student's CD 

Content
These lessons will involve the students in investigating and understanding:

- The properties of a square
- The properties of a rectangle(which is not a square)

Suggested class activities
Students might engage in the following activities relating to a square:

(In all instances they should be encouraged to explain their reasoning.)

Draw a square ABCD
Draw in the two diagonals AC and BD intersecting at E
Determine if the two diagonals equal in length
Mark in all the equal sides and angles in the triangles AED and BEC
Establish that triangles ADE and BEC are congruent
Determine if the triangles AEB and DEC congruent
Determine if there are two pairs of congruent triangles in the square?
Show that the diagonals perpendicular? Give a reason
Establish if the diagonals bisect the vertex angles of the square
Find how many axes of symmetry the square has
Determine if the square has a centre of symmetry and if it does, what is its location

Students might engage in the following activities about a rectangle:

(In all instances they should be encouraged to explain their reasoning.)

Draw a rectangle ABCD which is not a square
Draw in the two diagonals AC and BD intersecting at E and establish if the two diagonals are equal in length
Mark in all the equal sides and angles in the triangles AED and BEC
Establish that ADE and BEC are congruent
Determine if the triangles AEB and DEC congruent
Determine if there are two pairs of congruent triangles in the rectangle
Show that the diagonals are perpendicular
Determine if the diagonals bisect the vertex angles of the rectangle
Find how many axes of symmetry the rectangle has
Determine if the rectangle has a centre of symmetry and if it does, find its location

Possible extra activity:
Repeat these activities for the rhombus ABCD

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.34

One class period

Title

Theorem 11

Resources

Geometry Course for Post-Primary School Mathematics
NCCA Student Resources

A mathematical instruments set

Dynamic software package

Student's CD 

Content

These lessons will involve the students in investigating and understanding:

- **Theorem 11**: If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.35

Three class periods

Title

Theorem 12 and revision of Theorem 13

Resources

Geometry Course for Post-Primary School Mathematics
NCCA Student Resources

A mathematical instruments set

Dynamic software package

Student's CD 

Content
These lessons will involve the students in investigating and understanding:

- **Theorem 12**: Let ABC be a triangle. If a line l is parallel to BC and cuts [AB] in the ratio m: n, then it also cuts [AC] in the same ratio.
  Conversely, If line a l cuts the sides AB and AC of triangle ABC in the same ratio, then it is parallel to BC.

- The meaning of similar triangles and the difference between similar and congruent triangles.

- **Theorem 13**: If two triangles are similar, then their sides are proportional, in order.
  Conversely, if the sides of two triangles are in proportion, then the two triangles are similar.

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.36

Three class periods

Title
Revision of right-angled triangles and Pythagoras' Theorem

Resources
Geometry Course for Post-Primary School Mathematics
NCCA Student Resources

A mathematical instruments set

Dynamic software package

Student's CD  .

Content
These lessons will involve the students in investigating and understanding:

- **Construction 13**: Right-angled triangle, given length of hypotenuse and one other side.
- **Construction 14**: Right-angled triangle, given one side and one of the acute angles
- **Construction 15**: Rectangle given side lengths
- **Theorem 14:** Theorem of Pythagoras
- **Theorem 15**: [Converse to Pythagoras] If the square of one side of a triangle is the sum of the squares of the other two, then the angle opposite the first side is a right angle.
- **Proposition 9**: (RHS) If two right-angled triangles each have hypotenuse and one other side equal in length respectively, then they are congruent.

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.37

Three class periods

Title
Introduction to area

Resources
Geometry Course for Post-Primary School Mathematics
NCCA Student Resources

A mathematical instruments set

Dynamic software package

Student's CD 

Content
These lessons will involve the students in investigating and understanding:

- **Theorem 16:** For a triangle, the product of the base and height does not depend on the choice of base.
- **Definition 38:** The **area** of a triangle is half the base multiplied by the height.
- **Theorem 17:** A diagonal bisects the area of a parallelogram.
- **Theorem 18:** The area of a parallelogram is the base multiplied by the height.

Suggested class activities
Students might engage in the following activities:

In the case of each of these types of triangles: equilateral, isosceles, right-angled and obtuse-angled: draw three diagrams for each type of triangle showing each side as a base and the corresponding perpendicular height.

Students investigate the validity of the following statement and its converse: "Congruent triangles have equal areas".

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.38

Six class periods

Enlargements

Resources
Geometry Course for Post-Primary School Mathematics
NCCA Student Resources

A mathematical instruments set

Dynamic software package

Student's CD 

Content
These lessons will involve the students in investigating and understanding:

- Enlargements paying attention to
  - centre of enlargement , scale factor k, $k > 1$, $0 < k < 1, k \in \mathbb{Q}$
  - how to draw an enlargement given a scale factor when the centre of enlargement is outside the figure to be enlarged
  - how to draw an enlargement given a scale factor when the centre of enlargement is inside the figure to be enlarged
  - how to draw an enlargement given a scale factor when the centre of enlargement is a vertex of the figure to be enlarged or is a point on the figure
  - How to find the scale factor
- That when a figure is enlarged by a scale factor k , the area of the image figure is increased by a factor k^2
- How to solve problems involving enlargements

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Section 5 Probability and Statistics 1

Lesson Idea LCOL.39

Three class periods

Title

Fundamental Principle of Counting and Permutations

Resources

Statistics and Probability evening PMDT course 2010 modules 1 -5 (see www.projectmaths.ie)

Student's CD 

NCCA Student Resources

Content

These lessons will involve the students in investigating and understanding:

- The Fundamental Principle of Counting
- Arrangements of n distinct objects (*n*!)
- Arrangements of *n* distinct object taking *r* at a time

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/ www.censusatschool.ie www.cso.ie

Lesson Idea LCOL.40

Seven class periods

Title

Concepts of probability

Resources

Teaching and Learning Plans 1-5

Statistics and Probability PMDT evening course 2010 modules 1 -5 (see www.projectmaths.ie)

NCCA Student Resources

Report on the Trialling SEC 2010

Content

These lessons will involve the students in investigating and understanding:

- How to distinguish events where the outcome is certain from events where the outcome is uncertain
- How to represent the probability of an event as a fraction or decimal between 0 and 1 or as a percentage between 0% and 100 %
- How to list all possible outcomes for practical experiments such as rolling one die
- How to determine the probability of an event using the results of an experiment
- How to determine the relative frequency for each outcome by experiment and note how it approaches the theoretical probability as the number of trials increases
- Events that have equally likely outcomes in comparison to those that don't
- Probability as long term relative frequency
- The principle that, in the case of equally likely outcomes, the probability is given by the number of desirable outcomes divided by the total number of outcomes
- How to list all possible outcomes for throwing two dice using a two way table
- How to relate the number of outcomes of an experiment to the fundamental principle of counting
- Independent events
- A standard deck of playing cards, knowing the names of the cards and suits, the number of cards in a pack and in each suit
- How to calculate the theoretical probability of picking named cards from the deck
- How to calculate probabilities of events involving spinners or urns containing different coloured objects

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/ www.censusatschool.ie www.cso.ie

Lesson Idea LCOL.41

Three class periods

Title

Rules of probability

Resources

Statistics and Probability PMDT evening course 2010 modules 1 -5 (see www.projectmaths.ie)
NCCA Student Resources

Content

These lessons will involve the students in investigating and understanding:

- The basic rules of probability (AND/OR), mutually exclusive events, through the use of Venn diagrams
- The use of the formulae:
    1. Addition Rule (for mutually exclusive events only): $P(A \cup B) = P(A) + P(B)$
    2. Addition Rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
    3. Multiplication Rule(for independent events): $P(A \cap B) = P(A) \times P(B)$

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/ www.censusatschool.ie www.cso.ie

Lesson Idea LCOL.42

Four class periods

Title

Further problem solving with probability

Resources

Statistics and Probability evening course 2010 modules 1 -5 (see www.projectmaths.ie)
NCCA Student Resources

Content

These lessons will involve the students in investigating and understanding:

- The use of tree diagrams
- The use of set theory

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/ www.censusatschool.ie www.cso.ie

Lesson Idea LCOL.43

Seven class periods

Title

The purpose of Statistics and the Data Handling Cycle

Resources

The Data Handling Cycle
Appendix A of this document (How to use CensusAtSchool)
NCE-MSTL Summer Course in Statistics and Probability 2009
Statistics and Probability PMDT evening course 2010 modules 1 -5 (see www.projectmaths.ie)
NCCA Student Resources

Content

These lessons will involve the students in investigating and understanding:

- The purpose and uses of statistics and possible misconceptions and misuses of Statistics
- How to design a plan and collect data on the basis of the above knowledge
- The data handling cycle (Pose a question, collect data, analyse data, interpret the result and refine the original question if necessary)
- The Census at School (**CAS**) questionnaire as a means of collecting data
- Questionnaire designs
- Populations and samples
- The importance of representativeness so as to avoid biased samples
- Sample selection (Simple Random Sample)
- The extent to which conclusions can be generalised
- Primary sources of data (observational (including sample surveys) and experimental studies) and secondary sources of data
- The different ways of collecting data

The students will also engage in analysing the spreadsheet of class data returned from the Census at School questionnaire to include:

- Recognising different types of data – category (nominal /ordinal), numerical (discrete/ continuous)
- Recognising univariate/bivariate data
- Discuss possible questions which might be answered with the data

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/ www.censusatschool.ie www.cso.ie

Lesson Idea LCOL.44

Ten class periods

Title

Analysing data graphically and numerically, interpreting and drawing inferences from data

Resources

The Data Handling Cycle

NCE-MSTL Summer Course in Statistics and Probability2009 

Statistics and Probability PMDT evening course 2010 modules 1 -5 (see www.projectmaths.ie)

Student's CD 

Let's Investigate Strand 1

NCCA Student Resources

Report on the Trialling SEC 2010

Content

These lessons will involve the students in investigating and understanding:

- The selection and use of appropriate graphical methods to describe the sample taking account of data type: bar charts, pie charts, line plots, histograms(equal class intervals), stem and leaf plot (including back to back)
- The concept of the distribution of data
- The distribution of numerical data in terms of **shape** (concepts of symmetry, clustering, gaps, skewness)
- The selection and use of appropriate numerical methods to describe the sample:
    - The distribution of data in terms of **centre** (mean, median and mode and the advantages and disadvantages of each)
    - The relative positions of mean and median in symmetric and skewed data

- The distribution of numerical data in terms of **spread** (range, inter-quartile range)
  - The concept of inter-quartile range as a measure of spread around the median
- The distribution of data in terms of **spread** (standard deviation)
  - The concept of standard deviation as a measure of spread around the mean
  - The use of a calculator to calculate standard deviation

- How to analyse plots of data to explain differences in measures of centre and spread

- How to interpret a histogram in terms of distribution of data and make decisions based on the empirical rule (based on a normal distribution)
- The effect on the mean and of adding or subtracting a constant to each of the data points and of multiplying or dividing the data points by a constant

- Outliers and their effect on measures of centre and spread

- Interpreting and drawing inferences from data recognising how sampling variability influences the use of sample information to make statements about the population and relating the interpretation to the original question

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/ www.censusatschool.ie www.cso.ie

Proposed end of fifth year

Section 6 Length, Area and Volume

Lesson Idea LCOL.45

Ten class periods

Title
Length of the perimeter and area of plane figures, nets of solids, surface area and volume

Resources
A mathematical instruments set

Student's CD 

Content
These lessons will involve the students in investigating and understanding:

- How to solve problems involving the length of the perimeter and the area of a disc, triangle, rectangle, square, parallelogram, trapezium, sectors of discs and figures made from combinations of these
- The nets of prisms (polygonal bases), cylinders and cones
- How to solve problems involving the surface area of a rectangular block, cylinder, right cone, triangular based prism (right angle, isosceles and equilateral), sphere, hemisphere and solids made from combinations of these

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/ http://www.nce-mstl.ie/

Lesson Idea LCOL.46

Five class periods

Title
Trapezoidal rule

Resources
A mathematical instruments set

Student's CD 

Content
These lessons will involve the students in investigating and understanding:

- How to use the trapezoidal rule to approximate area
- How to calculate percentage error involved in using trapezoidal rule in e.g. the circle

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Section 7 Strand 5 Functions and differential calculus

Lesson Idea LCOL.47

Four class periods

Title

Relations without formulae

Resources

Student's CD 

Dynamic software package

Motion sensor

Workshop 4 booklet (graph matching)

Content

These lessons will involve the students in investigating and understanding:

- Graphs of motion
- Quantitative graphs and drawing conclusions from them
- The connections between the shape of a graph and the story of a phenomenon
- Quantity and change of quantity on a graph

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.48

Two class periods

Title

Revision of function concepts

Resources

Student's CD 

Content

These lessons will involve the students in investigating and understanding:

- Revision of function concepts

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.49

Twelve class periods

Title

Calculus

Resources

Student's CD 

Dynamic software package

Content

These lessons will involve the students in investigating and understanding:

Rate of change, average rate of change, instantaneous rate of change, the derivative

This will include:

- Calculus as the study of mathematically defined change How to use graphs and real life examples to analyse rates of change for:
  - Functions $f(x) = k$ where k is a constant
  - Linear functions - links should be established to the slope of a line from coordinate geometry
  - Functions where the rate of change varies. These will include quadratic and more complex functions.
- Instantaneous rate of change (what shows on a speedometer) as opposed to average rate of change say over the course of a journey
- The equality of the instantaneous and average rates of change for linear functions
- How to find the rate of change in situations where it is not constant - need to define it at every point

- The idea of average rate of change between two points on $f(x)=x^2$ and its calculation as the slope of the line connecting the two endpoints of the interval under consideration
- That the instantaneous rate of change is not the same as the average rate of change between two points for $f(x)=x^2$
- That the average rate of change approaches the instantaneous rate as the interval under consideration approaches zero (the concept of a limit)
- That the instantaneous rate of change is the slope of the tangent at the point

- The meaning of the first derivative as the instantaneous rate of change of one quantity relative to another and the use and meaning of the terms "differentiation" and notation such as $\frac{dy}{dx}$ and $f'(x)$
- How to find the first derivatives of linear functions using the equation $y=mx+c$ and observing the slope as the first derivative
- Use the rule to find the first derivative of $f(x)=x^2$
- How to find the first derivative of linear, quadratic and cubic functions by rule
- How to apply differentiation to real life examples of rates of change
- What it means when a function is increasing/decreasing/constant in terms of the slope and rate of change
- How to apply an understanding of the change in $\frac{dy}{dx}$ from positive to zero to negative around a local maximum in order to identify a local maximum
- How to apply an understanding of the change in $\frac{dy}{dx}$ from negative to zero to positive around a local minimum in order to identify a local minimum
- The meaning of the second derivative as the rate of change of a rate of change at any instant
- Real life examples of the rate of change of a rate of change, for example acceleration as a rate of change of velocity
- How to match a function with its first and second derivatives
- How to find second derivatives of linear, quadratic and cubic functions by rule
- The application of the second derivative to identify local maxima and local minima
- The second derivative and its connection with "concave up" and "concave down" sections of curves

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Section 8 Strand 1 Probability and Statistics 2

Lesson Idea LCOL.50

Four class periods

Title
Revision of counting and probability concepts from fifth year

Resources

Student's CD 

Content
These lessons will involve the students in investigating and understanding:

- The Fundamental Principle of Counting
- Arrangements of n distinct objects ($n!$)
- Arrangements of n distinct object taking r at a time
- Concepts of probability
- Rules of probability
- Use of tree diagrams, set theory and counting method in probability

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.51

Two class periods

Title
Bernoulli Trials

Resources
Statistics and Probability PMDT evening course 2010 modules 1 -5 (see www.projectmaths.ie)
Teaching and Learning Plan 4
NCCA Student Resources

Content
These lessons will involve the students in investigating and understanding:

- Bernoulli trials
  (A Bernoulli trial is a statistical experiment consisting of *n* repeated trials where each trial can result in just two possible outcomes. We call one of these outcomes a success and the other, a failure. The probability of success, denoted by *P*, is the same on every trial. The trials are independent; that is the outcome on one trial does not affect the outcome on other trials.)
- How to solve problems involving up to 3 Bernoulli trials
- Bernoulli Trials including calculating the probability that the 1^{st} success occurs on the n^{th} Bernoulli trial where *n* is specified

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/ www.censusatschool.ie www.cso.ie

Lesson Idea LCOL.52

Five class periods

Title
Random variables and expected value

Resources
Statistics and Probability PMDT evening course 2010 modules 1 -5 (see www.projectmaths.ie)
Let's Investigate Strand 1
NCCA Student Resources

Content
These lessons will involve the students in investigating and understanding:

- Expected value of probability distributions
- The calculation of expected value and the fact that this does not need to be one of the outcomes
- The role of expected value in decision making and the issue of fair games

Lesson Idea LCOL.53

Five class periods

Title
Revision of statistics concepts from 5th year

Resources
Statistics and Probability PMDT evening course 2010 modules 1 -5 (see www.projectmaths.ie)
NCCA Student Resources
Report on the Trialling SEC 2010

Content
These lessons will involve the students in participating in investigating and understanding:

- The data handling cycle
- Analysing data graphically and numerically, interpreting and drawing inferences from data

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/ www.censusatschool.ie www.cso.ie

Lesson Idea LCOL.54

Three class periods

Title
Bivariate data, scatter plots, correlation

Resources
Let's Investigate Strand 1
NCCA Student Resources
The Data Handling Cycle

Student's CD 
NCE-MSTL Summer Course in Statistics and Probability2009

Statistics and Probability PMDT evening course 2010 modules 1 -5 (see www.projectmaths.ie)
Report on the Trialling SEC 2010

Content
These lessons will involve the students in investigating and understanding:

- Bivariate data versus univariate data
- The use of scatter plots to determine the relationship between variables
- That correlation always has a value from -1 to +1 inclusive, and that it measures the extent of **linear relationship** between two variables
- How values of correlation coefficients match different scatter plots
- That correlation does not imply causality

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/ www.censusatschool.ie www.cso.ie

Section 9 Strand 2 Synthetic Geometry 2

Lesson Idea LCOL.55

Three class periods

Title

Revision of fifth year synthetic geometry

Resources

Geometry Course for Post-Primary School Mathematics
NCCA Student Resources
A mathematical instruments set

Dynamic software package

Student's CD 

Content

These lessons will involve the students in investigating and understanding:

- The synthetic geometry from 5th year

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.56

Two class periods

Title

Corollaries 3 and 4

Resources

Geometry Course for Post-Primary School Mathematics
NCCA Student Resources
A mathematical instruments set

Dynamic software package

Student's CD

Content
These lessons will involve the students in investigating and understanding:

- The term "corollary"
- **Corollary 3**: Each angle in a semi-circle is a right angle.
- **Corollary 4**: If the angle standing on a chord [BC] at some point of the circle is a right-angle, then [BC] is a diameter.

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.57

Two class period

Title
Theorem 20, Corollary 6 and Construction 19

Resources
Geometry Course for Post-Primary School Mathematics
NCCA Student Resources
A mathematical instruments set

Dynamic software package

Student's CD

Content
These lessons will involve the students in investigating and understanding:

- **Theorem 20:** (i) Each tangent to a circle is perpendicular to the radius that goes to the point of contact.

  (ii) If P lies on the circle S, and a line l is perpendicular to the radius to P, then l is a tangent to S.

- **Corollary 6:** If two circles intersect at one point only, then the two centres and the point of contact are collinear.

- **Construction 19**: Tangent to a given circle at a given point on it.

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.58

One class period

Title

Construction 18

Resources

Geometry Course for Post-Primary School Mathematics
NCCA Student Resources
A mathematical instruments set

Dynamic software package

Student's CD 

Content

This lesson will involve the students in investigating and understanding:

- **Construction 18**: Angle of 60^0 without using a protractor or set square.

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.59

Two class periods

Title

Theorem 21

Resources

Geometry Course for Post-Primary School Mathematics
NCCA Student Resources
A mathematical instruments set

Dynamic software package

Student's CD 

Content

These lessons will involve the students in investigating and understanding:

- **Theorem 21:** (i) The perpendicular from the centre of a circle to a chord bisects the chord.

  (ii) The perpendicular bisector of a chord passes through the centre of a circle.

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.60

Two class periods

Title

Construction 21

Resources

Geometry Course for Post-Primary School Mathematics
NCCA Student Resources
A mathematical instruments set

Dynamic software package

Student's CD

Content
These lessons will involve the students in investigating and understanding:

- **Definition 45:** medians and centroid
- **Construction 21:** centroid of a triangle

Suggested class activities
Students might engage in the following activities:

Draw the medians and centroid for an acute-angled triangle, a right-angled triangle and an obtuse-angled triangle.
In which instances is the centroid inside the triangle?

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Lesson Idea LCOL.61

Two class periods

Title
Constructions 16 and 17

Resources

Geometry Course for Post-Primary School Mathematics
NCCA Student Resources
A mathematical instruments set

Dynamic software package

Student's CD

Content
These lessons will involve the students in investigating and understanding:

- **Construction 16**: Circumcentre and circumcircle of a given triangle, using only straight edge and compass.
- **Construction 17**: Incentre and incircle of a given triangle, using only straight edge and compass.

Suggested class activities
Students might engage in the following activities:

Draw the circumcentre and incentre of an acute-angled triangle, a right-angled triangle, an obtuse-angled triangle.

They should then answer the following questions explaining their reasoning in each case:

In which instances is the circumcentre inside the triangle?

Where is the circumcentre in a right-angled triangle? (see Theorem 19, corollary 3)

In which instances is the incentre inside the triangle?

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Section 10 Strand 2 Coordinate geometry of the circle

Lesson Idea LCOL.62

Seven class periods

Title

Co-ordinate geometry of the circle

Resources

Graph paper, mathematical set, dynamic geometry software package, set of board drawing instruments

Student's CD 

Content

These lessons will involve the students in investigating and understanding:

- That $x^2 + y^2 = r^2$ represents the equation of a circle centre (0,0) and radius of length *r*
  (Link to Pythagoras' Theorem - distance from any point $p(x, y)$ on the circle to the centre of the circle is equal to the length of the radius of the circle.)
- That $(x-h)^2 + (y-k)^2 = r^2$ represents the equation of a circle centre (*h*, k) and radius of length *r*
  (Link to Pythagoras' Theorem - distance from any point $p(x, y)$ on the circle to the centre of the circle is equal to the length of the radius of the circle.)
- How to solve problems involving a line and a circle with centre (0,0)

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Section11 Strand 2 Trigonometry

Lesson Idea LCOL.63

Twelve class periods

Title

Trigonometry 1

Resources

Classroom model or visual aid for the theorem of Pythagoras, *Formulae and Tables* book, Graph paper, mathematical set, dynamic geometry software package, set of board drawing instruments

Teaching and Learning Plan 8: Introduction to Trigonometry

Student's CD 

Content

These lessons will involve the students in investigating and understanding:

- The use of Pythagoras' Theorem in right-angled triangles
- Trigonometric ratios in a right-angled triangle
- The use of the trigonometric ratios to solve problems involving right-angled triangles
- The use of similar triangles to find unknowns in right-angled triangles
- The use of the clinometer
- How to use trigonometry to calculate the area of a triangle
- How to solve problems using the sine and cosine rules (2D)
- How to define $\sin\theta$ and $\cos\theta$ for all values of θ
- How to use the unit circle to solve equations such as $\sin\theta = \frac{1}{2}, \cos\theta = 0$ etc.
- How to define $\tan\theta$
- How to solve problems involving the area of a sector of a circle and the length of an arc
- Work with trigonometric ratios in surd form

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Section 12 Strand 3 Complex numbers

Lesson Idea LCOL. 62

8 class periods

Title

Complex numbers

Resources

Graph paper, mathematical set, dynamic geometry software package, set of board drawing instruments

Teaching and Learning Plan:

- *Complex numbers*

Student's CD 

Content

These lessons will involve the students in investigating and understanding:

- The origin and need for complex numbers
- The use of complex numbers to model two dimensional systems as in computer games, alternating current and voltage etc.
- How to interpret multiplication by i as a rotation of 90^0 anticlockwise
- How to express complex numbers in rectangular form $a+ib$ and illustrate them on the Argand diagram
- How to investigate the operations of addition and subtraction of complex numbers in the rectangular form $(a+ib)$ using the Argand diagram
- How to interpret the modulus as distance from the origin on an Argand diagram
- How to interpret multiplication by a complex number as a "multiplication of" the modulus by a real number combined with a rotation
- How to interpret the complex conjugate as a reflection in the real axis
- Division of complex numbers in the rectangular form $(a+ib)$
- How to solve quadratic equations having complex roots and how to interpret the solutions

Useful websites

	www.projectmaths.ie http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Appendix A Geometry: Thinking at Different Levels The Van Hiele Theory

The **Van Hiele model** describes how students learn geometry. Pierre van Hiele and Dina van Hiele-Geldof, mathematics teachers from the Netherlands, observed their geometry students in the 1950's.The following is a brief summary of the Van Hiele theory. According to this theory, students progress through five levels of thinking starting from merely recognising a shape to being able to write a formal proof. The levels are as follows:

*Visualisation (Level 0)

The objects of thought are shapes and what they look like.

Students have an overall impression of a shape. The appearance of a shape is what is important. They may think that a rotated square is a "diamond" and not a square because it is different from their visual image of a square. They will be able to distinguish shapes like triangles, squares, rectangles etc but will not be able to explain, for example, what makes a rectangle a rectangle. **Vocabulary**: Students use visual words like "pointy", "curvy", "corner" as well as correct language like angle, rectangle and parallelogram.

*Analysis (Level 1)

The objects of thought are "classes" of shapes rather than individual shapes.

- Students think about what makes a rectangle a rectangle and can separate the defining characteristics of a rectangle from irrelevant information like size and orientation. They recognize its parts (sides, diagonals and angles) and compare their properties (similar, congruent)
- They understand that if a shape belongs to a class like "rectangle", then it has all the properties of that class (2 pairs of equal sides, right angles, 2 equal diagonals, 2 axes of symmetry).
- **Vocabulary:** words like parallel, perpendicular and congruent relating to properties within a figure and the words all, always, sometimes, never, alike, different.
- A concise definition of a figure, using a sufficient rather than an exhaustive list of properties is not possible at this level.
- They do not deal with questions like "Is a square a parallelogram?" but just look at the properties of each class of shape, without comparing the classes.

*Some visualisation and analysis is covered in Primary School.

Relational/ Ordering/Informal Deduction (Level 2)

The objects of thought are the properties of shapes.

- Students are ready to understand interrelationships of properties within figures and between figures. Opposite sides of a parallelogram are parallel and opposite angles are equal.

- A rectangle is a parallelogram since it has all the properties of a parallelogram as well as having all 90^0 angles.
- Students can recognise the difference between a statement and its converse. All squares are rectangles (true) is different from all rectangles are squares (not true).
- Capable of **"if –then"** thinking – if a shape is a rectangle then all the angles in it are right angles. If |<A |= |<B |and |<B| = |<C| then |< A| =|<C|
- They can select one or two properties to define a figure rather than an exhaustive list. If a quadrilateral has 4 equal sides and one right angle it must be a square.
- Students can discover new properties by simple deduction. The two acute angles in a right angled triangle add to 90^0 because all the angles in a triangle add up to 180^0. They can explain logically without having to measure everything.

Formal deduction (Level 3)

Students learn how to use an axiomatic system to establish geometric theory. This is the level at which proof of Theorems is learned. The sequence of theorems given in the appendix is arranged in such a manner that each theorem builds on the previous theorem(s).

Rigor (Level 4)

Comparing different axiomatic systems – not done at secondary level

Characteristics of these levels: Students cannot function at any particular level unless they are competent at all previous levels. The teacher's role is crucial in structuring activities to bring students from one level to the next.

How does the teacher bring students from any one level to the next?

5 phases of learning:

1. In an informal discussion of the topic, students are asked to give their initial observations.
2. The teacher provides structured activities such as drawing, making and measuring.
3. The students then verbalise and write down what they have learned and report back in groups to the class, which leads to a class discussion.
4. The teacher then provides an activity which will require students to apply what they have discovered
5. In the last stage students are required to summarise all they have learned and should be able to remember it as they have discovered it through guidance.

A PowerPoint presentation of the Van Hiele theory can be got at www.projectmaths.ie

2 examples are given on the PowerPoint slides

(1) Using similar triangles to show advancement between levels and

(2) Using an investigation of the rhombus to show how to progress from level 0 to level 1 with this figure using the 5 teaching phases.

A mind map of Van Hiele can be found at
http://agutie.homestead.com/files/mindmap/van_hiele_geometry_level.html

Appendix B

Guide to Theorems, Axioms and Constructions at all Levels*

This is intended as a quick guide to the various axioms, theorems and constructions as set out in the ***Geometry Course for Post-Primary School Mathematics.*** You can get this from the project maths website: www.projectmaths.ie

It is not intended as a replacement for this document, merely as an aid to reading at a glance which material is required to be studied at various levels. The sequence of theorems as given must be followed.

As stated in the heading, these theorems and constructions are underpinned by 46 definitions and 20 propositions which are all set out in the ***Geometry Course for Post-Primary School Mathematics***, along with many undefined terms and definable terms used without explicit definition.

*An **axiom** is a statement accepted without proof, as a basis for argument

*A **theorem** is a statement deduced from the axioms by logical argument. Theorems can also be deduced from previously established theorems.

* A **proposition** is a useful or interesting statement that could be proved at this point, but whose proof is not stipulated as an essential part of the programme. Teachers are free to deal with them as they see fit, but they should be mentioned, at least (Appendix p. 20, footnote).

*The instruments that may be used for **constructions** are listed and described on page 38 of the Appendix and are a straight edge, compass, ruler, protractor and set-square.

Terms

Students at **Junior Certificate Higher level and Leaving Certificate Ordinary level** will be expected to understand the meanings of the following terms related to logic and deductive reasoning:
Theorem, proof, axiom, corollary, converse, implies.

In addition, students at **Leaving Certificate Higher level** will be expected to understand the meanings of the following terms related to logic and deductive reasoning:

Is equivalent to, if and only if, proof by contradiction.

Synthetic Geometry

Guide to Axioms, Theorems and Constructions for all Levels

Information Technology is used whenever and wherever appropriate to help to present mathematical concepts effectively to students. In this document the symbol appears at the corresponding position of the content to indicate that an interactive IT module is available on the Project Maths Student's CD.

	Axioms and Theorems (supported by 46 definitions, 20 propositions) *proof required for JCHL and LCHL ** proof required for LCHL only	CMN Introd. Course	JC ORD	JC HR	LC FDN	LC ORD	LC HR
	Axiom 1: There is exactly one line through any two given points	√	√	√	√	√	√
	Axiom 2: [Ruler Axiom]: The properties of the distance between points.	√	√	√	√	√	√
	Axiom 3: Protractor Axiom (The properties of the degree measure of an angle).	√	√	√	√	√	√
1	Vertically opposite angles are equal in measure.	√	√	√	√	√	√
	Axiom 4: Congruent triangles conditions (SSS, SAS, ASA)	√	√	√	√	√	√
2	In an isosceles triangle the angles opposite the equal sides are equal. Conversely, if two angles are equal, then the triangle is isosceles.	√	√	√	√	√	√
	Axiom 5: Given any line l and a point P, there is exactly one line through P that is parallel to l.	√	√	√	√	√	√
3	If a transversal makes equal alternate angles on two lines then the lines are parallel. Conversely**,** if two lines are parallel, then any transversal will make equal alternate angles with them.	√	√	√	√	√	√
4*	The angles in any triangle add to 180^0.	√	√	√	√	√	√
5	Two lines are parallel if, and only if, for any transversal, the corresponding angles are equal.	√	√	√	√	√	√
6*	Each exterior angle of a triangle is equal to the sum of the interior opposite angles.	√	√	√	√	√	√
7	The angle opposite the greater of two sides is greater than the angles opposite the lesser. Conversely, the side opposite the greater of two angles is greater than the side opposite the lesser angle.					√	√
8	Two sides of a triangle are together greater than the third.					√	√
9*	In a parallelogram, opposite sides are equal, and opposite angles are equal. Conversely, (1) if the opposite angles of a convex quadrilateral are equal, then it is a parallelogram; (2) if the opposite sides of a convex quadrilateral are equal, then it is a parallelogram.		√	√	√	√	√
	Corollary 1. A diagonal divides a parallelogram into two congruent triangles.			√			√
10	The diagonals of a parallelogram bisect each other. Conversely, if the diagonals of a quadrilateral bisect one another, then the quadrilateral is a parallelogram.		√	√	√	√	√

	Axioms and Theorems (supported by 46 definitions, 20 propositions) *proof required for JCHL and LCHL ** proof required for LCHL only	CMN Introd. Course	JC ORD	JC HR	LC FDN	LC ORD	LC HR
11**	 If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.			√		√	√
12**	 Let ABC be a triangle. If a line l is parallel to BC and cuts [AB] in the ratio m:n, then it also cuts [AC] in the same ratio. Conversely, if the sides of two triangles are in proportion, then the two triangles are similar.			√		√	√
13**	 If two triangles are similar, then their sides are proportional, in order (and converse)		√	√	√	√	√
14*	 [Theorem of Pythagoras]In a right-angled triangle the square of the hypotenuse is the sum of the squares of the other two sides.		√	√	√	√	√
15	 [Converse to Pythagoras]. If the square of one side of a triangle is the sum of the squares of the other two, then the angle opposite the first side is a right angle.		√	√	√	√	√
	Proposition 9: (RHS). If two right-angled triangles have hypotenuse and another side equal in length respectively, then they are congruent.		√	√	√	√	√
16	 For a triangle, base x height does not depend on the choice of base.					√	√
	Definition 38: The area of a triangle is half the base by the height.					√	√
17	 A diagonal of a parallelogram bisects the area.					√	√
18	 The area of a parallelogram is the base x height.					√	√
19*	 The angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on the same arc.			√			√
	Corollary 2†: All angles at points of a circle, standing on the same arc are equal (and converse).			√			√
	Corollary 3: Each angle in a semi-circle is a right angle.		√	√	√	√	√
	Corollary 4: If the angle standing on a chord [BC] at some point of the circle is a right-angle, then [BC] is a diameter.		√	√	√	√	√
	Corollary 5: If ABCD is a cyclic quadrilateral, then opposite angles sum to 180^0.			√			√
20	 (i) Each tangent is perpendicular to the radius that goes to the point of contact. (ii) If P lies on the circle S, and a line l is perpendicular to the radius to P, then l is a tangent to S.					√	√
	Corollary 6: If two circles intersect at one point only, then the two centres and the point of contact are collinear.					√	√
21	 (i) The perpendicular from the centre to a chord bisects the chord. (ii) The perpendicular bisector of a chord passes through the centre.					√	√

† The corollaries are numbered as in the *Geometry for Post-primary School Mathematics*; corollary 2 is the first one relating to theorem 19

	Constructions (Supported by 46 definitions, 20 propositions, 5 axioms and 21 theorems)	CMN Introd. Course	JC ORD	JC HR	LC FN	LC ORD	LC HR
1	Bisector of an angle, using only compass and straight edge.	√	√	√	√	√	√
2	Perpendicular bisector of a segment, using only compass and straight edge.	√	√	√	√	√	√
3	Line perpendicular to a given line l, passing through a given point not on l.			√			√
4	Line perpendicular to a given line l, passing through a given point on l.	√	√	√	√	√	√
5	Line parallel to given line, through a given point.	√	√	√	√	√	√
6	Division of a line segment into 2 or 3 equal segments without measuring it.	√	√	√	√	√	√
7	Division of a line segment into any number of equal segments, without measuring it.			√			√
8	Line segment of a given length on a given ray.	√	√	√	√	√	√
9	Angle of a given number of degrees with a given ray as one arm.		√	√	√	√	√
10	Triangle, given lengths of 3 sides.		√	√	√	√	√
11	Triangle, given SAS data.		√	√	√	√	√
12	Triangle, given ASA data		√	√	√	√	√
13	Right-angled triangle, given length of hypotenuse and one other side		√	√	√	√	√
14	Right-angled triangle, given one side and one of the acute angles.		√	√	√	√	√
15	Rectangle given side lengths.		√	√	√	√	√
16	Circumcentre and circumcircle of a given triangle, using only straight edge and compass.					√	√
17	Incentre and incircle of a triangle of a given triangle, using only straight edge and compass.					√	√
18	Angle of 60^0 without using a protractor or set square.				√	√	√
19	Tangent to a given circle at a given point on it.				√	√	√
20	Parallelogram, given the length of the sides and the measure of the angles.				√	√	√
21	Centroid of a triangle.					√	√
22	Orthocentre of a triangle.						√

Appendix C Investigations of quadrilaterals and triangles

Investigating Quadrilaterals

Quadrilaterals	**Square**	**Rhombus** (not a square)	**Rectangle** (not a square)	**Parallelogram** (not a rectangle or a rhombus)	**Trapezium** (not a parallelogram) and not an isosceles trapezium which has the non parallel sides equal in length)
Describe it in words.					
Draw three examples in different orientations.					
How many axes of symmetry does it have? Show on a diagram.					
Does it have a centre of symmetry? Show on a diagram.					
Which sides are equal?					
What is the sum of all the angles?					
Are all angles equal?					
Which angles					

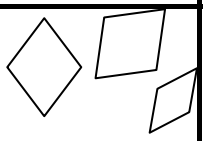
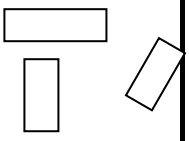
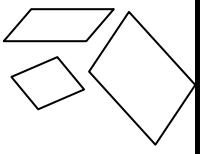
Quadrilaterals	Square	Rhombus (not a square)	Rectangle (not a square)	Parallelogram (not a rectangle or a rhombus)	Trapezium (not a parallelogram) and not an isosceles trapezium which has the non parallel sides equal in length)
are equal?					
What is the sum of two adjacent angles?					
Does a diagonal bisect the angles it passes through?					
Does a diagonal divide it into two congruent triangles?					
Given the length of its sides, can you calculate the length of a diagonal?					
Are the two diagonal s equal in length?					
Do the diagonals divide it into four congruent triangles?					
Do the diagonals divide it into					

Quadrilaterals	**Square**	**Rhombus** (not a square)	**Rectangle** (not a square)	**Parallelogram** (not a rectangle or a rhombus)	**Trapezium** (not a parallelogram) and not an isosceles trapezium which has the non parallel sides equal in length)
four triangles of equal area?					
Are the diagonals perpendicular?					
Do the two diagonals bisect each other?					
What information do you need to calculate its area? **How do you calculate it?**					
Does a diagonal bisect its area?					

Investigating triangles

Triangles	Equilateral	Isosceles	Right angled	Obtuse Angled
Describe it in words.				
Draw three examples in different orientations.				
How many axes of symmetry does it have? Show on a diagram.				
Does it have a centre of symmetry? Show on a diagram.				
What is the sum of the three angles?				
Are all angles equal?				
Are there any equal angles? Where?				
Can you say for certain what size the angles are?				
Apart from the isosceles triangles themselves which of the others could also be isosceles?				
What information do you need to				

Triangles	Equilateral	Isosceles	Right angled	Obtuse Angled
calculate its area? **How do you calculate it?**				
Draw 3 diagrams for each type of triangle showing each side as a base and the corresponding perpendicular height?				
How do you calculate the area?				
Is the centroid inside the triangle always?				
Is the circumcentre inside the triangle always?				
Is the incentre inside the triangle always?				

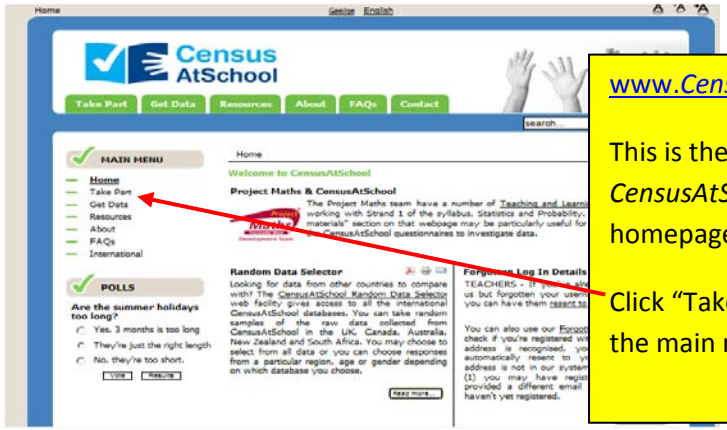
Quadrilaterals	Square	Rhombus (not a square)	Rectangle (not a square)	Parallelogram (not a rectangle or a rhombus)	Trapezium (not a parallelogram) and not an isosceles trapezium which has the non parallel sides equal in length)
Describe it in words.	A square is a quadrilateral in which all sides are equal in length and all angles are 90^0. (need only say that one angle is 90^0)	A rhombus is a quadrilateral with all sides equal and opposite angles equal. (a parallelogram with all sides equal in length.)	A rectangle is a quadrilateral with opposite sides equal and parallel and all interior angles equal to 90^0.	A parallelogram is a quadrilateral with opposite sides equal and parallel and opposite angles equal.	A trapezium is a quadrilateral which has 1 pair of parallel sides.
Draw three examples in different orientations.					
How many axes of symmetry does it have? Show on a diagram.	4	2	2	None	None if the non parallel sides are not equal in length.
Does it have a centre of symmetry? Show on a diagram.	✓	✓	✓	✓	No
Which sides are equal?	All	All	Opposite	Opposite	none
What is the sum of all the angles?	360^0	360^0	360^0	360^0	360^0
Are all angles equal?	✓	x	✓	x	x
Which angles are equal?	All angles	Opposite angles	All angles	Opposite angles	

Quadrilaterals	Square	Rhombus (not a square)	Rectangle (not a square)	Parallelogram (not a rectangle or a rhombus)	Trapezium (not a parallelogram) and not an isosceles trapezium which has the non parallel sides equal in length)
What is the sum of two adjacent angles?	180^0	180^0	180^0	180^0	180^0
Does a diagonal bisect the angles it passes through?	✓	✓	x	x	x
Does a diagonal divide it into two congruent triangles?	✓	✓	✓	✓	x
Given the length of its sides, can you calculate the length of a diagonal?	✓	No. Need to know an angle. Investigate using geostrips.	✓	No. Need to know an angle.	Need to know the lengths of two adjacent sides and the angle between them.
Are the two diagonal s equal in length?	✓	x	✓	x	x
Do the diagonals divide it into four congruent triangles?	✓	✓	x	x	x
Do the diagonals divide it into four triangles of equal area?	✓	✓	✓	✓	x

Quadrilaterals	**Square**	**Rhombus** (not a square)	**Rectangle** (not a square)	**Parallelogram** (not a rectangle or a rhombus)	**Trapezium** (not a parallelogram) and not an isosceles trapezium which has the non parallel sides equal in length)
Are the diagonals perpendicular?	✓	✓	x	x	x
Do the two diagonals bisect each other?	✓	✓	✓	✓	x
What information do you need to calculate its area? **How do you calculate it?**	One side length x. Area = x^2 (Base (*b*) and perpendicular height (*h*)from a vertex to that base Area = *b* x *h*)	Base (*b*) and perpendicular height (*h*)from a vertex to that base Area = *b* x *h* If you know the lengths of the diagonals *x* and *y* Area = ½ *x y*.	Lengths of 2 adjacent sides *l* and *b*. Area = *l* x *b*. (Base (*b*) and perpendicular height (*h*)from a vertex to that base Area = *b* x *h*)	Base (*b*) and perpendicular height (*h*)from a vertex to that base Area = *b* x *h*	The lengths of its parallel sides (*a* and *b*)and the perpendicular distance between them. Area = ½ (*a*+*b*) *h*
Does a diagonal bisect its area?	✓	✓	✓	✓	

Appendix D

How to register for *CensusAtSchool*, complete online questionnaires and retrieve class data



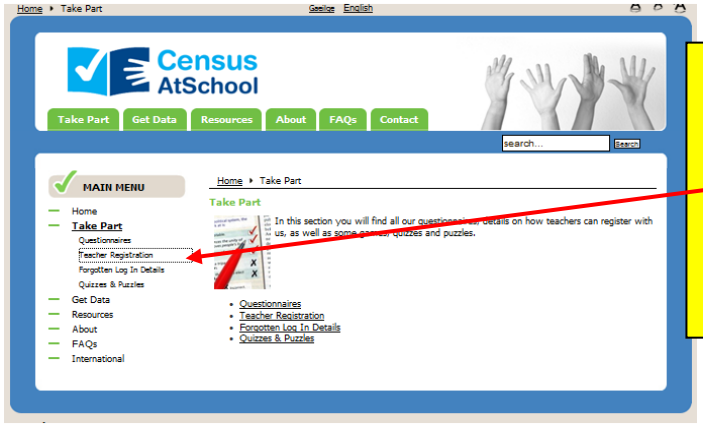
www.*CensusAtSchool*.ie

This is the Irish *CensusAtSchool* homepage.

Click "Take Part" under the main menu.

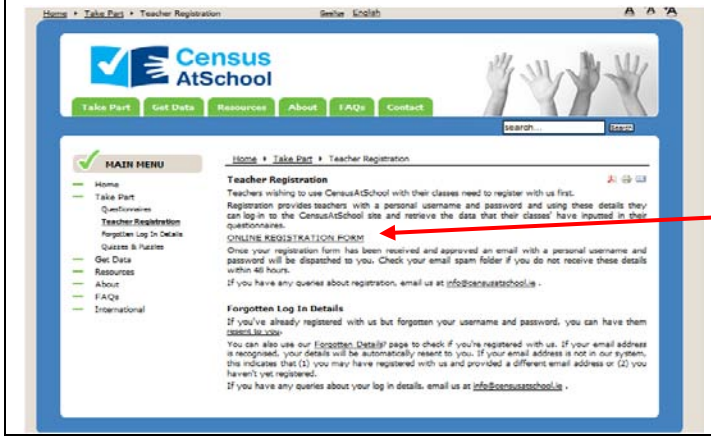
1

Part 1: Registration



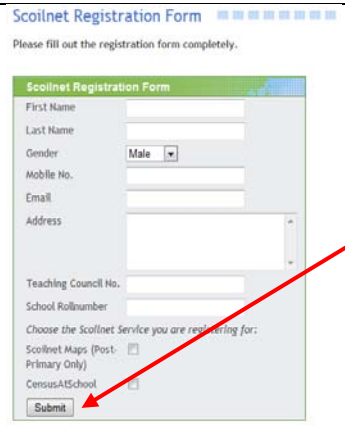
Click "Teacher Registration" under "Take Part" in the main menu

2



Click "ONLINE REGISTRATION FORM"

3



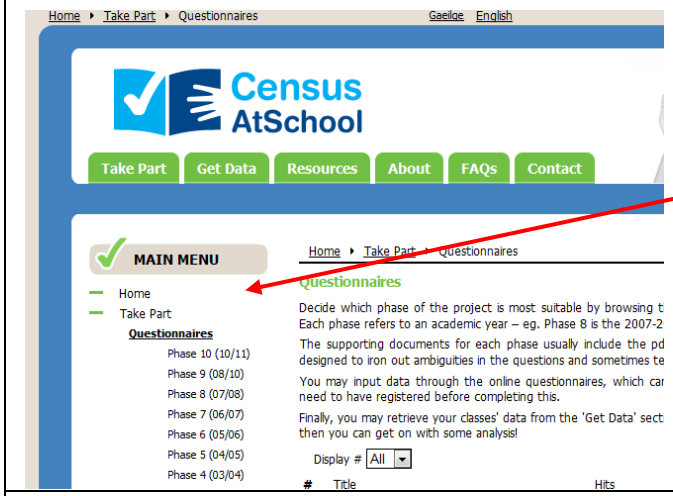
Fill out and click on "Submit"

Once your registration form has been received and approved an email with a personal username and password will be dispatched to you.

Check your email spam folder if you do not receive these details within 48 hours.

4

Part 2: Accessing the questionnaires -use the highest phase available



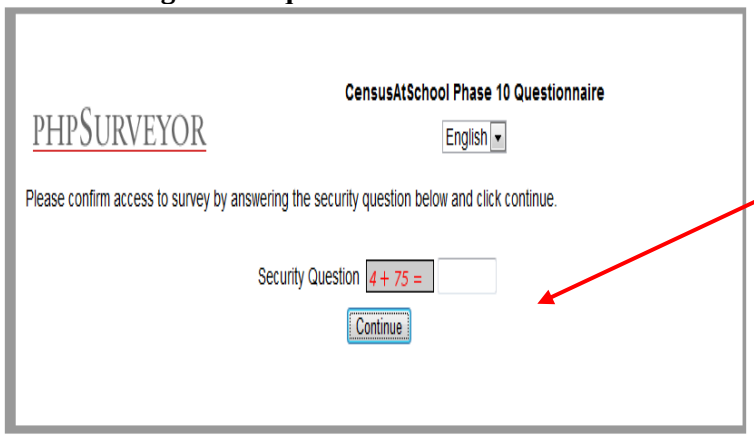
Click "Take Part", "Questionnaires" and then click "Phase 10" or highest phase.

5

Click "Take the Phase 10 questionnaire"

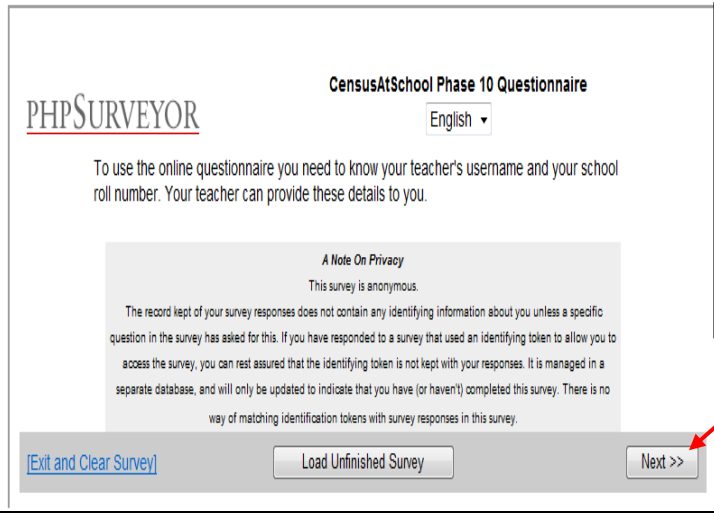
6

Part 3: Filling out the questionnaire



Fill in and click on "Continue"

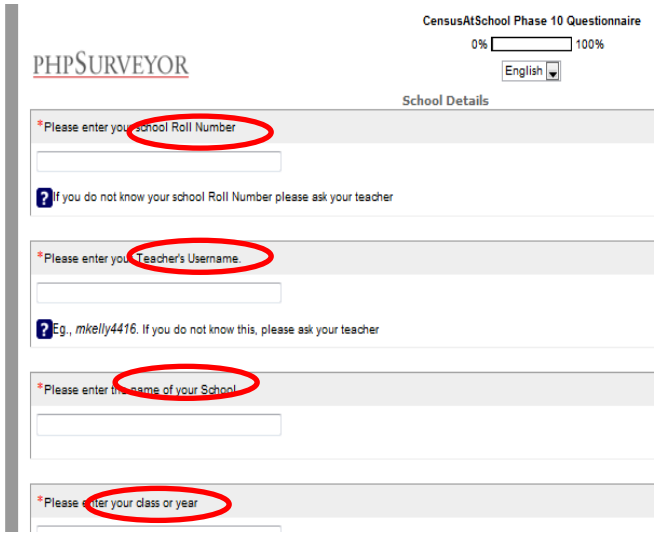
7



Ensure you have your teacher's username and the school roll number.

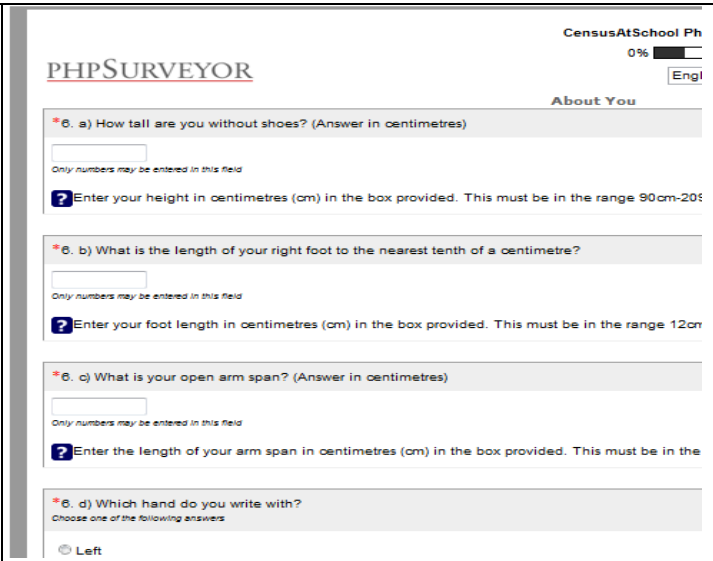
click on "Next"

8



Have each student input the school roll number and your teacher username (sent to you by email). School name and class or year details must also be input.

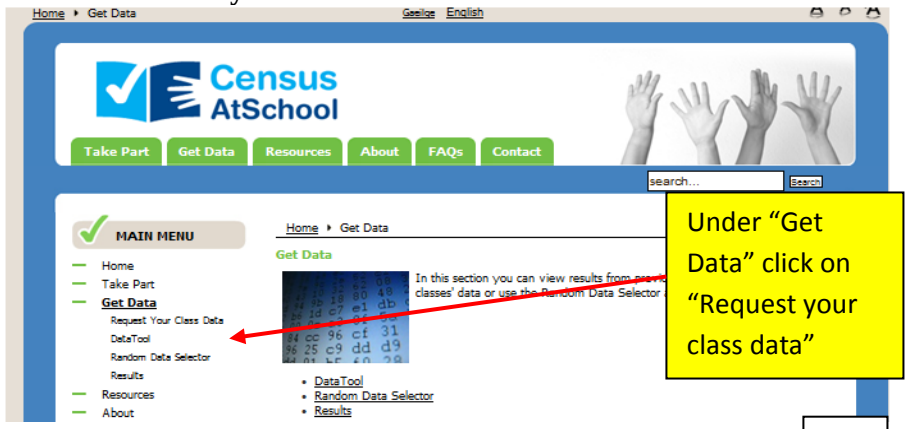
9



Sample questions from the questionnaire

10

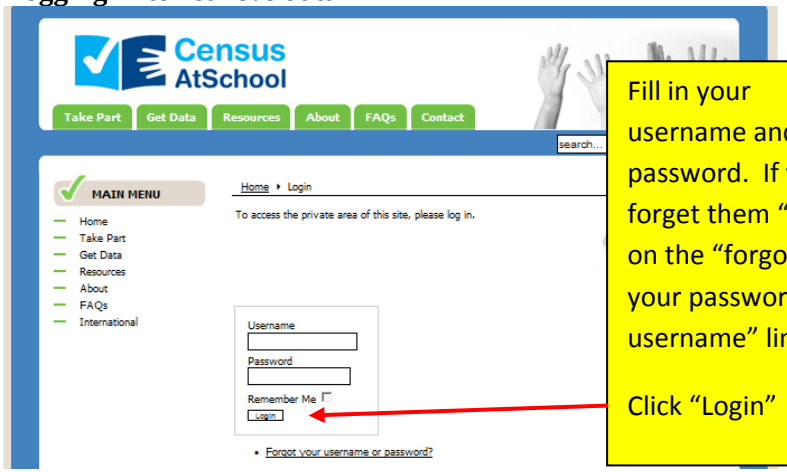
Part 4 Now that my students have filled in the data – What next?



Under "Get Data" click on "Request your class data"

11

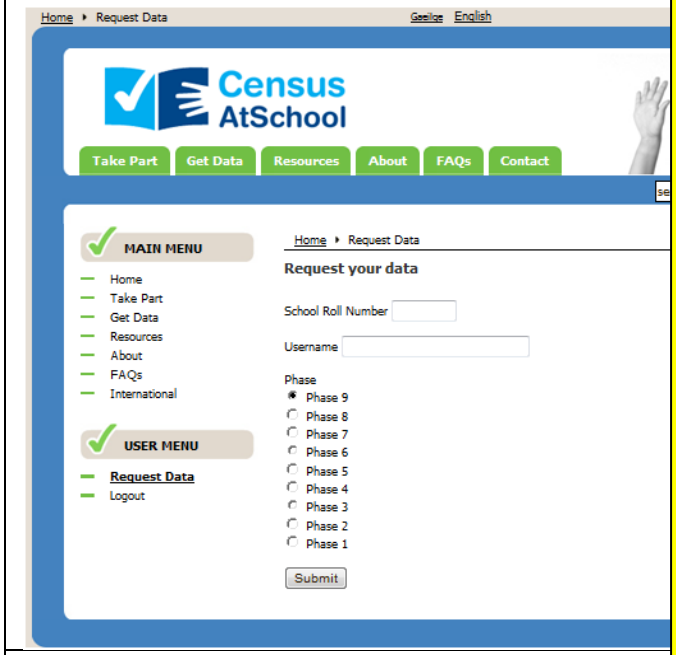
Logging in to retrieve data



Fill in your username and password. If you forget them "click on the "forgot your password or username" link.

Click "Login"

12



Once logged in you will be asked for your school roll number and username.

Select whichever phase your students have completed. Click on "Submit" and your data will be returned to you immediately in spreadsheet form.

13

Your data will look similar to the spreadsheet below.

Sex	BrthMnth	BrthYear	BrthPlce	BedRms	PeoplHme	InetAxs	NumPts	PtsNone	PtsDog	PtsCat	PtsMouse	PtsRabt
Female	July	1992	New Sout	4	5	Yes - broa	4	No	Yes	Yes	No	No
Female	February	1996	Australian	4	5	Yes - broa	4	No	Yes	Yes	No	No
Male	Septembe	1994	Victoria	4	5	Yes - dial-	5	No	Yes	No	No	No
Female	Septembe	1991	Fiji	4	4	No - Interi	0	Yes	No	No	No	No
Male	August	1993	Queenslai	3	4	No - Interi	0	No	No	No	No	No
Female	Decembei	1993	Tasmania	3	5	Yes - dial-	3	No	No	Yes	No	No
Female	August	1990	Queenslai	4	5	Yes - broa	3	No	Yes	No	No	No
Female	Novembe	1991	New Sout	11	5	Yes - dial-	4	No	Yes	No	No	Yes
Male	June	1995	Victoria	3	5	Yes - othe	6	No	Yes	No	No	No
Female	March	1993	Western A	30	122	No - Interi	2	No	Yes	No	No	No
Female	Decembei	1989	Queenslai	3	3	Yes - broa	2	No	No	No	No	No
Female	April	1993	South Aus	3	5	No - Interi	4	No	Yes	Yes	No	No
Male	March	1996	New Sout	3	2	No - Interi	0	Yes	No	No	No	No
Male	Septembe	1993	Queenslai	4	4	Yes - dial-	18	No	Yes	No	No	No
Male	May	1995	Tasmania	3	4	Yes - broa	2	No	No	No	No	Yes
Female	March	1994	Western A	4	4	Yes - dial-	4	No	Yes	Yes	No	No

14

Students can now begin numerical and graphical analysis of their own class data and make interpretations based on the results.