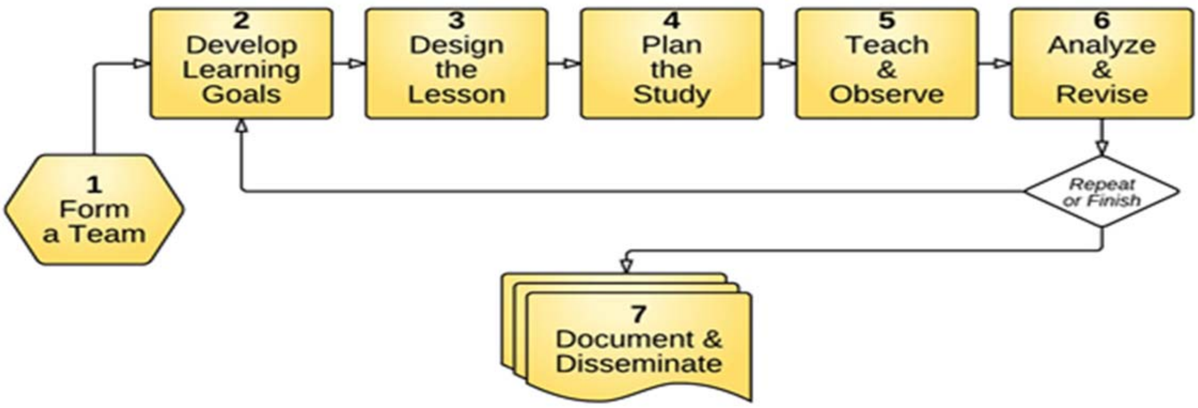
Reflections on Practice



Project Maths
Tionscadal Mata
Development Team

Lesson Plan for Second, third, fifth and 6th years. Introducing problem solving

For the lesson on 22nd October, 2014

At Coláiste Phádraig, Lucan

Teacher: Kevin Carey

Lesson plan developed by: Kevin Carey

1. **Title of the Lesson:** Introducing problem solving: Can differentiated learning be used when introducing problem solving.

2. **Brief description of the lesson** Can a single lesson plan provide differentiated instruction to a wide range of mathematical abilities, while engaging all participants in the problem solving process.

3. **Aims of the Lesson:**

- To encourage students to question what appears to be an "easy" solution to a problem.
- To introduce students to non -traditional mathematical problems
- To engage all participants in the problem solving process.
- To harness skills from each group member through differentiated learning

4. **Learning Outcomes:**

- Students should begin to question if a pattern continues for more than a few specific cases
- Students start to test solutions using critical experimentation.
- Students should begin to realise that there can be a structured approach to mathematical problem solving

5. **Background and Rationale**

Problem solving has become an integrated part of the new project maths curriculum. Despite this, the "art" of mathematical problem solving has not become part a normal part of the solving of problems.

In developing this lesson I drew on research that suggests that the progress in a student's ability to solve a mathematical problem generally passes through a number of stages. Simple empirical experimentation with various combinations of examples gives way to critical thinking and finally towards a deeper understanding of how to approach and solve mathematical problems.

When students are given a problem, especially on that contains a geometric component, they often begin by drawing various examples to look visually for a pattern. The point at which they abandon the visual and embrace the abstract mathematics can depend on the self-efficacy of the student involved and on that of the group they are working with.

In planning this lesson I am cognisant of the fact that in order for students to move away from the idea of naïve empiricism, they must discover the flaw in the pattern most of them will no doubt initially discover.

6. Research

Polya, G. (1957) *How to solve it: a new aspect of mathematical method*. London. Penguin.

Stylianides, A. J. (2009) 'Breaking the equation "empirical argument = proof."' *Mathematics Teaching* 213, 9-14.

Mason, J., Burton, L. and Stacey, K. (1985) *Thinking mathematically. Revised edition*. Wokingham. Addison-Wesley.

Chazan, D. (1993). 'High school geometry students' justification for their views of empirical evidence and mathematical proof. *Educational Studies in Mathematics*, 24, 359-387.

7. About the Lesson

Only previous knowledge is to have done patterns in the 1st year CIC strand 4 patterns.

"Circle and dots" problem

If you place dots around the circumference of a circle
and join each pair of dots by straight lines,
is there a relationship between the number of dots?
and the maximum number of non-overlapping regions produced?

Can you figure out how many regions there are with 14 dots?

8. Flow of the Lesson

Teaching Activity	Points of Consideration
1. Introduction Explain the class that they are going to work in groups on a task. Avoid using terminology like ‘maths problem’ or ‘problem-solving’ Explain that during the task you will be asking them to write down a commentary on how they are working on the task (adventurous teachers might like to get them to ‘tweet’ their progress) Tell them the task will last an hour with a break half way through for a quick discussion	Split the class into groups, making sure there is a good spread of mathematical ability in each group. Impress on them the need to record their decisions and comments Make sure everyone has mathematical sets and plenty of paper
2. Posing the Task Introduce the problem. Make sure to not mention that you are looking for a formula, but rather you want them to explore the relationship between the dots and circles.	Watch for any confusion as the what some of the phrases mean e.g. ‘non-overlapping’ or ‘pairs of points’
3. Anticipated Student Responses Students notice a ‘doubling’ pattern.	

Students draw a lot of pictures. Students come to the conclusion that there is a pattern that has something to do with 2 squared	
4. Comparing and Discussing After 30 mins ask a spokesperson from each group to talk about the relationship they have found (if any). Also ask about 14 dots Check with other groups to see if this is relationship, making sure to tease out if anyone has put this relationship into some formal structure i.e. a formula (2^{n-1}) **5. Posing the Task (part 2)** Ask if anyone noticed anything about when there are 6 dots?	 Listen to the feed back If an answer is given ask how they found it Make sure that all the various relationships are displayed for all the other groups Watch for any signs that groups might know but are afraid to speak out
6. Anticipated Student Responses They noticed it was different but they though they counted wrong They noticed it was different but everyone else got the “right” answer so they must have made a mistake They skipped 6 and did 7 or 8 etc.	 Ask them to discuss among themselves why 6 dots are different? Ask them to check 6 dots now.

7. Comparing and Discussing Ask them to describe how they approached the task Ask them did they have confidence once they figured out a pattern Ask them how did 6 dots affect their idea for 14 dots	Listen to the feedback Prompt them to give reasons for that confidence Was there a way of checking for other "exceptions"
8. Summing up What has this taught us about patterns? Would you be confident in predicting 14 dots if the 6 dot example had worked like all the others?	

8. Evaluation

- **What is your plan for observing students?**

  Allowing them to work on their own and coming to a conclusion is essential for them to begin to leave behind empiricism as a method of problem solving.

- **Discuss logistical issues such as who will observe, what will be observed, how to record data, etc.**

  It is important that they keep a record of the steps that they take when solving the problem. Ideally one of them should act as a record keeper keeping 'minutes' as the progress through the task

- **What types of student thinking and behaviour will observers focus on?**

Look for signs of empirical problem solving strategies and also for critical thinking strategies. If students are using visual or abstract methods of solving the problem, ascertain the amount of each type they use

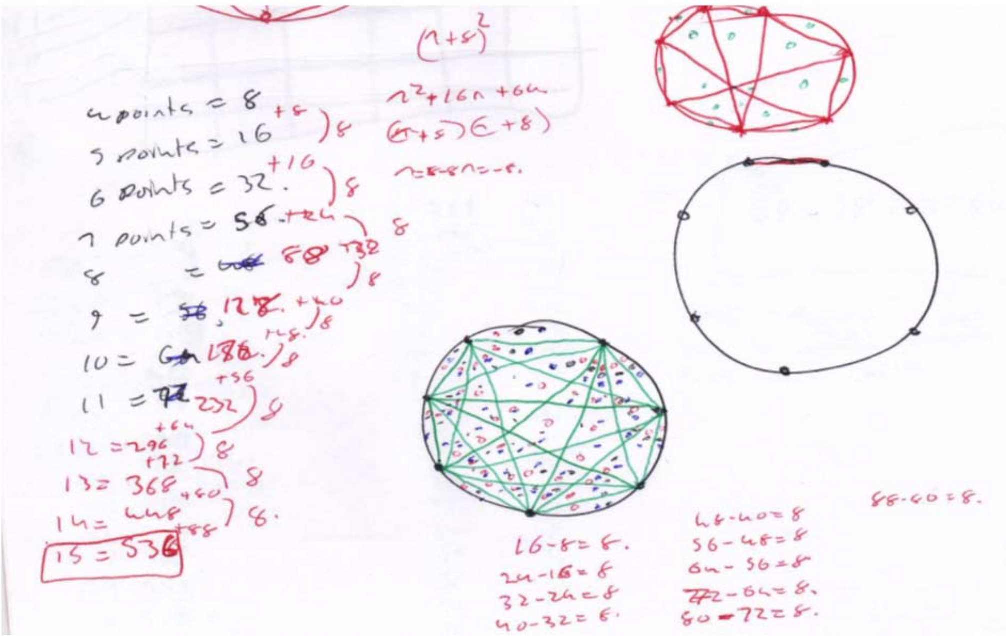
Post-lesson reflection

- **What are the major patterns and tendencies in the evidence? Discuss**

There was a mixture of strategies used by the students. Some worked on visual proofs while some start on visual proofs before trying algebraic proofs. . In all cases, naïve empiricism was evident at the initial stage and all groups moved on to some form of critical experimentation.

- **What are the key observations or representative examples of student learning and thinking?**

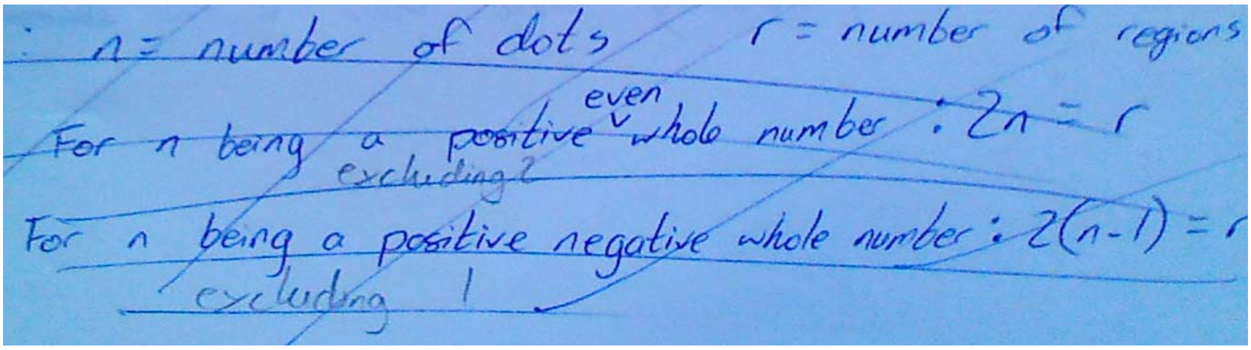
Within about 10 minutes of giving them the task, I started to notice the formula, 2^{n-1}, appearing in a number of groups work. A number of students had noticed the “doubling” pattern. Philip, for example, had begun a more sophisticated approach and was looking for the quadratic progression as you can see below



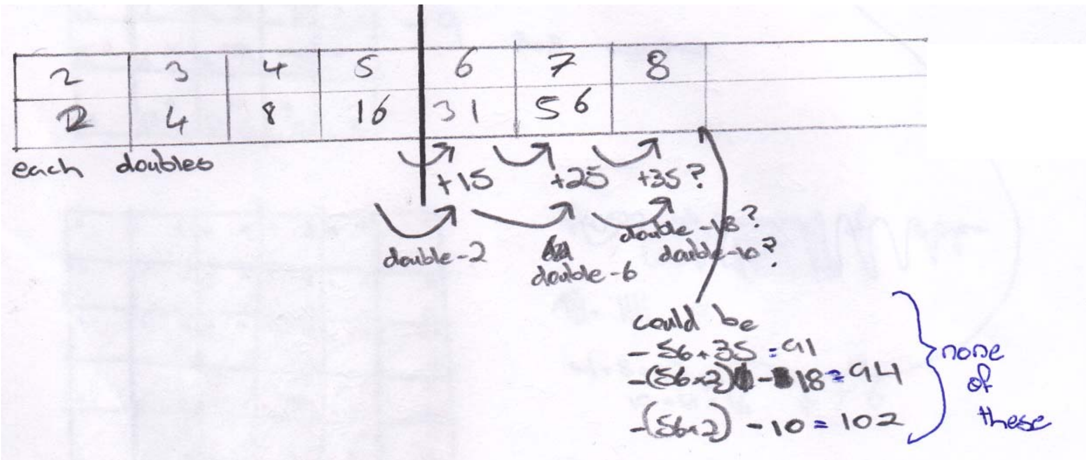
Philip’s method is important because it shows that he has begun to use the tools of the new Project Maths to find a pattern. This particular method of finding a quadratic pattern is a new skill to this cohort of students and while he made a mistake in n=6, it showed what Poyla describes in his

“Devising a plan” phase (Poyla, 1957, p.xvi). Philip has used prior knowledge of a similar problem and applied it to the current one.

After the 15 minutes I asked the students were they confident they knew the answer to n=14. They all said 2^{13} with the exception of one student called Eoin. I had noticed Eoin struggling during the task (Eoin is one of the strongest maths students in the class), and when I asked him what he got he said he couldn’t find a solution.

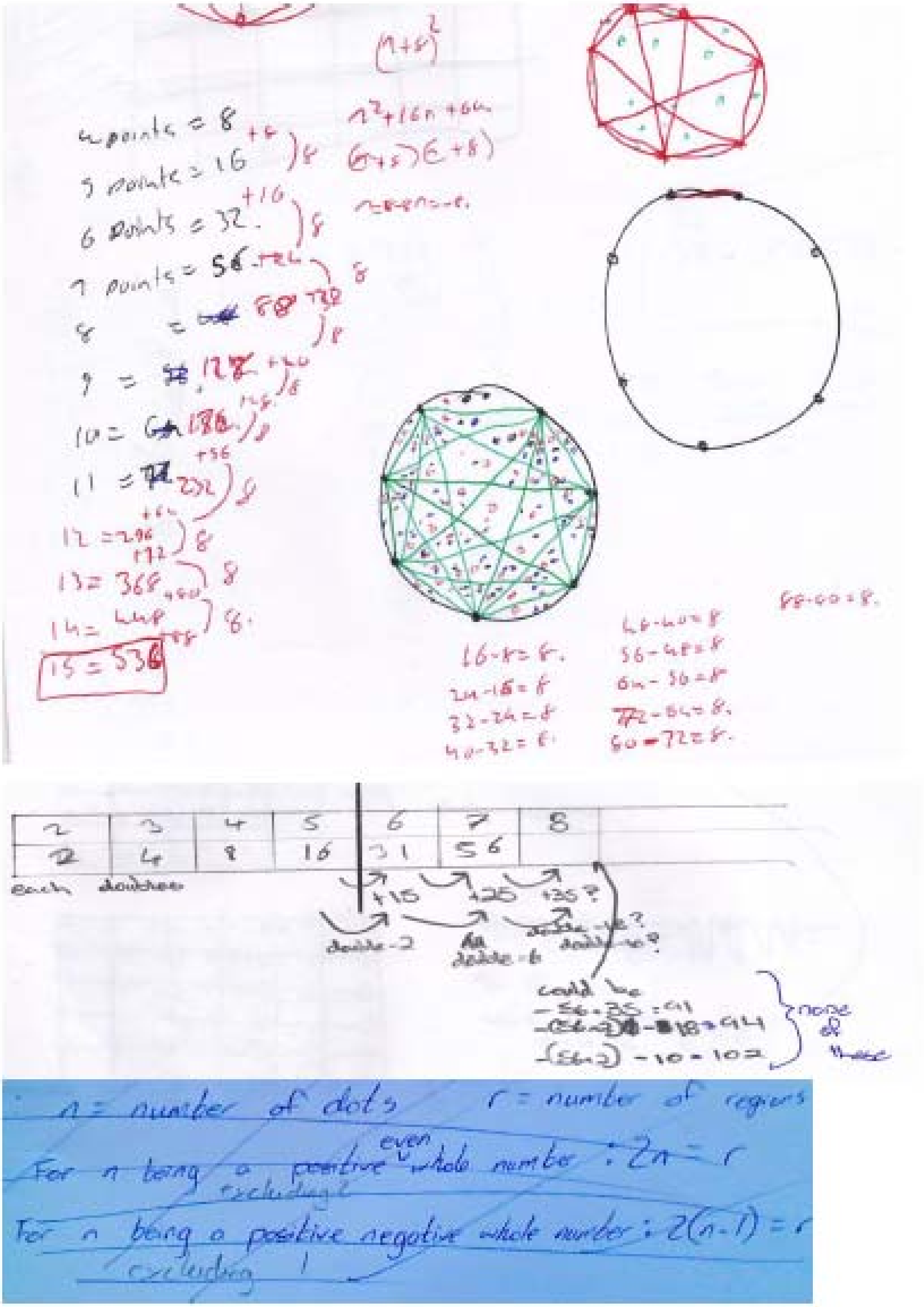


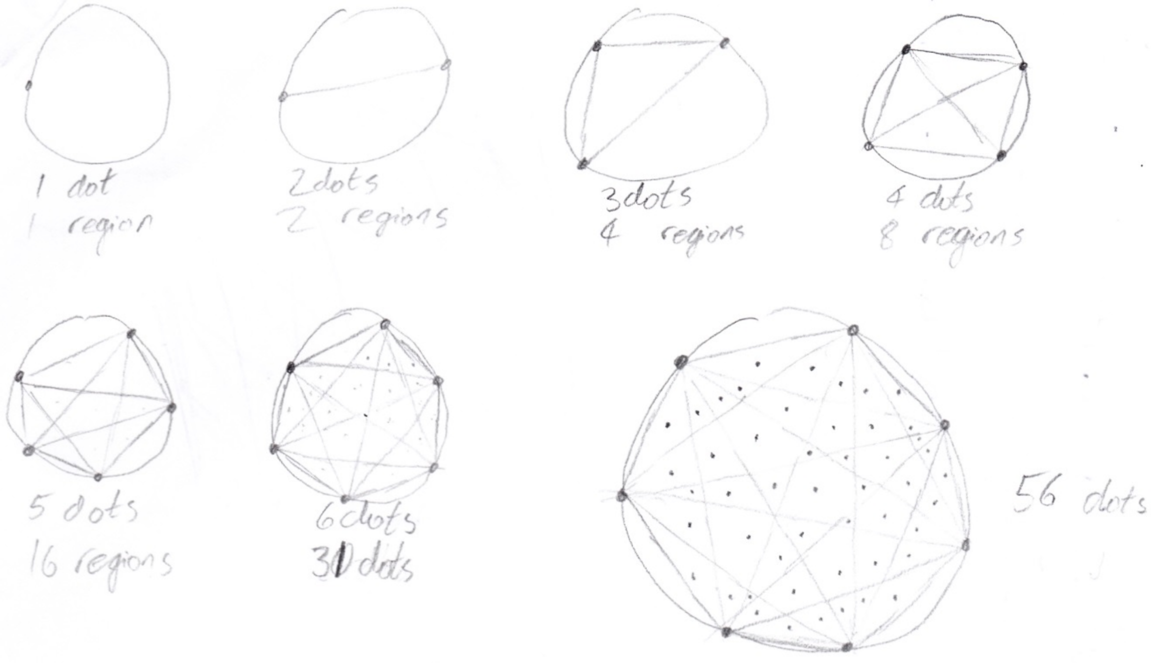
Eoin’s attempt is a perfect example of what Styliandes (2009) calls a ‘crucial experiment’ form of empirical argument. Eoin has moved on from the ‘naive empiricism’ which was prevalent in all the students work for the task and has attempted to look for counter- examples. I asked the students had anyone checked n=6, and one group, Mark and Sean, said it didn’t work but they just assumed they were wrong and had missed an area when counting.



I asked the students how they felt about 2^{13} being the answer for n=14, now that 2^5 was not the answer for n=6. They all agreed that finding an exception should make the rule invalid.

Student Work:





Q1) Not particularly as the circle problem was evidently much more difficult from the start

Q2) Yes, it appeared as if the pattern was 2^{n-1}

Q3) It can be incredibly difficult to prove something as there is always the possibility that there is one exception.

Mark & Sean
Questions

Q.1. Yes, we felt we could solve the circle question

Q2) We did think there was a pattern from the first attempts.

Q3) Exeptions can mess up your ideas for a proof, so can counting errors

6 spots
region.
12

4 regions
3 spots

4 spots
8 regions.

5 spots
16 regions

8 regions
4 spots

~~12 regions~~
~~7 spots~~
22 regions
7 spots

36 regions
9 spots.

1	1
2	2
3	4
4	8
5	8
6	12
7	12
8	16
9	16
10	20
11	20
12	24
13	24
14	25
15	25.

10 dots
20 regions