PDST Professional Development Service for Teachers | An tSeirbhís um Fhorbairt Ghairmiúil do Mhúinteoirí

POST PRIMARY MATHS

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Maths Counts 2019

Resources Booklet

Name: ______________________________

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Teaching & Learning Workshops
Financial Maths

Task 1 – Modelling Investments

A bank offers a yearly compound interest rate of 20%. Paul wishes to invest €100. What will the value of this investment be at the end of n years?

Each Box represents €10, colour in Paul's initial investment.

In a different colour, add in his interest received after 1 year.

Now repeat the steps above using the same two colours but also add in his interest received after 2 years in a new colour. (Any fractional parts should be represented by a partially completed box)

Repeat the above steps adding in his interest received after 3 years.

What do you notice about the pattern?

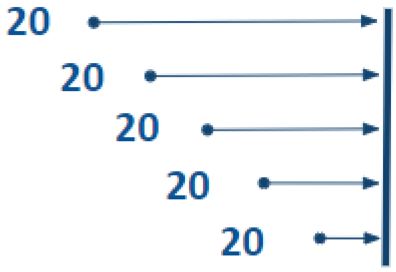
Fill in the blanks.

This is a/an ___________ sequence with a common _____________ of ________.

In the yellow boxes provided, write an expression to describe each of John's investments in the form $€100\ x\ 1.2^{t}, t \epsilon N$.

Task 2 – Investments

Jenny wishes to invest some money. She invests €20 at the start of each month for five months, in a savings account which offers a monthly rate of 0.5%. What will the value of this investment be at the end of five months?



Task 3 - Modelling

Jenny wishes instead to invest her money for 5 years She invests €20 at the start of each month for five years, into a savings account which offers a monthly rate of 0.5%. Model this investment as a group of individual deposits.

Task 3 - Extension

Jenny invests €20 at the start of each month for three years. She then reduces her investment to €15 for the remaining two years. What will the value of this investment be at the end of five years?

Complex Numbers

Task 1

If A and B are integers, what is the effect on A of multiplying by B?

Describe the effect of multiplication on a real number by another real number. Refer to the scale factor, location and rotation.

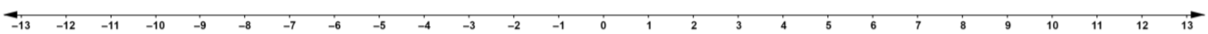
What is the effect of raising A to the power of n?

If A is a real number and b is a positive integer, what is the effect on A of multiplying by each
of the following?

$\boldsymbol{A\ x\ (-B)^n}$	**Rotation**	**Number of Rotations**	**Scale Factor**
$A\ x\ (-3)^1$	180°		
$A\ x\ (-3)^2$	$180^{\circ} + 180^{\circ}$		
$A\ x\ (-3)^3$			
$A\ x\ (-3)^4$			
$A\ x\ (-3)^n$			
$A\ x\ (-1)^1$			
$A\ x\ (-1)^2$			
$A\ x\ (-1)^3$			
$A\ x\ (-1)^n$			
$A\ x\ (-B)^n$			

What is the effect on A of multiplying by $(-1)^{\frac{1}{2}}$? Why might we ask students to consider this?

Choose a value for A. Can you suggest a suitable position for $A\ x\ (-1)^{\frac{1}{2}}$ on the diagram below?



Notes:

Task 2

Open the GeoGebra file:
https://tinyurl.com/WS4task2-2

z_1 and z_2 are complex numbers with a product z_3. By moving z_1 and observing z_3 consider the effect on z_1 of multiplication by z_2. Use the check boxes to support this investigation.
Can you suggest in words a rule for multiplying complex numbers?

$z_1 = -1 + i$ is a complex number with a modulus of $\sqrt{2}$ and argument of 135°.
Set $z_2 = z_1$,now $z_3 = (-1+i)^2$. Describe the effect of squaring z_1, referring to the moduli and the angles. Can you describe the effect on a complex number z, of raising z to the power of n?

Task 3
This task should be approached from the perspective of a higher level 5th year student.
Prior Knowledge:

- JC Number Operations
- Modulus of a complex number
- Multiplication of complex numbers using modulus and angles.

Link to Syllabus (LC OL):

– interpret the modulus as distance from the origin on an Argand diagram and calculate the complex conjugate

Link to Syllabus (LC HL):

– calculate conjugates of sums and products of complex numbers
– use De Moivre's Theorem

Open the GeoGebra file "complex Conjugate"
https://www.geogebra.org/classic/wrxxyrsf

z_1, z_2 and z_3 are complex numbers. Move z_1. What is the effect on z_2 of moving z_1? Z_3 is the product of z_1 and z_2. Why does z_3 always lie on the real axis?

Click checkbox |z|. Choose a value for z_1, and record the modulus of z_1, z_2 and z_3. Repeat this process with a different value for z_1.
Can you describe the relationship between the moduli?

Click the checkbox "Angle"
Change the position of z1 and note the effect on the angles α, β and γ.
Write down in words any relationship you can find between α, β and γ.

Construct a line segment from z_1 to z_2.

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What questions could you ask students to connect Pythagoras' Theorem to the modulus of a complex number?

Using the trigonometric ratios, write down the coordinates of the point z_1 and z_2.
Why might this investigation help students understand their study of De Moivre's Theorem?

Why might this investigation help students to understand the role of the conjugate in complex division?

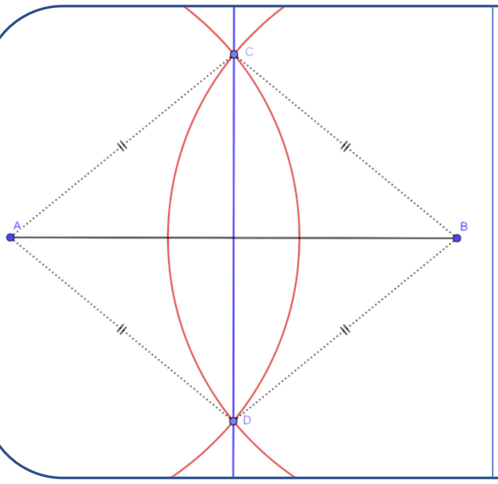
Resources

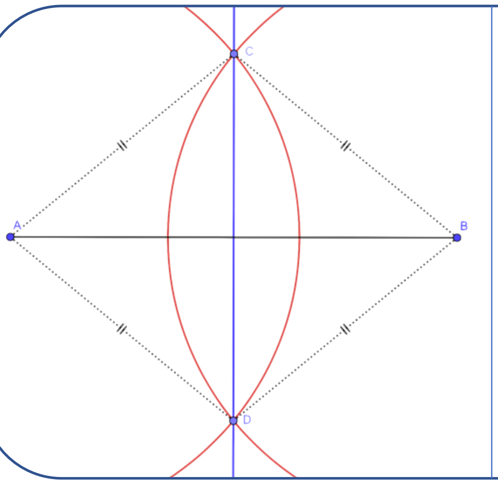
Addition of Complex Numbers
https://www.geogebra.org/classic/hapyyrhd

Multiplication of Complex Numbers
https://www.geogebra.org/classic/bmedez74

Exponents of i
https://www.geogebra.org/classic/r9tb7ftj

Conjugate
https://www.geogebra.org/classic/wrxxyrsf


C
A
B
D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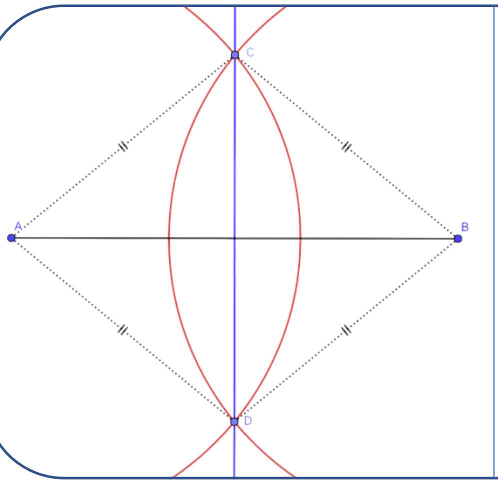
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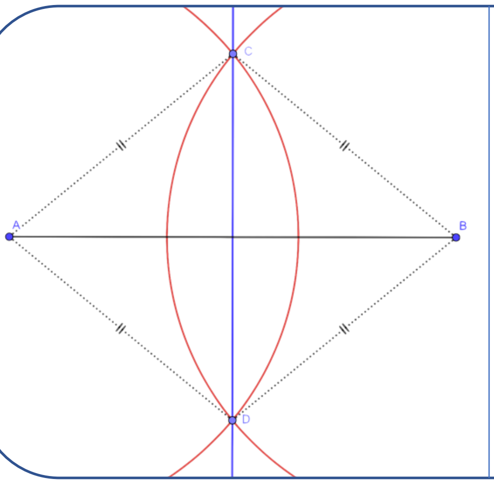
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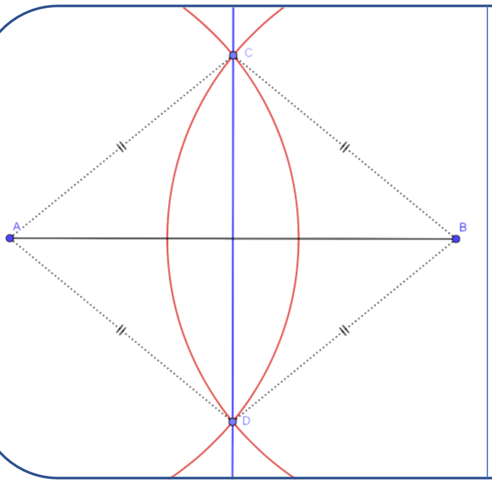
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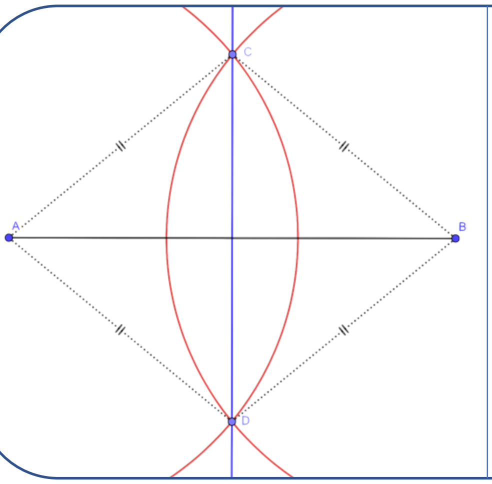
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Synthetic Geometry

Guide to Axioms, Theorems and Constructions for all Levels

Information Technology is used whenever and wherever appropriate to help to present mathematical concepts effectively to students. In this document the symbol appears at the corresponding position of the content to indicate that an interactive IT module is available on the Project Maths Student's CD.

	Axioms and Theorems (supported by 46 definitions, 20 propositions) *proof required for JCHL and LCHL ** proof required for LCHL only	Suggested for 1st year	JC ORD	JC HR	LC FDN	LC ORD	LC HR
	Axiom 1: There is exactly one line through any two given points	√	√	√	√	√	√
	Axiom 2: [Ruler Axiom]: The properties of the distance between points.	√	√	√	√	√	√
	Axiom 3: Protractor Axiom (The properties of the degree measure of an angle).	√	√	√	√	√	√
1	Vertically opposite angles are equal in measure.	√	√	√	√	√	√
	Axiom 4: Congruent triangles conditions (SSS, SAS, ASA)	√	√	√	√	√	√
2	In an isosceles triangle the angles opposite the equal sides are equal. Conversely, if two angles are equal, then the triangle is isosceles.	√	√	√	√	√	√
	Axiom 5: Given any line l and a point P, there is exactly one line through P that is parallel to l.	√	√	√	√	√	√
3	If a transversal makes equal alternate angles on two lines then the lines are parallel. Conversely**,** if two lines are parallel, then any transversal will make equal alternate angles with them.	√	√	√	√	√	√
4*	The angles in any triangle add to 180º.	√	√	√	√	√	√
5	Two lines are parallel if, and only if, for any transversal, the corresponding angles are equal.	√	√	√	√	√	√
6*	Each exterior angle of a triangle is equal to the sum of the interior opposite angles.	√	√	√	√	√	√
7	The angle opposite the greater of two sides is greater than the angles opposite the lesser. Conversely, the side opposite the greater of two angles is greater than the side opposite the lesser angle.					√	√
8	Two sides of a triangle are together greater than the third.					√	√
9*	In a parallelogram, opposite sides are equal, and opposite angles are equal. Conversely, (1) if the opposite angles of a convex quadrilateral are equal, then it is a parallelogram; (2) if the opposite sides of a convex quadrilateral are equal, then it is a parallelogram.		√	√	√	√	√
	Corollary 1. A diagonal divides a parallelogram into two congruent triangles.			√			√
10	The diagonals of a parallelogram bisect each other. Conversely, if the diagonals of a quadrilateral bisect one another, then the quadrilateral is a parallelogram.		√	√	√	√	√
11**	If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.			√		√	√
12**	Let ABC be a triangle. If a line l is parallel to BC and cuts [AB] in the ratio m:n, then it also cuts [AC] in the same ratio. Conversely, if the sides of two triangles are in proportion, then the two triangles are similar.			√		√	√

	Axioms and Theorems (supported by 46 definitions, 20 propositions) *proof required for JCHL and LCHL ** proof required for LCHL only	Suggested for 1st year	JC ORD	JC HR	LC FDN	LC ORD	LC HR
13**	If two triangles are similar, then their sides are proportional, in order (and converse)		√	√	√	√	√
14*	[Theorem of Pythagoras]In a right-angled triangle the square of the hypotenuse is the sum of the squares of the other two sides.		√	√	√	√	√
15	[Converse to Pythagoras]. If the square of one side of a triangle is the sum of the squares of the other two, then the angle opposite the first side is a right angle.		√	√	√	√	√
	Proposition 9: (RHS). If two right-angled triangles have hypotenuse and another side equal in length respectively, then they are congruent.		√	√	√	√	√
16	For a triangle, base x height does not depend on the choice of base.					√	√
	Definition 38: The area of a triangle is half the base by the height.					√	√
17	A diagonal of a parallelogram bisects the area.					√	√
18	The area of a parallelogram is the base x height.					√	√
19*	The angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on the same arc.			√			√
	Corollary 2†: All angles at points of a circle, standing on the same arc are equal (and converse).			√			√
	Corollary 3: Each angle in a semi-circle is a right angle.		√	√	√	√	√
	Corollary 4: If the angle standing on a chord [BC] at some point of the circle is a right-angle, then [BC] is a diameter.		√	√	√	√	√
	Corollary 5: If ABCD is a cyclic quadrilateral, then opposite angles sum to 180º.			√			√
20	(i) Each tangent is perpendicular to the radius that goes to the point of contact. (ii) If P lies on the circle S, and a line l is perpendicular to the radius to P, then l is a tangent to S.					√	√
	Corollary 6: If two circles intersect at one point only, then the two centres and the point of contact are collinear.					√	√
21	(i) The perpendicular from the centre to a chord bisects the chord. (ii) The perpendicular bisector of a chord passes through the centre.					√	√

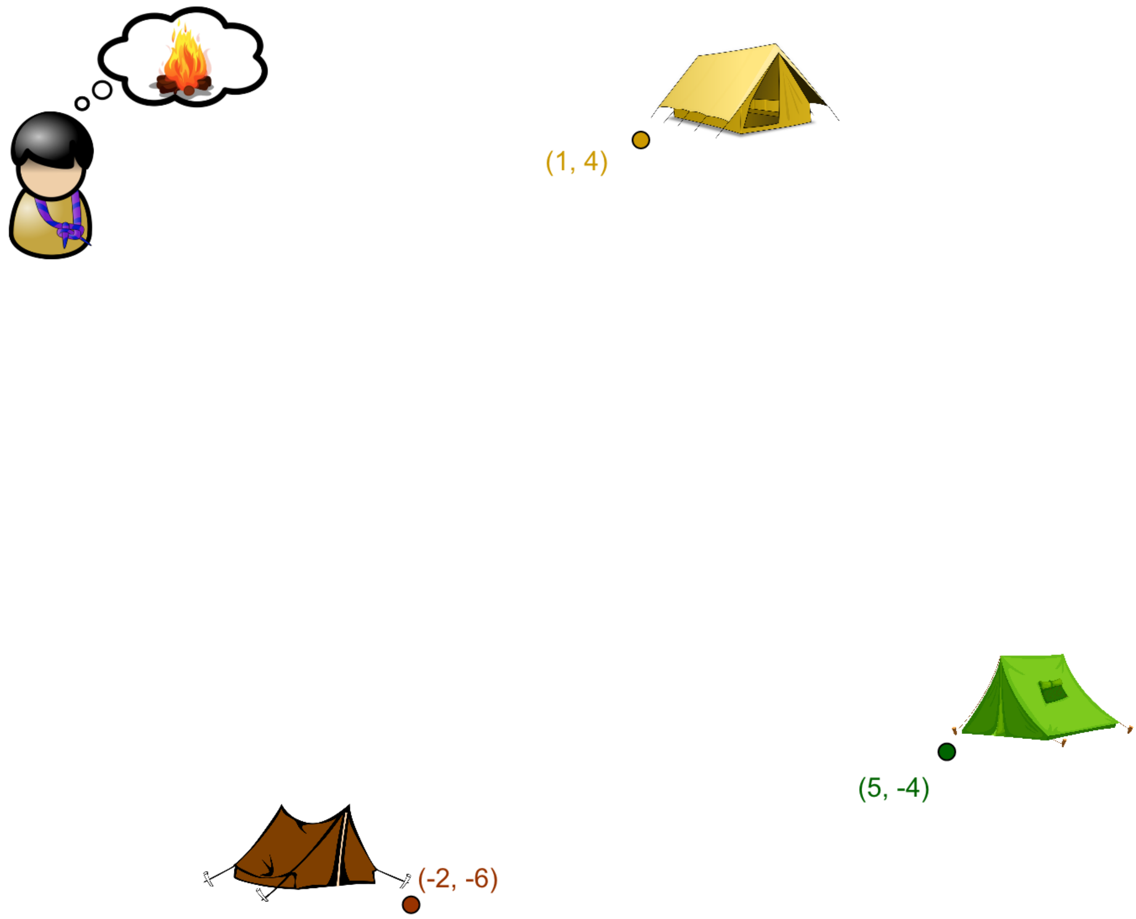
† The corollaries are numbered as in the appendix; corollary 2 is the first one relating to theorem 19

	Constructions (Supported by 46 definitions, 20 propositions, 5 axioms and 21 theorems)	Suggested for 1st year	JC ORD	JC HR	LC FN	LC ORD	LC HR
1	Bisector of an angle, using only compass and straight edge.	√	√	√	√	√	√
2	Perpendicular bisector of a segment, using only compass and straight edge.	√	√	√	√	√	√
3	Line perpendicular to a given line l, passing through a given point not on l.			√			√
4	Line perpendicular to a given line l, passing through a given point on l.	√	√	√	√	√	√
5	Line parallel to given line, through a given point.	√	√	√	√	√	√
6	Division of a line segment into 2 or 3 equal segments without measuring it.	√	√	√	√	√	√
7	Division of a line segment into any number of equal segments, without measuring it.			√			√
8	Line segment of a given length on a given ray.	√	√	√	√	√	√
9	Angle of a given number of degrees with a given ray as one arm.		√	√	√	√	√
10	Triangle, given lengths of 3 sides.		√	√	√	√	√
11	Triangle, given SAS data.		√	√	√	√	√
12	Triangle, given ASA data		√	√	√	√	√
13	Right-angled triangle, given length of hypotenuse and one other side		√	√	√	√	√
14	Right-angled triangle, given one side and one of the acute angles.		√	√	√	√	√
15	Rectangle given side lengths.		√	√	√	√	√
16	Circumcentre and circumcircle of a given triangle, using only straight edge and compass.					√	√
17	Incentre and incircle of a triangle of a given triangle, using only straight edge and compass.					√	√
18	Angle of 60º without using a protractor or set square.				√	√	√
19	Tangent to a given circle at a given point on it.				√	√	√
20	Parallelogram, given the length of the sides and the measure of the angles.				√	√	√
21	Centroid of a triangle.					√	√
22	Orthocentre of a triangle.						√

Effective GeoGebra

Rich Task 1 – Problem

A Scout Troop have pitched 3 tents to sleep in and wish to build one fire to cook with. Where is the fairest location for the fire?



Rich Task 2 – Option 1

Task to investigate effect of *a, b* and *c* in the function of

$$g(x) = a + b * sin(c * x)$$

1. Use GeoGebra to graph the function $f(x) = sin(x)$

2. Using sliders to control the values of *a, b* and *c,* graph the function of

$$g(x) = a + b * sin(c * x)$$

3. Write down the equation of as many functions as you can that have a maximum value of 3 and a minimum value of -3.

4. Write down the equation of as many functions as you can that have a maximum value of 3 and a minimum value of 1.

5. Write down the equation of as many functions as you can that intersect with roots of $f(x) = sin(x)$

- Two points to bear in mind while you're doing this activity
  - How could this activity be used with other types of functions?
  - What do the sliders in this activity represent mathematically?

Rich Task 2 – Option 2

Task to investigate effect of *a, b* and *c* in the function of

$$h(x) = a * (x+b)^2 + c$$

1. Use GeoGebra to graph the function
$$h(x) = a * (x+b)^2 + c$$

2. Using sliders to control the value of a, b and c, graph
$$h(x) = a * (x+b)^2 + c$$

3. Write down the equation of as many functions as you can that have a minimum y-value of -1.

4. Write down the equation of as many functions as you can that have a turning point at the origin.

5. Write down the equation of as many functions as you can that have roots of 2 and 6.

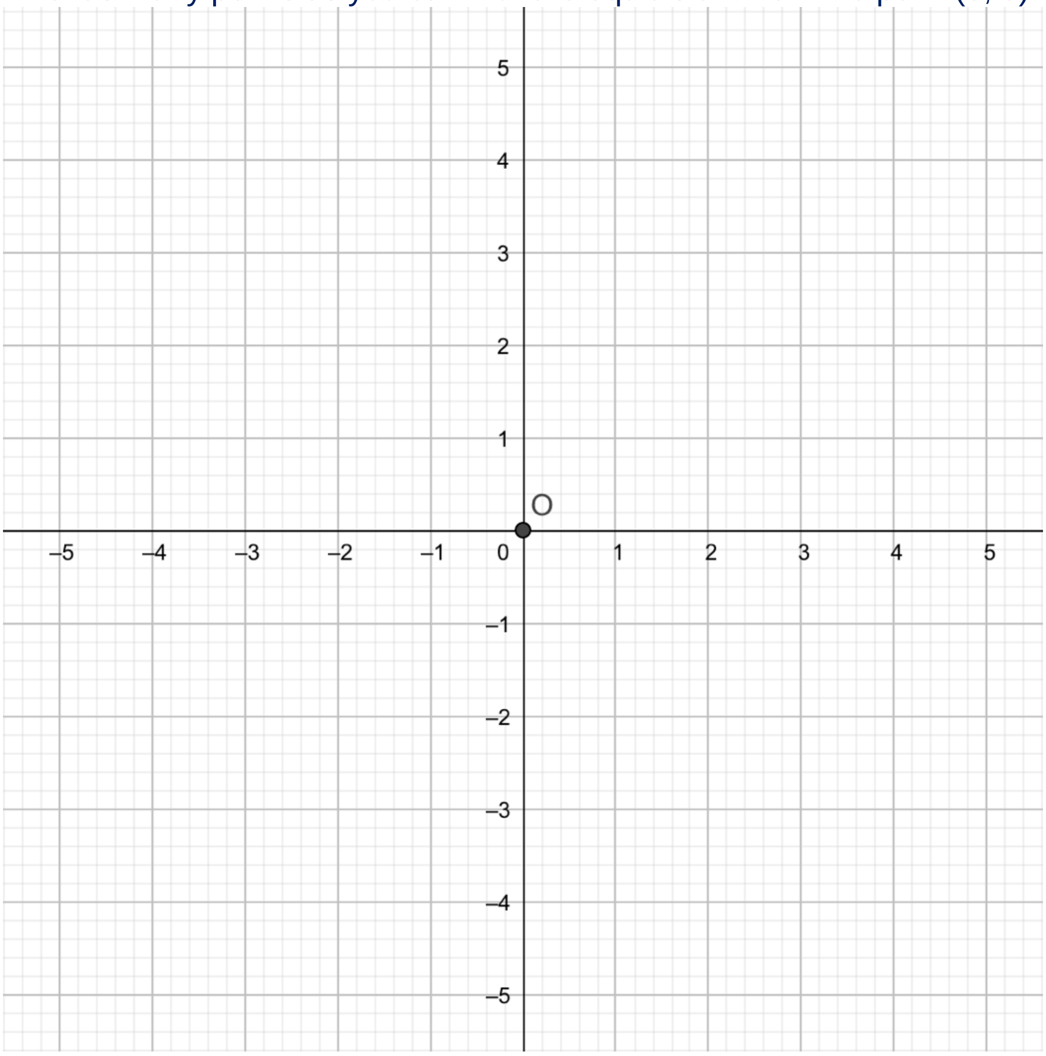
- Two points to bear in mind while you're doing this activity
    - How could this activity be used with other types of functions?
    - What do the sliders in this activity represent mathematically?

Extension Questions:

1. Write down the equation of as many functions as you can that have no roots.
2. What changes would you make to the function to make it invertible?

The Circle

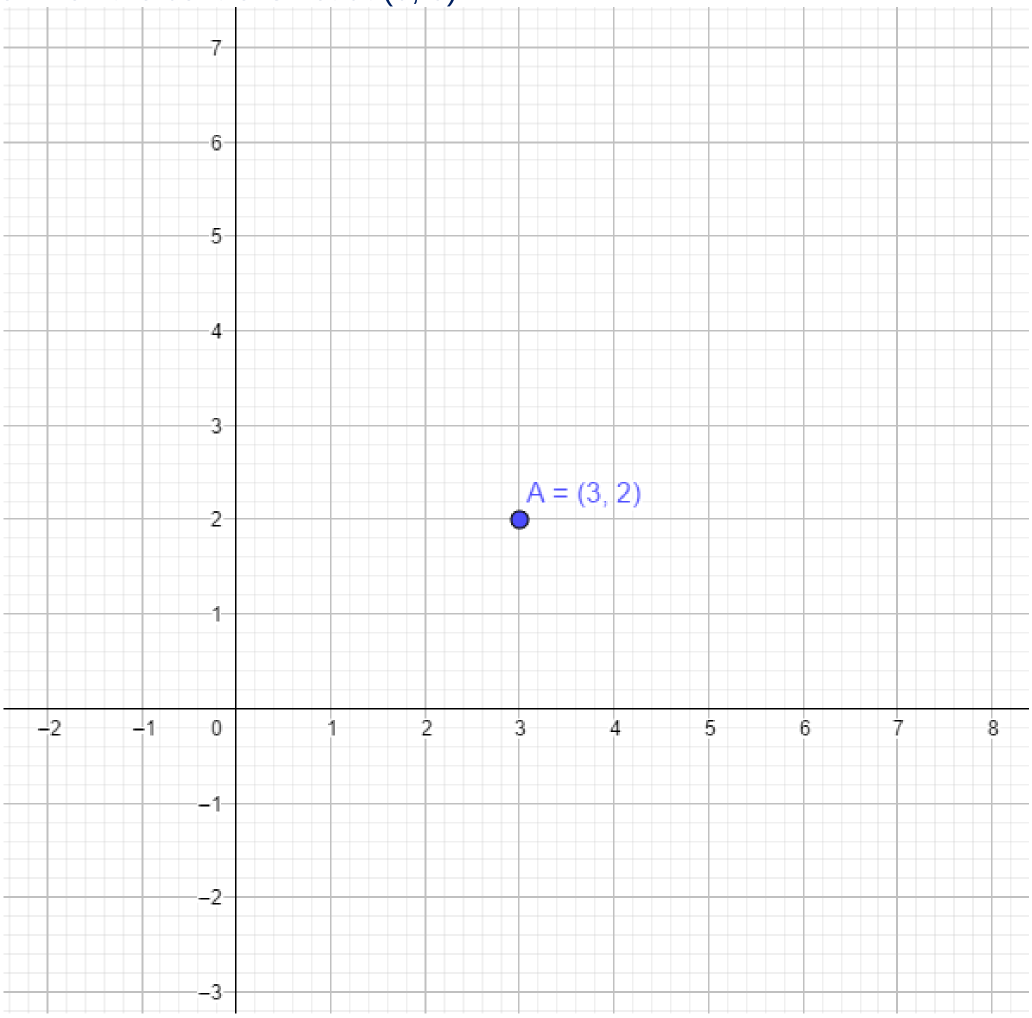
Plot as many points as you can that are equidistant from the point (0, 0)



Point	x-coordinate	y-coordinate	radius

https://www.geogebra.org/classic/jr2steag

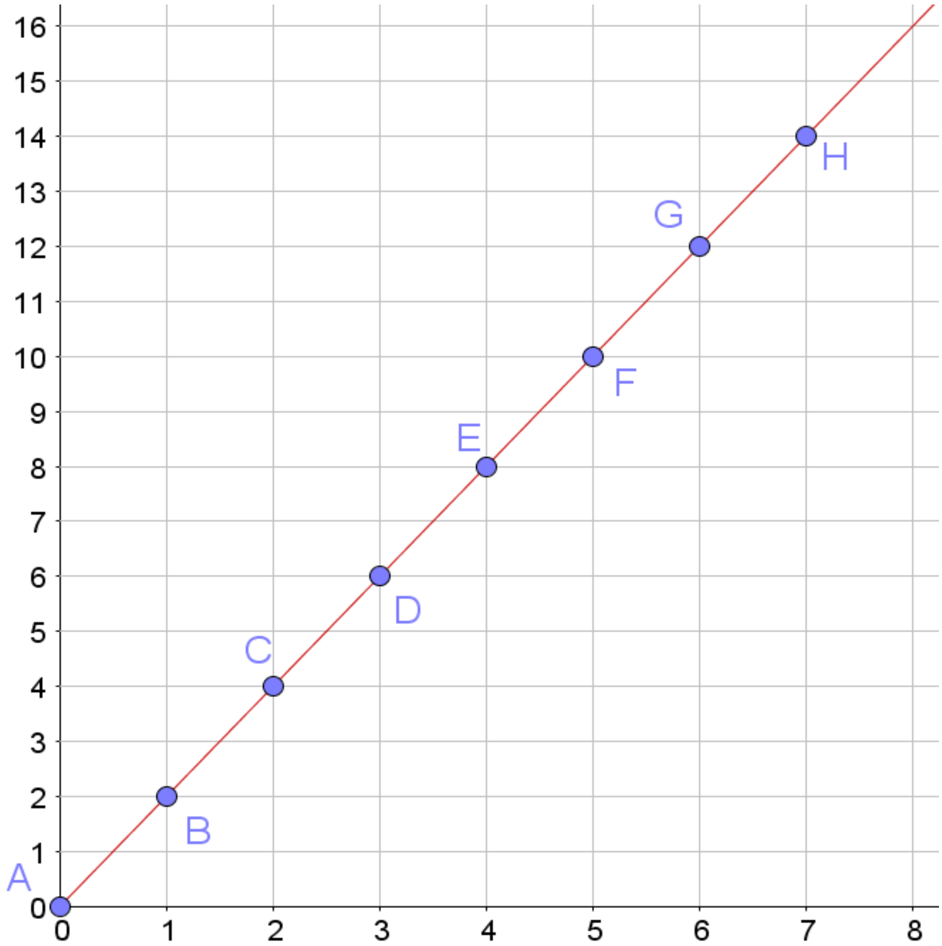
How would you use this diagram to guide students to the discovery of the equation of a circle when the centre is not at (0, 0)?

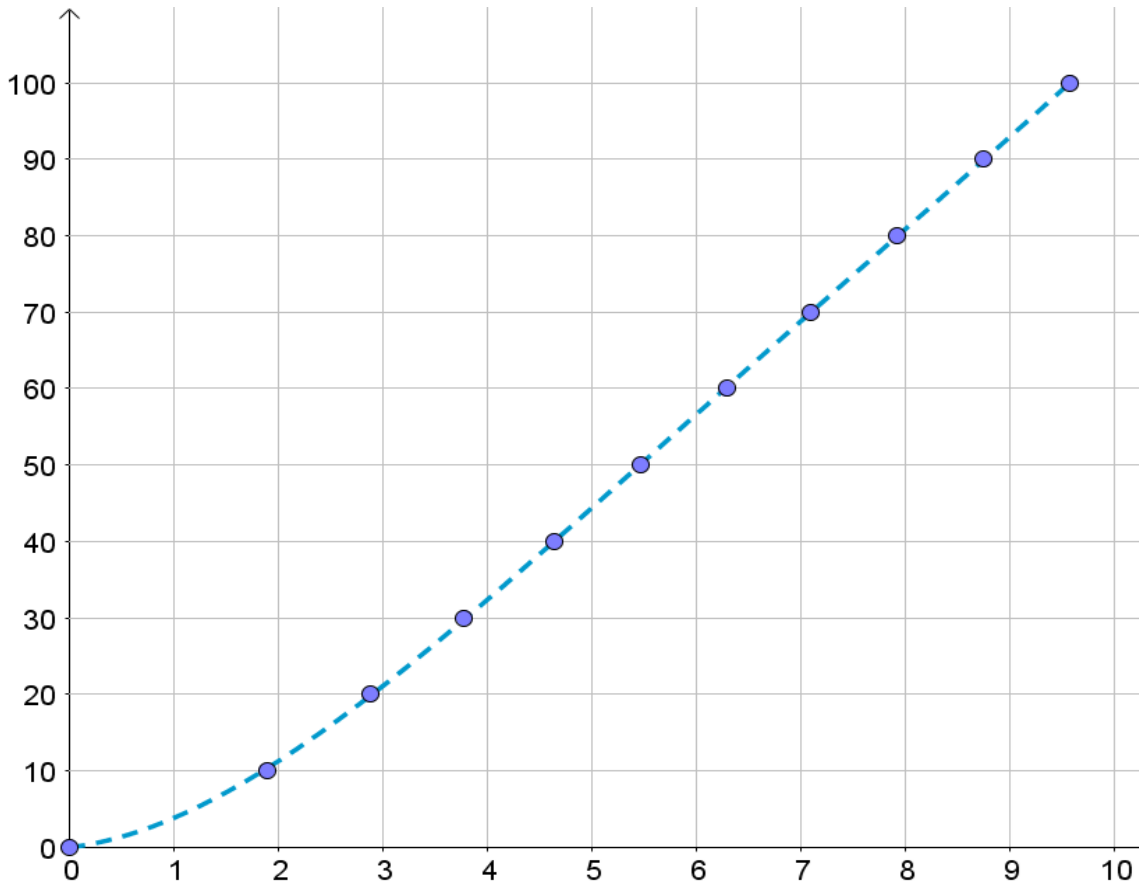


Point					radius

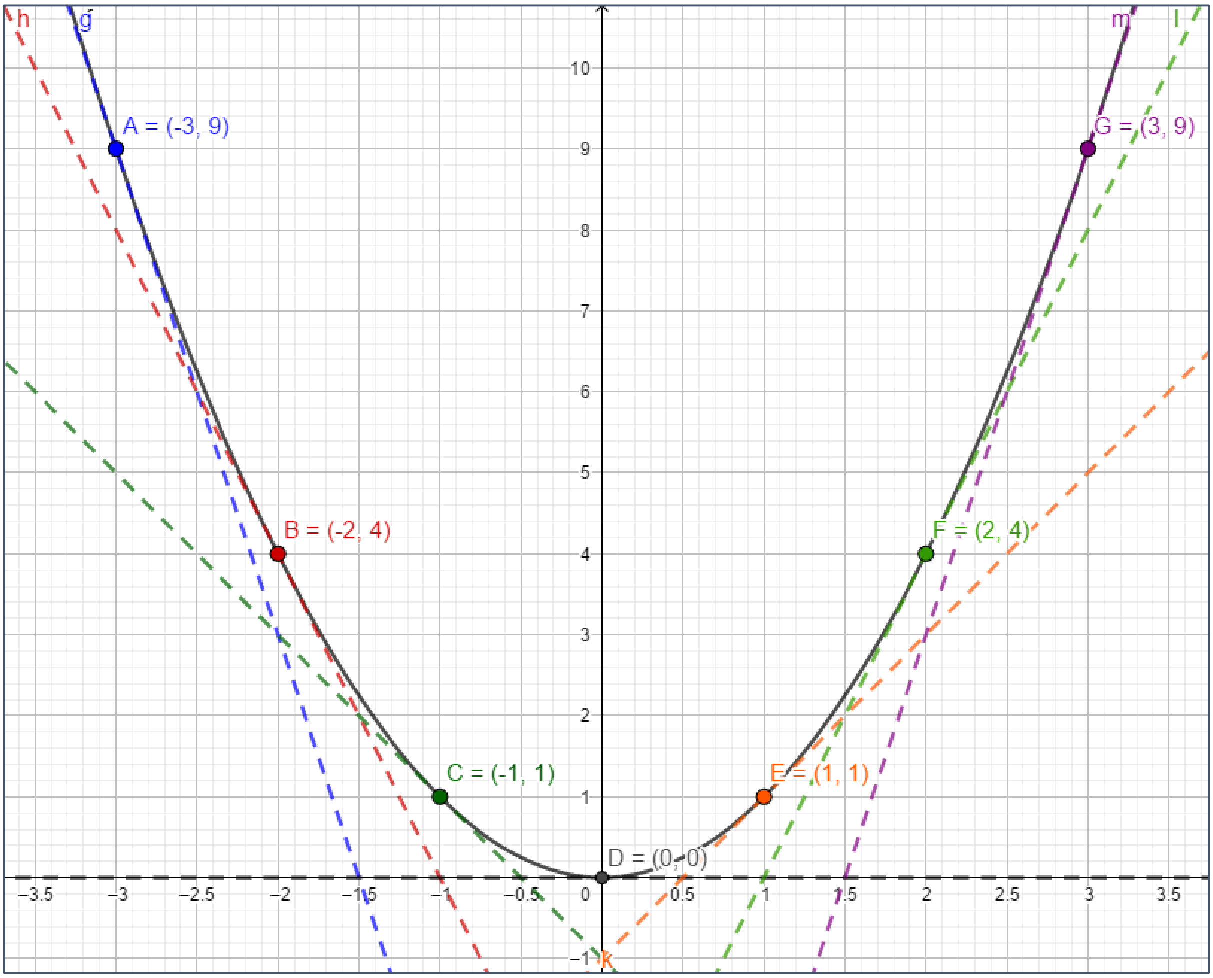
https://www.geogebra.org/classic/ukufacyx

Calculus

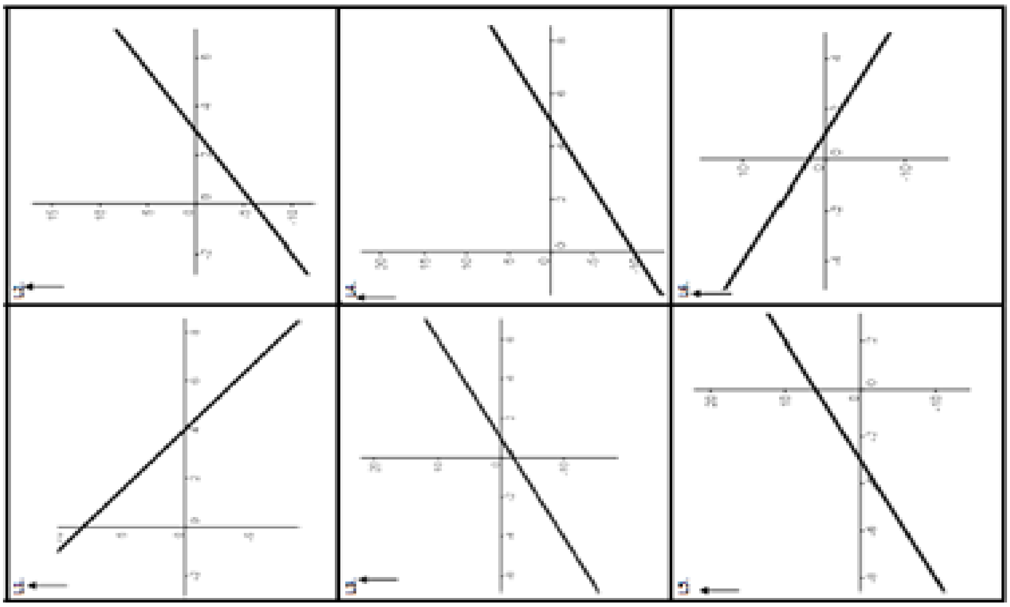


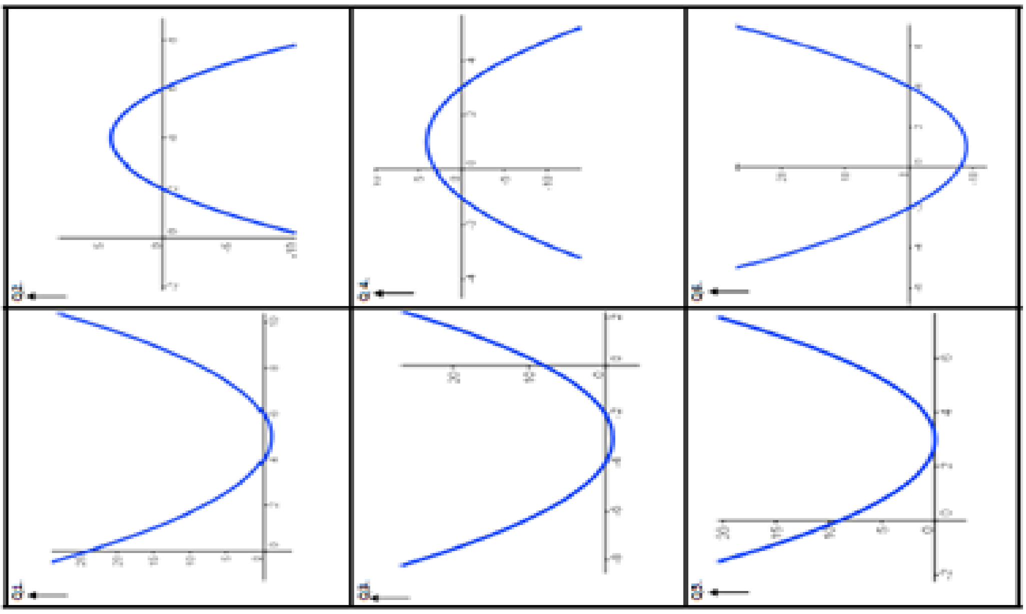


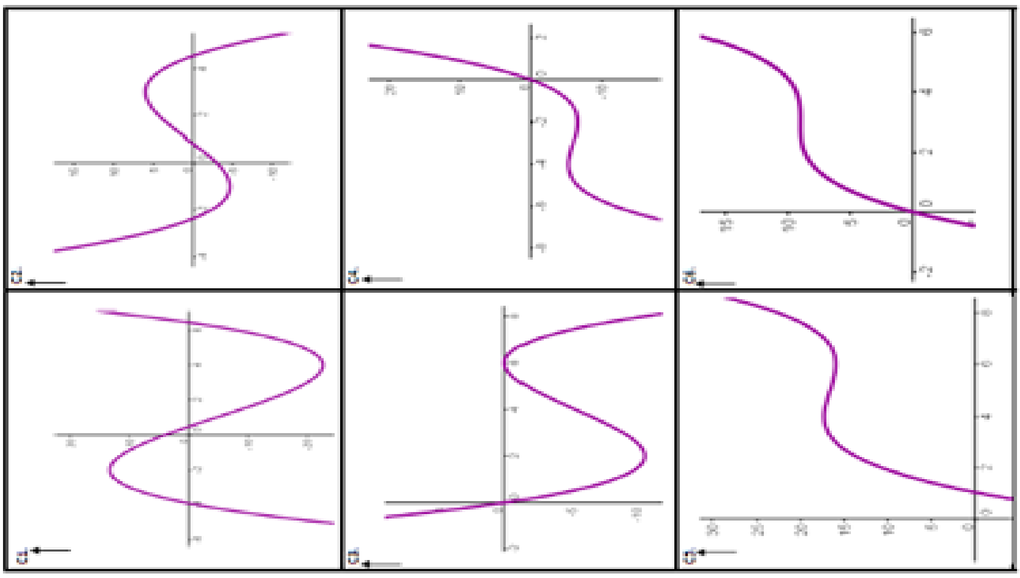
GeoGebra File: **https://www.geogebra.org/classic/eghtazfn**



GeoGebra File: **https://www.geogebra.org/classic/k55z2yfh**







Lesson Study Workshops

The X Factor – St. Mark's Community School

Task 1

For the next 5 minutes, work on your own and factorise the terms in as many ways as possible

3x

6

4y

8

Task 2

Students are given handouts with 2 copies of 3x + 6 and 2 copies of 4y + 8

3x	6

4y	8

Students will be asked to write down ways to describe the diagram

Task 3

Factorise the boxes and fill in the blanks.

4y	8

_____ + ______ = _____ (__________)

5a	20ab

_____ + ______ = _____ (__________)

2xy	+4x	+8

_____ + ______ + _______ = _____ (__________)

The Power that lies beneath – Woodbrook College

Every year the amount of plastic in the ocean triples!!
In 2019, there is 8 million tons of plastic in our ocean. That is the equivalent to 5 grocery bags filled with plastic for every foot of the coastline in the world!

(i) How many tons of plastic will be there in 15 years?

(ii) How many tons of plastic were there 15 years ago?

In 2019, there is 8 million tons of plastic in our ocean. That is the equivalent to 5 grocery bags filled with plastic for every foot of the coastline in the world!

(i) How many grocery bags could you fill with plastic in 15 years?

(ii) 1 ton = 1000kg. How many kg of plastic was there in the ocean 5 years ago? Give your answer correct to 2 decimal places.

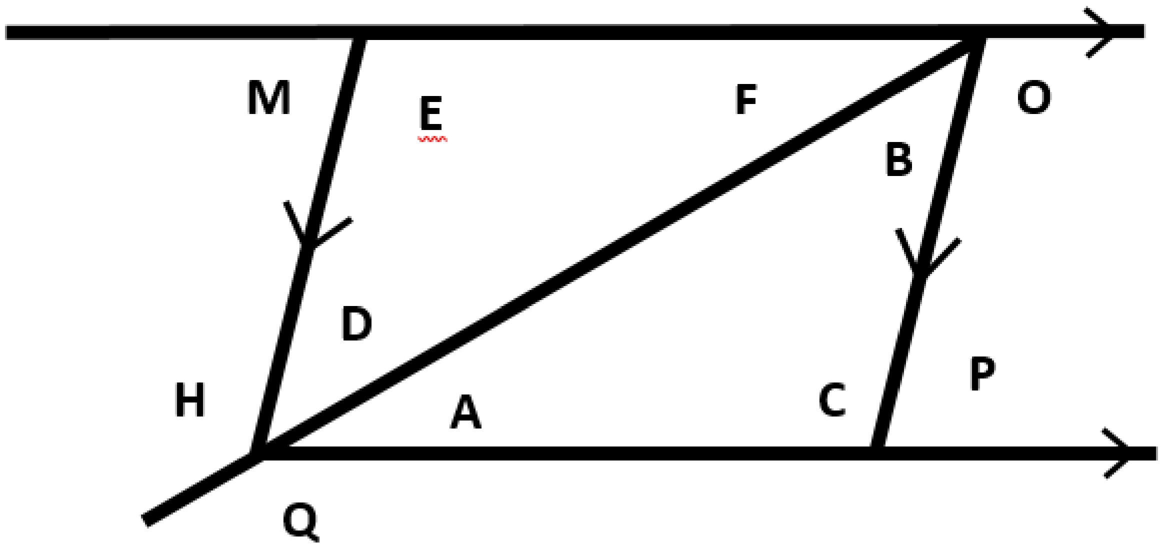
(iii) 1 ton = 2240 lbs (pounds). How many lbs of plastic will there in 15 years? Give your answer correct to 1 decimal place.

1 foot = 30.48cm and 1 mile = 5280 feet. There are 598675.97km of coastline in the world. How many miles of coastline is there? Give your answer to the nearest mile.

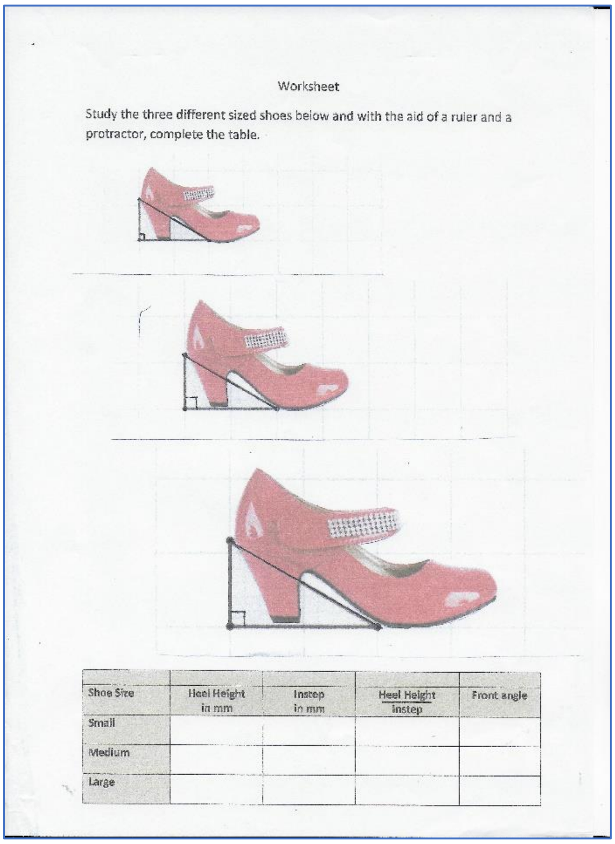
Think outside the triangle! – St. Gerald's College

<u>GROUP TASK:</u>

List all the connections that you can identify in the diagram below.



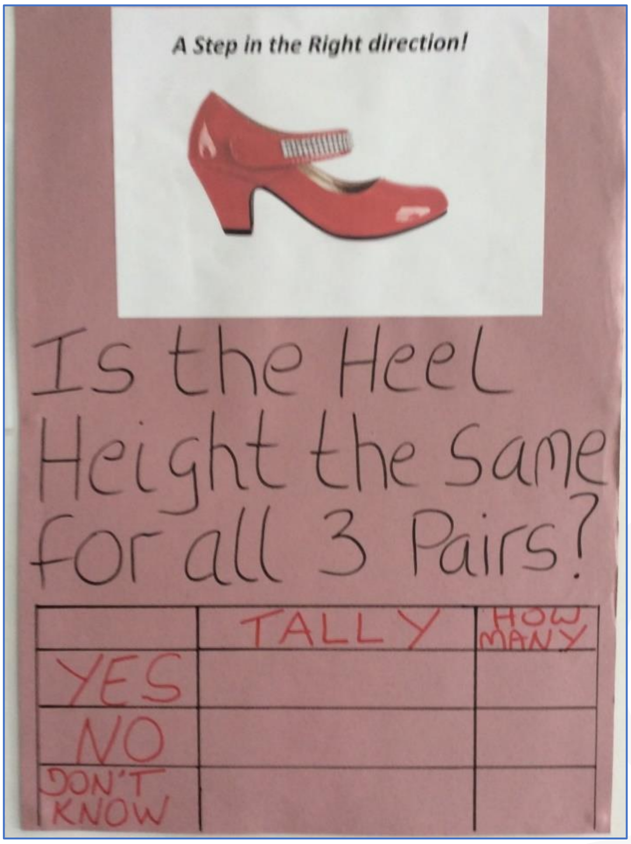
A step in the right direction – Presentation Secondary School, Castleisland



Worksheet

Study the three different sized shoes below and with the aid of a ruler and a protractor, complete the table.

Shoe Size	Heel Height in mm	Instep in mm	Heel Height / Instep	Front angle
Small				
Medium				
Large				



Get Un-Snookered – Holy Family Community School, Rathcoole

Problem Posed to the Students:

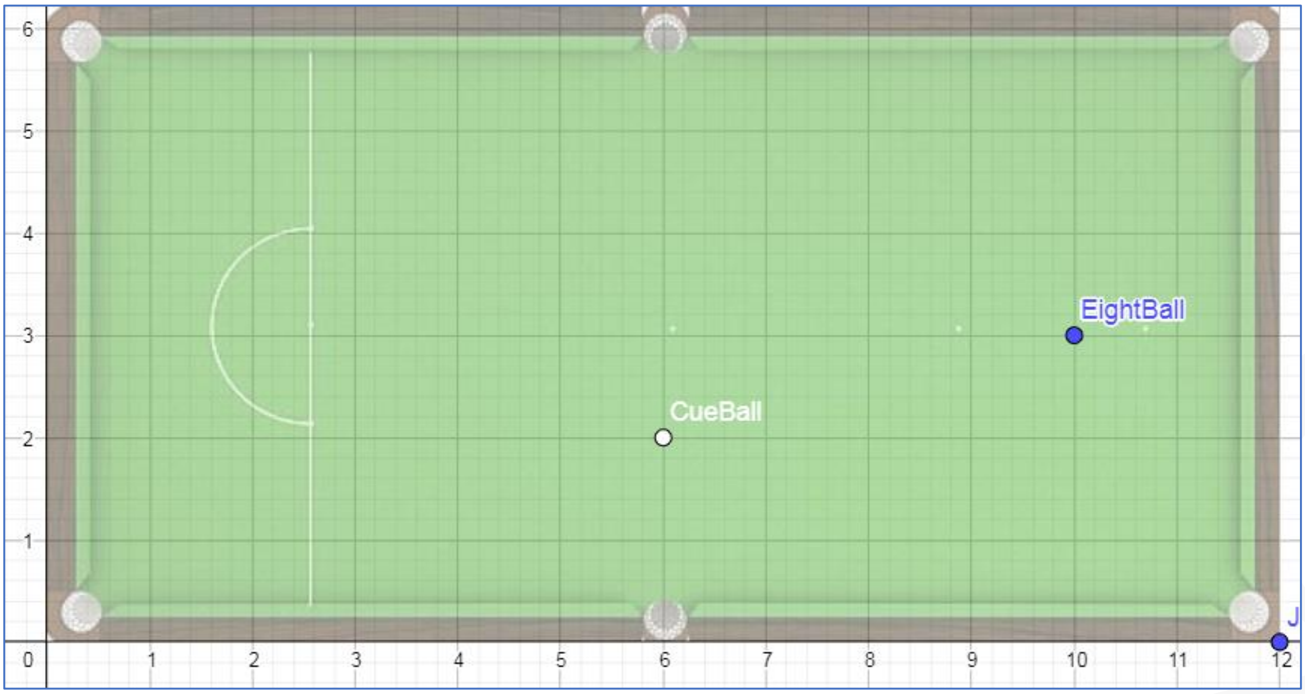
GET UN-SN🎱🎱KERED!

Matthew and his friends are playing pool. The pool table has the following dimensions 12 units by 6 units. There is no direct path to the Eight Ball, so Matthew must take a shot that rebounds off the cushion first before hitting the Eight Ball into the pocket. He knows that the white ball has the coordinates (6,2) and the Eight Ball has the coordinates (10,3). His selected pocket is positioned at (12,0).

He wants to calculate the required angle to complete this shot as necessary.

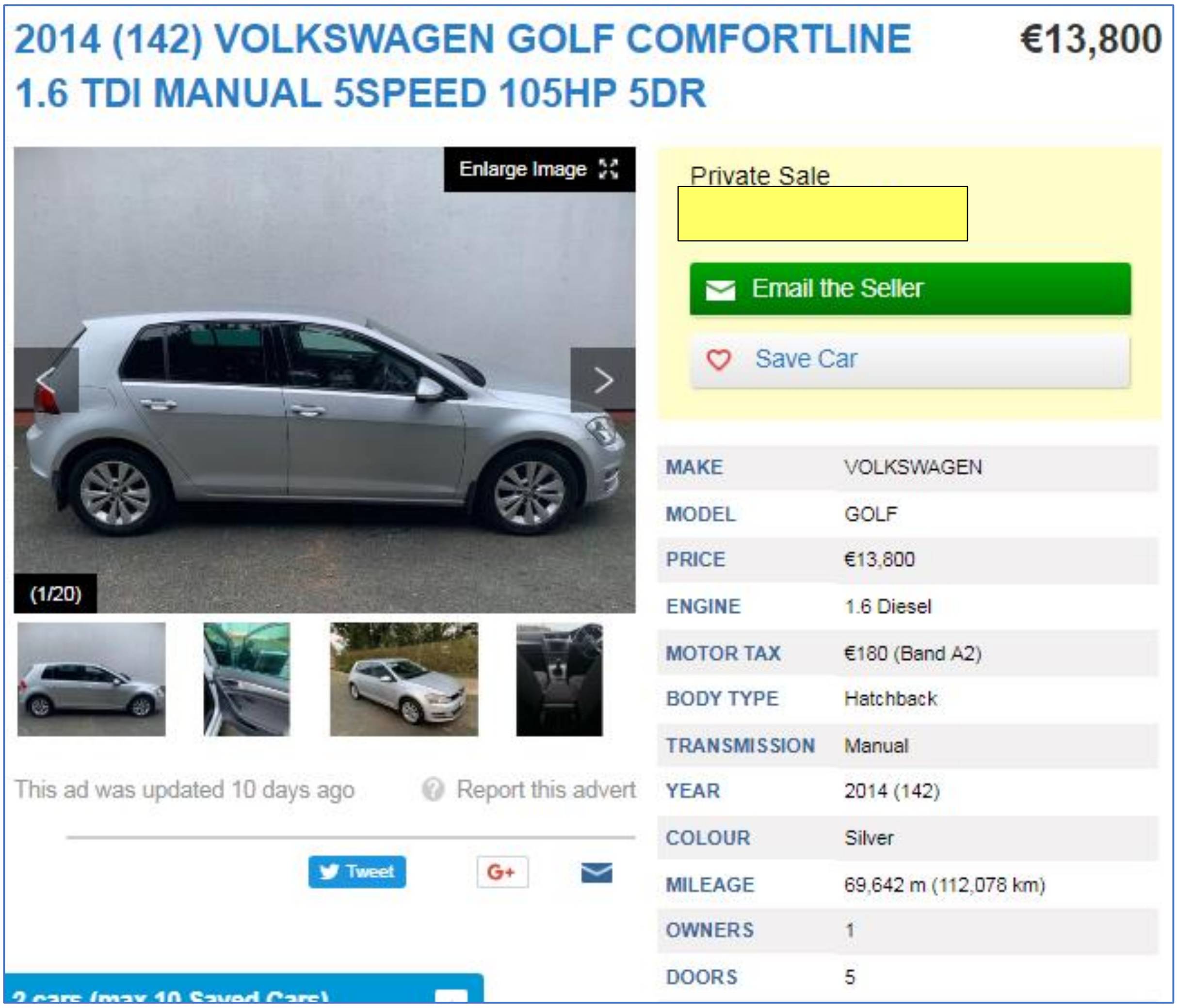
Find this angle in as many ways as possible.

Support Diagram if needed:



A journey in finance – St. Mary's College

Lesson Question



On the 7th February it is Alex's 18th Birthday. On Alex's 21st birthday he wants to buy a car. Alex's parents have agreed to pay for his fist year's insurance as a 21st birthday gift. He decides he would like a 2014 Golf TDI equivalent to the one above. Depreciation for this model is 17% per annum. He opens a bank account which will give him an AER of 3%.

How much per month will he have to save in order to afford the car, and the first year of tax, if he makes his first lodgment on his 18th birthday?

Extension:
If Alex later decides that he would prefer to simply take out a loan of the full amount on his 21st birthday. If the APR is also 3% on the loan, what will his monthly repayments be?

Straight to the nth game – Ardee Community School

Question:

Johnny is 13 years of age and he wants to join an online gaming community because his friends say it's "cool". €3 is the joining fee for under 15's plus a subscription of €2 a month thereafter.

His mother allows him to join on a trial basis for 10 months, buying him the subscription for his birthday. Show how much this would cost in as many ways as you can.

Part 2.

If Johnny were to join the gaming community for 2 years how much would it cost? Remember that it costs €3 to join the community and there is a monthly subscription of €2.

This time you may not simply add or multiply.

Part 3.

If he remained a member until he is 21? How much would this cost overall?

Can we work out the total cost of membership for any particular month in an easier way?

Circling "Longitude" – St. Mary's Secondary School, Charleville

A drone is rented to fly over longitude and capture aerial photographs. Will I Am is operating the drone and is given an aerial map of the festival with the best location for an aerial photograph of each stage marked with a large dot.

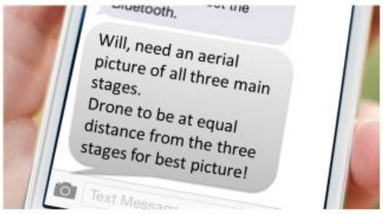
Task 1

Will wants a picture of each of the three stages (individually).

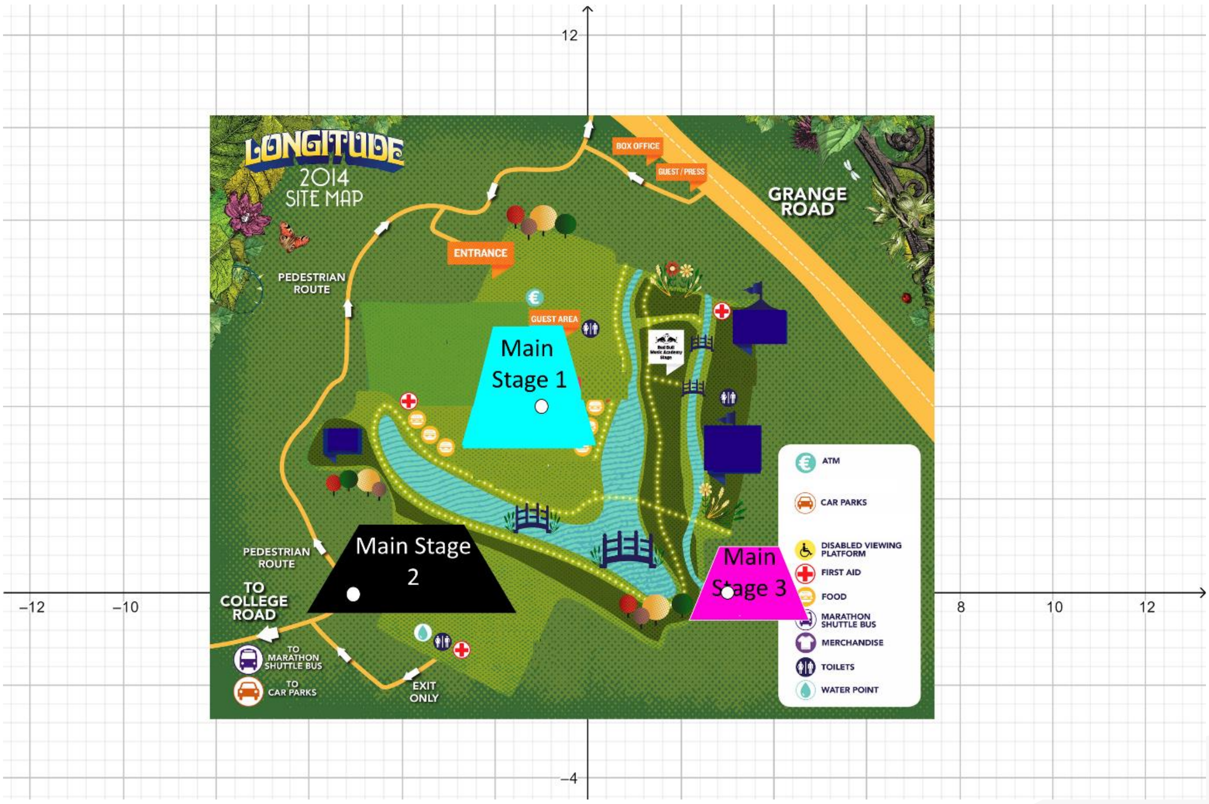
Can you identify the coordinates of each stage which he will input to the drone to take these pictures and hence form a graphical representation of the stage locations?

Task 2

The organisers of the festival have text Will and requested the following:



Find, **in as many ways as you can,** the co-ordinate he needs for the drone.

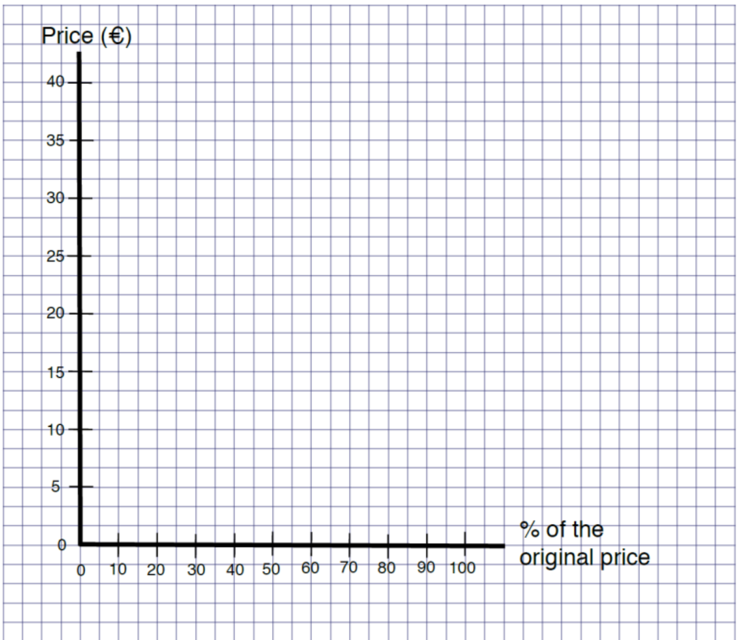


Percentage Paradox – Coláiste Bríde, Enniscorthy

Nike t–shirts are reduced by 85% in a sale. If the sale price of a t–shirt is €5, what was its original price?
Fill in the table and then show this relationship on a graph.

% of original price	Price (€)
0	
15%	
100%	

What type of relationship is this?



Student:

Useful Links / Notes