



Coimisiún na Scrúduithe Stáit
State Examinations Commission

Report on the Trialling
of
Leaving Certificate Sample Papers
for
Phase 1 of Project Maths
in the twenty-four initial schools

January 2010

Acknowledgements

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1. Introduction

This report deals with the trialling of draft Leaving Certificate sample papers for phase 1 of *Project Maths* in twenty-four schools. The purpose of the trialling process was to measure the effectiveness of the draft sample papers and the marking schemes, rather than to test current levels of candidate achievement. As a result of the trialling process and feedback received from relevant parties, the present report was prepared and amendments were made to the drafts. The finalised versions of the sample papers are being issued to coincide with this report. Feedback from the trialling exercise has also informed the work of the NCCA in providing further assessment examples for school use.

The *Project Maths* curricular development initiative involves the phased introduction of new syllabuses in Mathematics for both Junior Certificate and Leaving Certificate students. The syllabuses at all levels are divided into five strands. There are three phases in the implementation of the project. The first phase involves two of the five strands, the second phase involves another two and the third phase involves the final strand. With each new phase, revised examination arrangements are required at all levels.

The project also involves an initial group of twenty-four schools using syllabus materials two years ahead of all other schools. The experiences of these schools are taken into account in finalising the various syllabus strands. Appropriate examination arrangements are required for these initial schools when the relevant candidates reach the examination stage.

The initial schools began phase 1 (involving strands 1 and 2 of the syllabuses) in September 2008. In June 2010, the Leaving Certificate candidates from these schools will have followed two strands of the revised syllabuses. These students will therefore have different examination arrangements from all other candidates.

The Leaving Certificate examination in Mathematics is offered at three levels (Higher, Ordinary, and Foundation) and each level involves two written papers. The content areas involved in the two strands in phase 1 have traditionally been examined on Paper 2, and this will continue to be the case as the new syllabuses are rolled out. Accordingly, in 2010, the candidates in the twenty-four initial schools will sit the same Paper 1 as all other candidates, and a different Paper 2 from all other candidates.

2. Structure of this report

Chapters 3 and 4 of this report deal with the preparation of the sample papers and the arrangements for their trialling. Chapter 5 outlines the overall structure of the marking schemes used. These marking schemes are different in structure from those that the State Examinations Commission has used in the recent past for Mathematics, and Chapter 5 is therefore important in interpreting the chapters that follow it.

Chapters 6, 7, and 8 give a detailed analysis of each question at each of the three levels (Higher, Ordinary, and Foundation respectively). In relation to each question, the content area and assessment objective are stated. The question is given, as it was presented on the trialled draft, along with one or more model solutions. These model solutions are not exhaustive, and there may be other correct solutions. The detailed marking notes for examiners are then presented. These indicate how the relevant marking scale was to be applied to candidates' work. Commentary on candidate answering follows. Where relevant, "candidate exemplars" are given. These are intended to illustrate and clarify some of the issues in the marking notes and to give concrete examples of how the scheme was applied. It should be noted that these are actual examples of work received. Although typed, they have been reproduced as faithfully as was feasible, with diagrams rendered as closely as possible to the originals. Finally, other points worthy of note are included where relevant. There are occasional references to instances where the final version of the sample paper has been amended as a result of the trialling process.

Chapter 9 presents the conclusions of the report. These are based on the trialling process, on the feedback received, and on subsequent discussions between the State Examinations Commission (SEC), the Department of Education and Science (DES), and the National Council for Curriculum and Assessment (NCCA). A number of recommendations are included in this chapter.

Finally, Chapter 10 presents some points that should be noted regarding the finalised sample papers and the extent to which certain aspects of the 2010 papers may vary from these.

Appendix 1 shows the assessment grid for the examination at all three levels. This is the framework used in the design and analysis of the sample papers. The same framework will be used for the real examination papers in 2010.

A separate report on the analysis of the feedback from teachers and students is being issued.

3. Preparation of the sample papers

3.1 Project context

There are a number of features of *Project Maths* that make the preparation of sample papers different from other circumstances. Heretofore, the introduction of a new syllabus has followed a sequential process. The National Council for Curriculum and Assessment (NCCA) prepares a draft syllabus and submits it along with related assessment advice, including suggested specimen examination materials, to the Minister for Education and Science. If and when the Minister accepts this syllabus, it is no longer “draft”. Responsibility for the implementation of the syllabus falls to the Department of Education and Science (DES), and the implementation of the examination arrangements in accordance with DES policy passes to the State Examinations Commission (SEC). The SEC prepares sample papers for issue to schools. Following this, the ongoing implementation of the examination, including maintaining the agreed standards, is the responsibility of the SEC.

The *Project Maths* initiative is different. In relation to the initial schools in particular, an examination is being implemented on a draft version of a syllabus. The work of the NCCA is ongoing, and the examination process is inextricably linked with that ongoing work. Furthermore, the DES, since it has not yet signed off on a final and complete syllabus, also has a different role to play, ensuring that the syllabus and examinations as they emerge are satisfactory. Sample papers must therefore be prepared with a significant degree of consultation. Furthermore, since various drafts of the syllabus are in existence for use by different student cohorts, care must be taken to ensure clarity for teachers and candidates regarding which version applies to each group.

3.2 The syllabus and specifications for the examination

The version of the syllabus that applies for candidates in the initial schools for the Leaving Certificate examination of June 2010 is the one issued to those schools by the NCCA in October 2008. The examination arrangements were amended and clarified by two information notes issued to the schools by the NCCA in October 2009 (Information Note #1 and Information Note #2).

This October 2008 draft of the syllabus, subject to the two information notes, is the version that was used as the basis for the preparation of the draft sample papers.

3.3 Preparation procedure

The draft sample papers were prepared in accordance with normal SEC protocols. The drafters received appropriate training and a copy of the SEC's *Manual for Drafters, Setters and Assistant Setters*, which is available for download from the *Publications* section of the SEC website¹. Setters were not appointed, as the Chief Examiner took direct control of bringing the drafts to readiness. An assessment grid for the examination was drawn up by the Chief Examiner in consultation with NCCA officers. This was used by the drafters and the Chief Examiner in preparing the sample papers. A copy of this assessment grid is included in Appendix 1.²

Drafts of the sample papers were forwarded to relevant officers in the NCCA and the DES in advance of the trialling. Following receipt of their observations, further amendments were made before the drafts were finalised for trialling.

¹ The *Publications* section is currently accessed via the *About Us* link in the navigation panel. The direct link to the document is: http://www.examinations.ie/about/Setting%20Manual_rev3.pdf

² The same grid was used for all three levels. In the case of Ordinary and Foundation levels, the column referring to *Vectors* was ignored.

4. The trialling process

4.1 The purpose of trialling

The trialling of sample papers is a quality-assurance procedure that the SEC may use when preparing to assess a new syllabus. It is conducted in order to test various aspects of the sample papers and the examining process. It helps the designers of the syllabus and the examination to decide whether the examination will do what it is supposed to do. Its purpose, therefore, is not to gain information about levels of candidate achievement, but to gain information about the examination itself.

Before trialling, draft sample papers are prepared with a great deal of care to ensure that they will indeed work as they are supposed to. Trialling is part of the final stage of a long developmental process, which is completed when the sample papers are subsequently finalised. In this regard, trialling gives valuable information about the following specific aspects of the sample papers and their associated marking criteria:

- whether the tasks set are appropriate for testing the particular objectives and learning outcomes at the particular level
- whether these tasks are properly understood by the candidates
- whether the time taken to complete the tasks is as expected
- whether there are any unexpected difficulties for candidates
- whether the marking criteria accurately identify evidence of the achievement of the objectives and reward such achievement appropriately
- whether there are any unanticipated difficulties for examiners
- whether the format of the papers is appropriate.

In addition to testing these aspects of the examination itself, the outcomes of the trialling also assist with the implementation of the syllabus itself. It allows the SEC to provide comprehensive information on marking criteria, including examples of candidates' work and comments thereon. This helps to clarify the objectives and learning outcomes of the syllabus for teachers, candidates and other interested parties, and gives valuable information about how candidates' work will be assessed in the real examination.

Trialling also serves to improve the quality of feedback from teachers and candidates regarding the sample papers. Whereas opinions on the papers can be sought even without trialling, the trialling process helps candidates and teachers to engage with the examination papers in as realistic a way as is feasible. The resulting feedback is therefore more soundly based.

Finally, trialling facilitates the training of examiners in advance of the real examinations in the summer.

4.2 Implementation in schools

In order to gain the maximum possible benefit, all twenty-four of the initial schools were asked to trial the sample papers with all students, if possible. All of them agreed to do so, and the SEC appreciates this full co-operation from all. The trial sitting was held on the morning of Wednesday 21 October, which was in the week immediately before the mid-term break.

Given that the trialling took place early in the second year of a two-year course, it was clear that many schools would not have completed the relevant syllabus strands. Schools were therefore asked to adjust the examination arrangements accordingly. That is, schools were told in advance which strands were tested in which questions, and were asked to direct students to answer only the questions that related to the material they had covered. Schools were asked to report back to the SEC on these arrangements.

Questionnaires were also provided for candidates and teachers in order to capture their opinions on the sample papers. Schools were asked to arrange for the candidates to complete their questionnaires immediately after sitting the examination and before leaving the examination hall.

Schools were asked to return all completed scripts, questionnaires, and the reports on examination arrangements by post to the SEC.

Marked scripts will be returned to schools shortly after this report is issued.

4.3 Marking arrangements

Most aspects of the marking arrangements were the same as for the written papers each summer. Not all quality assurance procedures were the same, as outlined below. This is because the purpose of this exercise was very different from that which normally applies in the summer. In this marking process, resources were particularly focussed on reviewing and analysing the questions, the marking criteria, and the outcomes.

Teachers from the twenty-four initial schools were invited to apply to act as examiners. Preference was given to those with previous examining experience, and almost all had some such experience. The examining team also included others with a familiarity with *Project Maths*, such as members of the support service (Regional Development Officers) and two experienced Chief Advising Examiners. Five examiners and an Advising Examiner were appointed at Higher Level. Eleven examiners and an Advising Examiner were appointed at Ordinary Level. Two examiners were appointed at Foundation Level. At Higher and Ordinary levels, the Advising Examiners fulfilled many of the functions normally fulfilled by a Chief Advising Examiner. The anonymity of the students was maintained throughout the marking process,

Marking conferences were held during the mid-term break. A one-day joint pre-conference was held between the Chief Examiner and the two Advising Examiners. This was followed by separate one-day marking conferences for all of the examiners at each of three levels. Leaving Certificate marking

conferences (including pre-conferences) are normally of two days' duration. Accordingly, the focus of the conferences was primarily on explaining and clarifying the marking scheme, rather than working on exemplar scripts.

Examiners marked the scripts over the following three weeks. To test consistency of application of the scheme, some selected scripts were copied and sent to all examiners to mark; these marks were compared to those awarded by the Advising Examiners and the Chief Examiner.

Examiners submitted data on the marks awarded as the work progressed, and each submitted a report at the end of the marking process. The reports from examiners and from the Advising Examiners contributed significantly to the relevant chapters in this report.

5. Overview of the marking schemes used

Candidate responses to the various questions were marked according to a number of *marking scales*, depending on the types of response that were anticipated. In the case of scales labelled “A”, candidate responses are divided into two categories: *correct* and *incorrect*. In the case of scales labelled “B”, responses are divided into three categories: *correct*, *partially correct*, and *incorrect*. Scales C, D and E have four, five and six categories respectively. The scales and the marks that they generated are summarised in this table:

Scale label	A	B	C	D	E
No of categories	2	3	4	5	6
5 mark scale	0, 5	0, 2, 5	0, 2, 4, 5		
10 mark scale	0, 10	0, 5, 10	0, 3, 7, 10	0, 2, 5, 8, 10	
15 mark scale	0, 15	0, 7, 15	0, 5, 10, 15	0, 4, 7, 11, 15	
20 mark scale	0, 20	0, 10, 20	0, 7, 13, 20	0, 5, 10, 15, 20	
25 mark scale		0, 12, 25	0, 8, 17, 25	0, 6, 12, 19, 25	0, 5, 10, 15, 20, 25

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the body of the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response (no credit)
- correct response (full credit)

B-scales (three categories)

- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

C-scales (four categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

D-scales (five categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

E-scales (six categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response almost half-right (lower middle partial credit)
- response more than half-right (upper middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

6. Detailed analysis – Higher Level

Summary of the Higher Level marking scheme

– mark allocations and scales applied

Section A

Question 1

- (a) 5B
- (b) 10B
- (c) 10B

Question 2

- (a) skewness: 5B
averages: 5B
mode : 5A
- (b) 10B

Question 3

- (a) 5C
- (b) 5B
- (iii) 5B
- (iv) 10D

Question 4

- (a) 5A
- (b) 20D

Question 5

- (i) 5B
- (ii) 15C
- (iii) 5B

Question 6

- (i) 5B
- (ii) 5B
- (iii) 15C

Section B

Question 7

- (i) 5B
- (ii) 5B
- (iii) 10C
- (iv) 5A
- (v) 5B
- (vi) 5B
- (vii) 5B
- (viii) 10C

Question 8

- (a) 20D
- (b) intermediate calc.: 15C
height: 15C

Question 9A

- (i) 10B
- (ii) 20D
- (iii) hypoth. test: 15C
contextualise: 5A

Question 9B

- (a) diag. & given: 5B
construction: 5B
main body: 15C
“similarly...” 5A
- (b) (i) |AF| 5C
(ii) angle 10C
length 5C

Question 1**(25 marks)****Content area:** Strand 1 – Probability**Main assessment objective(s):**

1. Execute routine procedures in a mathematical context
2. Demonstrate knowledge of notation and terminology

Question & model solution

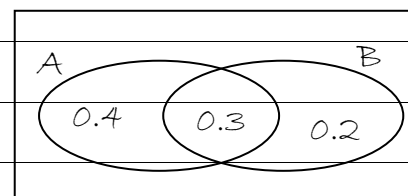
The events A and B are such that $P(A) = 0.7$, $P(B) = 0.5$ and $P(A \cap B) = 0.3$.

(a) Find $P(A \cup B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad \text{OR}$$

$$= 0.7 + 0.5 - 0.3$$

$$= 0.9$$



$$P(A \cup B) = 0.9$$

(b) Find $P(A|B)$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.3}{0.5} = 0.6$$

(c) State whether A and B are independent events, and justify your answer.**OR**

Not independent,

since $P(A|B) \neq P(A)$

$$P(A) \cdot P(B) = (0.7)(0.5)$$

$$= 0.35$$

$$\neq P(A \cap B)$$

$$\therefore \text{not independent}$$

Marking scheme & related notes

Question 1

(a) Scale 5B

Partial credit:

Correct expression, but not executed accurately (provided answer lies in $[0,1]$)

(b) Scale 10B

Partial credit:

Correct expression, but not executed accurately (provided answer lies in $[0,1]$)

or

$P(B|A)$ found (correctly)

(c) Scale 10B

Partial credit:

States that events are not independent, but with incomplete or unsatisfactory justification.

Note: if using $P(A \cap B) \neq P(A).P(B)$ then must state value of $P(A).P(B)$ for full credit.

Comment on answering

In part (c), of the candidates who correctly stated that A and B are not independent, very few correctly justified the assertion. There were only three fully correct responses to this part.

Candidate exemplars

The following example of a candidate response to part (c) received **partial credit**:

They're not independent because they affect each other.

Comment: The assertion is true and reflects a proper intuitive understanding of the concept. However, there is no reference to the relevant calculations that support the conclusion. Therefore, this is not a satisfactory justification.

Other points to note

Proper understanding and awareness dictates that answers offered must be sensible in the context of the question. In this case, no credit was awarded in part (a) or (b) if the candidate offered an answer outside the interval $[0, 1]$, irrespective of what work is done. This is because such an answer indicates a fundamental lack of understanding of the meaning of probability.

Question 2

(25 marks)

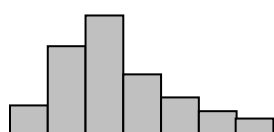
Content area: Strand 1 – Statistics

Main assessment objective(s):

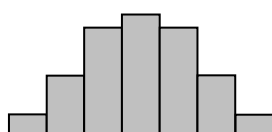
1. Demonstrate understanding of concepts and connections
(viz. concepts and measures used in describing distributions (not contextualised); connection between the shape of the distribution and those measures.)

Question & model solution

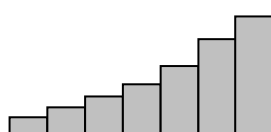
The shapes of the histograms of four different sets of data are shown below.



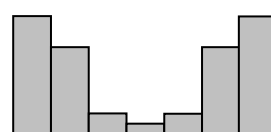
A



B



C



D

- (a) Complete the table below, indicating whether the statement is correct (✓) or incorrect (✗) with respect to each data set.

	A	B	C	D
The data are skewed to the left	✗	✗	✓	✗
The data are skewed to the right	✓	✗	✗	✗
The mean is equal to the median	✗	✓	✗	✓
The mean is greater than the median	✓	✗	✗	✗
There is a single mode	✓	✓	✓	✗

- (b) Assume that the four histograms are drawn on the same scale. State which of them has the largest standard deviation, and justify your answer.

Answer: D

Justification:

A lot of the data are far from the mean in set D

or

Set D has a lot of extreme values

(etc.)

Marking scheme & related notes

Question 2

- (a) Skewness (rows 1 and 2): scale 5B

Partial credit: skewness reversed. (i.e. row 1 and 2 swapped).

Mean/median (rows 3 and 4): scale 5B

Partial credit: order reversed. (i.e. row 3 correct AND row 4 reads: ✗ ✗ ✓ ✗)

Mode (row 5): scale 5A

- (b) Scale 10B

Partial credit:

States D, but with incomplete or unsatisfactory justification. (Justification must show understanding of standard deviation as a measure of spread, such as referring to extreme values, or saying that this distribution has more data that are far from the mean.)

Comment on answering

Whereas many candidates were able to correctly distinguish between the skewed and symmetric distributions, the ability to correctly identify the direction of skewness was no better than would be observed by chance. That is, of those who correctly identified the skewed distributions, fewer than half correctly identified which was skewed left and which skewed right.

Few candidates correctly identified that the mean would equal the median in the case of the two symmetric distributions and not in the skewed ones. Of those who did, the majority correctly identified the distribution in which the mean would be the greater.

Of the candidates who correctly identified the distribution with the largest standard deviation, about half gave a satisfactory justification for their choice.

Candidate exemplars

The following example of a candidate response to **part (b)** received **full credit**. The candidate identified dataset D and gave the following justification:

The are a greater number of outliers on Data set D. The data is not grouped around the Median but closer to the outliers.

Comment: The data in the outermost categories here should not properly be referred to as outliers.

However, it is clear from this response that the candidate understands that having a high proportion of the data farther from the centre is the feature that causes the standard deviation to be greater than in the other sets. Therefore, this was treated as a satisfactory justification.

The following example of a candidate response to **part (b)** received **partial credit**. The candidate identified dataset D and gave the following justification:

There is more of a variety of figures in this histogram.

Comment: This is not a satisfactory justification; any effort to read a correct understanding of the relevant concepts into this statement would be unjustified.

Other points to note

Note that the marking scheme for part (a) is **not** structured according to how many individual cells in the table happen to be correct or incorrect. Credit is awarded for clear evidence that some relevant portion of the target skill or concept has been acquired.

Question 3**(25 marks)****Content area:** Strand 2 – Co-ordinate geometry**Main assessment objective(s):**

1. Execute routine procedure in familiar mathematical context.
2. Apply understanding within a mathematical context.
(viz. use $\frac{1}{2}$ base $\times$ height in a less familiar context – intra-mathematical)

Question & model solution

The co-ordinates of three points A , B , and C are: $A(2, 2)$, $B(6, -6)$, $C(-2, -3)$.
(See diagram on facing page.) [On the actual paper, a page break occurred after part (a)]

- (a) Find the equation of AB .

$$\text{slope } AB = \frac{-6 - 2}{6 - 2} = \frac{-8}{4} = -2$$

Equation: $y - 2 = -2(x - 2)$

$$2x + y = 6 \quad \text{OR} \quad y = -2x + 6$$

- (b) The line AB intersects the y -axis at D .
Find the coordinates of D .

$$2x + y = 6$$

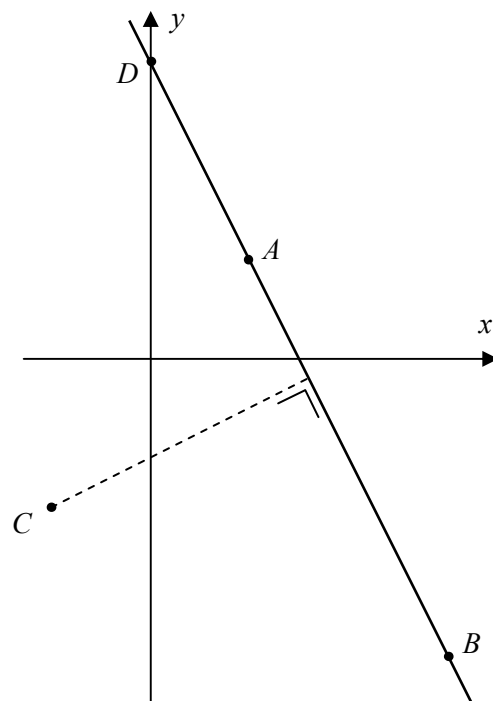
$$x = 0 \Rightarrow y = 6$$

$$\therefore D(0, 6)$$

- (iii) Find the perpendicular distance from C to AB .

$$\begin{aligned} & \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} \\ &= \frac{|2(-2) + (-3) - 6|}{\sqrt{2^2 + 1^2}} \\ &= \frac{|-13|}{\sqrt{5}} = \frac{13}{\sqrt{5}} \end{aligned}$$

$$\text{(or } \frac{13\sqrt{5}}{5} \text{)}$$



(iv) Hence, find the area of the triangle ADC .

$$|AD| = \sqrt{(2-0)^2 + (2-6)^2}$$

$$= \sqrt{4+16} = \sqrt{20}$$

$$\text{Area} = \frac{1}{2} \text{ base} \times \text{height}$$

$$= \frac{1}{2} \cdot \sqrt{20} \cdot \frac{13}{\sqrt{5}}$$

$$= 13$$

Marking scheme & related notes

Question 3

- (a) Scale 5C
High partial credit: correct answer, but not in standard form.
Low partial credit: one error
- (b) Scale 5B
Partial credit: finds $x = 0$ and $y = 6$ but fails to write as co-ordinates.
- (iii) Scale 5B
Partial credit: correct method completed with one or two errors
- (iv) Scale 10D
High partial credit: correct method, correctly executed, but fails to properly simplify
Mid partial credit: correct method, completed, but with some errors
Low partial credit: $|AD|$ correctly found
incomplete solution
area correctly found by other method (i.e., not “hence”)
uses $|AB|$ or $|BD|$ as base and completes correctly
correct solution by other method (i.e. not “hence”)

Comment on answering

This question was of a relatively familiar type for those used to the previous syllabus.

Answering on the first three parts was comparatively good.

Few candidates were able to link their work in the previous part to the perpendicular height of the triangle concerned and undertake the remaining steps to finish.

Candidate exemplars

The following example of a candidate response to **part (a)** received **low partial credit**

$$\begin{aligned}
 m &= \frac{y_2 - y_1}{x_2 - x_1} & \frac{-6 - 2}{2 - 6} & m = \frac{-8}{-4} = 2 \\
 y - y_1 &= m(x - x_1) \\
 y - 2 &= 2(x - 2) \\
 y - 2 &= 2x - 4 \\
 y - 2 + 4 - 2x &= 0 \\
 2x - y - 2 &= 0
 \end{aligned}$$

Comment: note that there is just one error in the work, (made while calculating the slope). Given that this is a very straightforward question at this level, the focus of the assessment is on the candidates' ability to execute it accurately. Any error in finding the equation therefore resulted in low partial credit.

The following example of a candidate response to **part (a)** received **high partial credit**

$$\begin{aligned}
 &\begin{matrix} (2,2) & (6,-6) \\ x_1 & y_1 & x_2 & y_2 \end{matrix} & y - y_1 &= m(x - x_1) \\
 &\frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{-6 - 2}{6 - 2} = \frac{-8}{4} = -2 \\
 &y - 2 = -2(x - 2) \Rightarrow y - 2 = -2x + 4 \\
 &2x - y + 6 = 0
 \end{aligned}$$

Comment: the correct equation was found in unsimplified form. Since the candidate would have received high partial credit had they stopped there, the subsequent error does not reduce the credit below that level.

The following example of a candidate response to **part (iv)** received **high partial credit**:

$$\begin{aligned}
 &\text{Distance from D} \rightarrow \text{C} & \begin{matrix} x_1 & y_1 & x_2 & y_2 \\ (0,6) & \rightarrow & (-2,-3) \end{matrix} \\
 &\sqrt{(-2)^2 + (-3 - 6)^2} \\
 &\sqrt{4 + 81} \\
 &\sqrt{85} \\
 &|AD| \Rightarrow \begin{matrix} x_1 & y_1 & x_2 & y_2 \\ (2,2) & - & (0,-6) \end{matrix} \\
 &\sqrt{(-2)^2 + (6 - 2)^2} = \sqrt{4 + 16} \\
 &= \sqrt{20} \\
 &\text{area} = \frac{1}{2}bh \\
 &= \frac{1}{2}(\sqrt{20})\left(\frac{13}{\sqrt{5}}\right) \\
 &\text{area} = 26 \text{ units}^2
 \end{aligned}$$

Comment: finding $|DC|$ is not relevant. However, the candidate ignores this and continues correctly. The final step is not executed correctly, probably due to a mishandling of the square root.

The following example of a candidate response to **part (iv)** received **low partial credit**:

$$\begin{aligned} & \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| \\ & \frac{1}{2} |-2(2 - 6) + 2(6 + 3) + 0(-3 - 2)| \\ & \frac{1}{2} |-4 + 12 + 12 + 6| \\ & A = 13 \end{aligned}$$

Comment: although the area of the triangle has been correctly found, the candidate has not followed the direction “hence” in the question. (See note below.)

Other points to note

There are other ways to find the area of the triangle. The direction “hence” required the candidate to bring to bear an understanding of specific concepts in geometry. Candidates who did not follow this instruction therefore circumvented the target skill. They were awarded some credit if they executed their method accurately.

Question 4**(25 marks)****Content area:** Strand 2 – Co-ordinate geometry**Main assessment objective(s):**

1. Demonstrate knowledge of results (*viz.* knowledge of form of equation of circle).
2. Demonstrate understanding within a familiar mathematical context.
(*viz.* know and apply criterion for tangency)
3. Execute routine procedures within a mathematical context.
(*viz.* accurately apply algebraic skills of substitution, manipulation of equations involving moduli and square roots, and solving quadratic equations.)

Question & model solution

- (a) Write down the equation of the circle with centre $(-3, 2)$ and radius 4.

$$(x + 3)^2 + (y - 2)^2 = 4^2$$

- (b) A circle has equation $x^2 + y^2 - 2x + 4y - 15 = 0$.

Find the values of m for which the line $mx + 2y - 7 = 0$ is a tangent to this circle.

$$\text{centre: } (1, -2), \text{ radius} = \sqrt{20}$$

Distance from centre to line must equal radius:

$$\frac{|m(1) + 2(-2) - 7|}{\sqrt{m^2 + 4}} = \sqrt{20}$$

$$|m - 11| = \sqrt{20(m^2 + 4)}$$

$$m^2 - 22m + 121 = 20m^2 + 80$$

$$0 = 19m^2 + 22m - 41$$

$$0 = (19m + 41)(m - 1)$$

$$m = \frac{-41}{19} \quad \text{or} \quad m = 1$$

Marking scheme & related notes

Question 4

(a) Scale 5A

(b) Scale 20D

High partial credit:

correct method, largely completed, but with one or two errors (unless oversimplified)

Mid partial credit:

correct method, largely completed, but with more than two errors (unless oversimplified)

Low partial credit:

Substantial progress in correct method. (e.g. sets up substantially correct equation in m .)

Comment on answering

This question was of a relatively familiar type for those used to the previous syllabus.

Part (a) was answered very well.

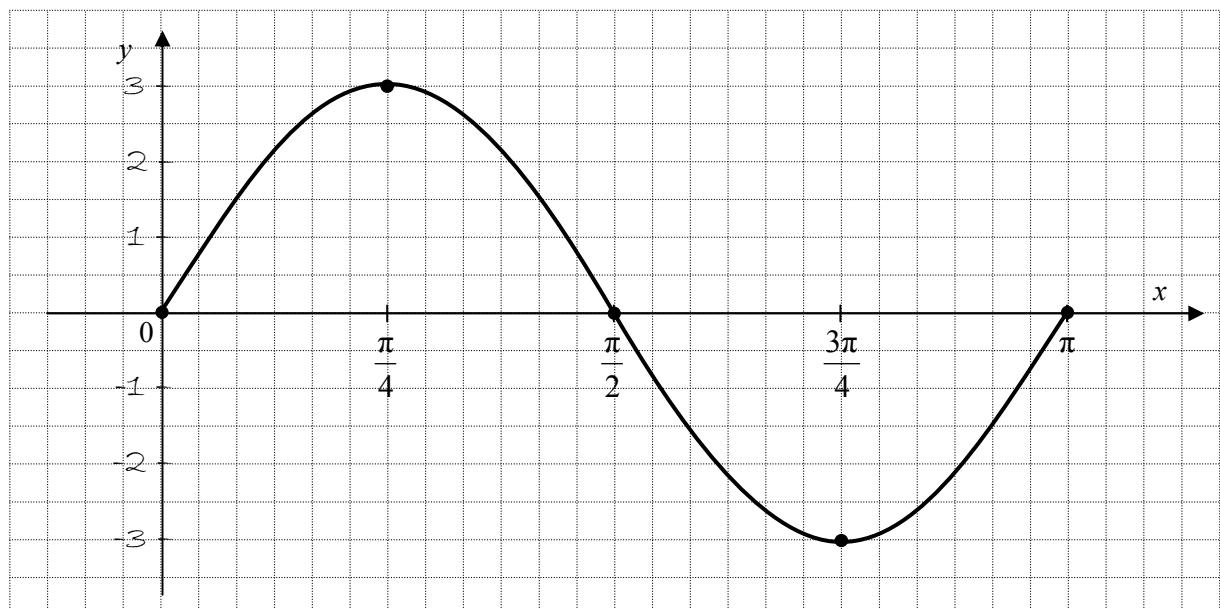
Approximately half of the candidates who attempted part (b) were able to make a meaningful start. Few, however, were able to see the work through to completion with accuracy.

Question 5**(25 marks)****Content area:** Strand 2 – Trigonometry**Main assessment objective(s):**

1. Execution routine procedure (*viz.* evaluate and graph a trigonometric function).
2. Demonstrate knowledge of terminology (*viz.* Identify the *range* and *period* of a trigonometric function.)

Question & model solution**(a)** The function $f(x) = 3\sin(2x)$ is defined for $x \in \mathbb{R}$.**(i)** Complete the table below

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$2x$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin(2x)$	0	1	0	-1	0
$3\sin(2x)$	0	3	0	-3	0

(ii) Draw the graph of $y = f(x)$ in the domain $0 \leq x \leq \pi$, $x \in \mathbb{R}$.**(iii)** Write down the range and the period of f .Range = $[-3, 3]$ Period = π

Marking scheme & related notes

Question 5

(i) Scale 5B

Partial credit: one (or more) error(s) in table
one row or one column fully correct

(ii) Scale 15C

High partial credit:

correct graph without correct scaling on y-axis.

or

graph with corners or straight segments, or other such errors of form (such as flattening at the endpoints) but otherwise correct (including scale)

Low partial credit:

incorrect or no scale **and** graph has corners or straight segments, etc.

or

Graph correctly created from incorrect table.

(Note that, for example, if the candidate's table gives y-values 0,3,0,3,0, then the maximum available for the graph is low partial credit, unless the candidate ignores the table and draws a correct graph.)

No credit:

one or more points incorrectly plotted; points not joined, etc.

(iii) Scale 5B

Partial credit:

period **or** range correct

or

period **and** range consistent with candidate's incorrect graph (provided graph has merited at least partial credit).

Note 1: "6" is not the correct answer for the range. Must write $[-3, 3]$ or $-3 \leq y \leq 3$ or "From -3 to 3" or similar.

Note 2: 180° is not the correct answer for the period.

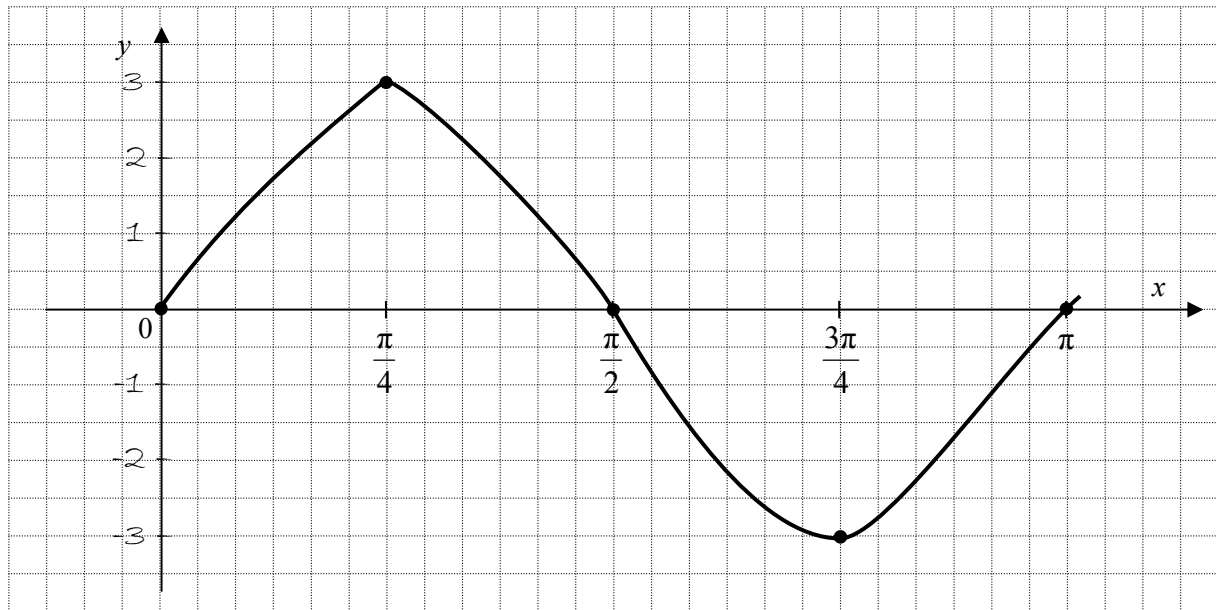
Comment on answering

This was the best answered question on the paper. Candidates had clearly acquired the target skills for parts (i) and (ii).

Nevertheless, few candidates correctly identified the range and period.

Candidate exemplars

The following example of a candidate response to part (ii) received **high partial credit**



Comment: this is a clear example of the descriptor given in the scheme above for high partial credit: “graph with corners or straight segments, or other such errors of form (such as flattening at the endpoints)”.

Other points to note

Proper understanding of the topic requires that candidates be aware of the shape of graphs of functions of this form. If computation errors lead to a graph that is not of the expected shape, candidates are expected to realise this and resolve the matter, rather than proceed regardless. Accordingly, little credit was awarded for a graph drawn from an incorrect table.

Question 6**(25 marks)****Content area:** Strand 2 – vectors**Main assessment objective(s):**

1. Demonstrate knowledge of terminology and results
(viz. knowledge of basic identities involving vectors)
2. Apply understanding of concepts and connections (viz. geometric properties of vectors).
3. Execute routine procedure in familiar context (viz. find angle between two given vectors).

Question & model solution

$ABCD$ is a parallelogram in which $[AC]$ is a diagonal.

$\vec{a} = 2\vec{i} - \vec{j}$, $\vec{b} = 5\vec{i} + 3\vec{j}$, and $\vec{c} = -\vec{i} - \vec{j}$.

- (i) Express $\vec{d}$ in terms of $\vec{i}$ and $\vec{j}$.

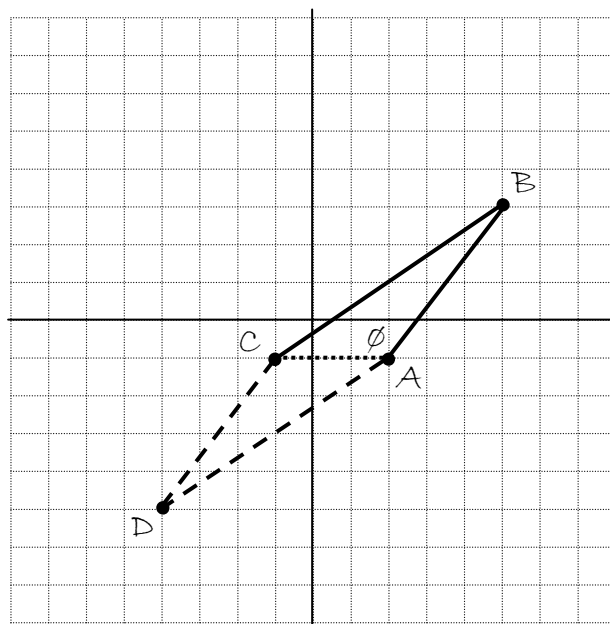
$$\vec{CD} = \vec{BA}$$

$$\vec{d} - \vec{c} = \vec{a} - \vec{b}$$

$$\vec{d} = \vec{c} + \vec{a} - \vec{b}$$

$$= -\vec{i} - \vec{j} + 2\vec{i} - \vec{j} - 5\vec{i} - 3\vec{j}$$

$$= -4\vec{i} - 5\vec{j}$$



- (ii) Find $\vec{AC}$ and $\vec{AB}$.

$$\vec{AC} = \vec{c} - \vec{a} = -\vec{i} - \vec{j} - 2\vec{i} + \vec{j} = -3\vec{i}$$

$$\vec{AB} = \vec{b} - \vec{a} = 5\vec{i} + 3\vec{j} - 2\vec{i} + \vec{j} = 3\vec{i} + 4\vec{j}$$

- (iii) Hence, find $|\angle CAB|$, correct to the nearest degree.

$$\vec{AC} \cdot \vec{AB} = |\vec{AC}| \cdot |\vec{AB}| \cos \phi$$

$$-9 = 3.5 \cos \phi$$

$$\cos \phi = \frac{-3}{5}$$

$$\phi = \cos^{-1}\left(\frac{-3}{5}\right)$$

$$\phi \approx 127^\circ$$

$$[\text{OR: } \text{slope } AB = \frac{4}{3} \Rightarrow \theta = \tan^{-1}\left(\frac{4}{3}\right) \approx 53^\circ \Rightarrow \phi \approx 127^\circ]$$

Marking scheme & related notes

Question 6

(i) Scale 5B

Partial credit:

States $\overrightarrow{AB} = \overrightarrow{DC}$ or $\overrightarrow{BC} = \overrightarrow{AD}$ or $\overrightarrow{BA} = \overrightarrow{CD}$ or $\overrightarrow{CB} = \overrightarrow{DA}$ or $\vec{d} = \vec{a} + \vec{c} - \vec{b}$.
or

$\vec{d} = 2\vec{i} + 3\vec{j}$ or $\vec{d} = 8\vec{i} + 3\vec{j}$ with work

No credit: Correct answer with no work shown.

(ii) Scale 5B

Partial credit:

One or other correct.

(iii) Scale 15C

High partial credit:

Correct method, completed, with one or two errors

or

Stops at $\cos \theta = \frac{-3}{5}$ or fails to correctly finish out.

Low partial credit:

Correct relevant equation (first line of solution or equivalent)

or

Correctly evaluates dot product or **both** relevant moduli.

Comment on answering

This question was of a relatively familiar type for those used to the previous syllabus.

Whereas there was a considerable number of very low scores, these appeared to be from candidates who had not yet covered the topic.

Those who appeared to have covered the topic performed comparatively well.

Answer Question 7, Question 8, and **either** Question 9A **or** Question 9B.

Question 7

(50 marks)

Content area: Strand 1 – statistics

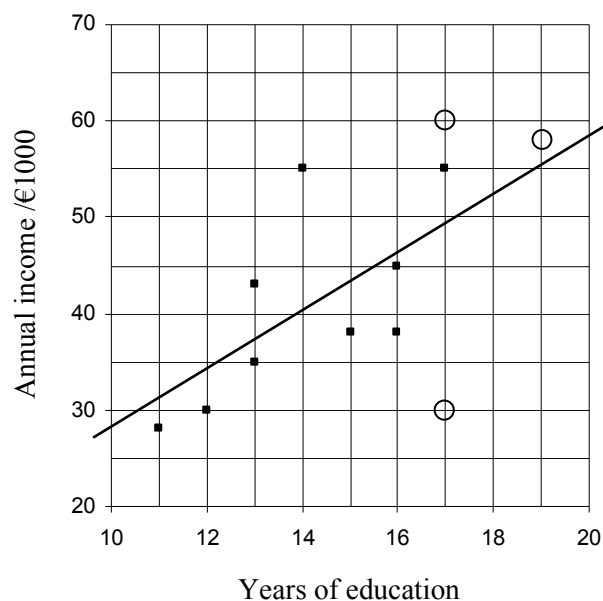
Main assessment objective(s):

1. Present information in graphical form [parts (i) & (iv)]
2. Apply routine procedures (in non-mathematical context) [parts (ii) (v) & (vi)].
3. Interpret solutions of routine procedures in context [parts (iii) & (v)].
4. Demonstrate understanding of concepts, connections, conditions and implications [parts (vii), (viii)]

Question & model solution

An economics student is interested in finding out whether the length of time people spend in education affects the income they earn. The student carries out a small study. Twelve adults are asked to state their annual income and the number of years they spent in full-time education. The data are given in the table below, and a partially completed scatter plot is given.

Years of education	Income /€1,000
11	28
12	30
13	35
13	43
14	55
15	38
16	45
16	38
17	55
17	60
17	30
19	58



- The last three rows of data have not been included on the scatter plot. Insert them now.
- Calculate the correlation coefficient.

Answer:

0.623

(iii) What can you conclude from the scatter plot and the correlation coefficient?

There is a moderate positive correlation between the variables. That is, those with more education tend to have higher incomes

(iv) Add the line of best fit to the completed scatter plot above.

(v) Use the line of best fit to estimate the annual income of somebody who has spent 14 years in education.

Answer:

€40,000

(vi) By taking suitable readings from your diagram, or otherwise, calculate the slope of the line of best fit.

line passes through (10, 28) and (20, 58).

$$\text{slope} = \frac{58 - 28}{20 - 10} = \frac{30}{10} = 3 \quad [\text{or } €3000]$$

(vii) Explain how to interpret this slope in this context?

It is the expected (average) increase in income per additional year of education. That is, each additional year of education corresponds to an average increase of €3000 in annual income.

(viii) The student collected the data using a telephone survey. Numbers were randomly chosen from the Dublin area telephone directory. The calls were made in the evenings, between 7 and 9 pm. If there was no answer, or if the person who answered did not agree to participate, then another number was chosen at random.

List **three** possible problems regarding the sample and how it was collected that might make the results of the investigation unreliable. In each case, state clearly why the issue you mention could cause a problem.

Problem 1:

The sample is very small. Bigger samples give more reliable results.

Problem 2:

They used the Dublin directory only. People in Dublin might not be representative of the population as a whole.

Problem 3:

Those who agree to participate may not be representative of the population. For example, those with lower income might be less inclined to answer questions about such things. (etc. – other problems exist.)

Marking scheme & related notes

Question 7

(i) Scale 5B

Partial credit:

Two points correctly plotted

(Note: to count as correct, the vertical position of the last point must clearly be between 55 and 60, and must not be closer to 55 than to 60.)

(ii) Scale 5B:

Full credit:

Correct answer (0.623...) to one or more decimal places

Partial credit:

Answer in the interval [0.4, 0.8].

(iii) Scale: 10C

Full credit:

Correct statement, properly contextualized (explicitly refers to meaning of variables)

E.g. "Those with more education tend to have higher incomes", "There is a reasonably strong (positive) association between income and education."

High partial credit:

Correct statement, referring to direction **and** strength of correlation, but not contextualized. (e.g. "There is a moderate positive correlation between the variables.")

or

Clearly asserts causality. (e.g. "Getting more education gives you more income").

or

Overly deterministic conclusion. (e.g. "People who have more education have higher incomes", without saying "in general", "on average", "tend to" or similar)

Low partial credit:

Uncontextualised answer that correctly identifies the direction **or** the strength of relationship but not both. (e.g. "there is a positive relationship between the variables").

(iv) Scale 5A

Accept:

Line whose slope is between 1.5 and 5, (i.e. rises by at least three and no more than ten grid squares across the full width of the diagram) **and** has at least four data points on either side.

(v) Scale 5B

Full credit:

€39,000 or €40,000 (note that the correct answer by calculator is €39,917.)

or

Answer correctly read from own diagram, to within €2000.

Partial credit:

Correct answer (as above) but omits to multiply by 1000.

or

Answer more than €2000 and less than €5000 from correct answer by own graph.

(vi) Scale 5B

Full credit:

Correct answer (3 or 3000), with or without work

or

Answer correctly calculated from own diagram, with work.

Partial credit:

Slope correctly calculated from assumption that horizontal and vertical grid squares are equal units. (e.g. slope = rise / run = 6/10 = 0.6).

or

correctly writes down the co-ordinates of **two** points on the line of best fit.

(vii) Scale 5B

Partial credit:

Correct interpretation with implied failure to take account of thousands. (e.g. if slope has been given as 3, then “It is the average increase in income per additional year of education.” is only partially correct.)

or

Asserts that it is a ratio of income to education (i.e. implies intercept = 0).

(viii) Scale 10C

High partial credit

two properly developed problems

Low partial credit

one properly developed problem

or

two problems stated but not properly developed.

(Note, for example: “They shouldn’t have used the Dublin phone book only.” is a problem correctly identified but not properly developed. Stating “People living in Dublin may not be representative of the population as a whole” is properly developed. That is, the latter statement has clarified *why* the use of the Dublin phone book might be an issue.)

Comment on answering

Candidates coped well with most parts of this question and scores were higher than on any other question.

The best answered parts were parts (i), (iv), (v) and, to a slightly lesser extent, (vi) and (viii).

Part (vii) required a conceptual understanding of slope and its meaning in context. It was not well done, with many candidates giving an interpretation of the correlation coefficient instead.

Part (viii) is interesting in having been quite well answered. It was handled significantly better than other text-intensive questions on the paper. In exploring the possible reasons for this, it is worth observing that this part did not require connections to be made between the mathematics and the context. That is, the potential difficulties with the sampling process described are identifiable without referring to the actual calculations or data presented. Other text-based answers tended to require interpretation of some mathematics that the candidates had themselves done. This may indicate that candidates are relatively comfortable describing and understanding “real world” issues and concepts, but not as yet able to meaningfully relate these to the mathematics they are engaged in.

Candidate Exemplars

The following candidate response to **part (iii)** received **low partial credit**:

It has a strong positive correlation.

The following candidate response to **part (iii)** received **high partial credit**:

There is a strong correlation between their annual income and the number of years they spent in full-time education.

Comment: it is dubious to describe a correlation of 0.623 as “strong” in these circumstances, and this is the reason for not awarding full credit. However, there is no universally accepted criterion for applying the adjectives “strong”, “moderate” and “weak” to correlation coefficients. The application of a penalty here might be considered harsh, and this matter will be kept under review.

The following candidate response to **part (iii)** received **full credit**:

That generally there is the increase in the annual income as the years spent in education increase, however there are some exceptions to this.

The following candidate response to **part (vii)** received **partial credit**:

Every 1 year that we go across, we go up 5 of the annual income.
(The above candidate had calculated the slope to be 5.)

The following candidate response to **part (vii)** received **full credit**:

That for each year of education a persons income should increase by 5 thousand.
(The above candidate had calculated the slope to be 5.)

The following candidate response to **part (viii)** is an example of a problem that was regarded as stated and “just about” properly developed (borderline).

Most people with average wages work from 9 to 5. Some people that earn more may work later so they wouldn't have been at home to answer the call.

Comment: whether or not the likelihood of being in at the relevant time is in reality dependent on income is not relevant to assessing the response. What is correct is that this is a *potential* difficulty. It would have been preferable to see a somewhat clearer statement to the effect that the issue (if true) biases the sample in favour of lower earners. Nonetheless, there is an implication that this is the case and the statement was therefore regarded as satisfactory.

The following candidate responses to **part (viii)** are examples of problems that were “stated but not properly developed”. That is, that candidate did not “state clearly why the issue you mention could cause a problem”.

A lot of people like to socialise or “chill out” at this time therefore possible participants are less likely to participate.

Comment: the problem identified will only bias the sample if some relevant categories of people are more likely to be affected by this than others; the candidate made no reference to this.

Not everybody has their number in the telephone book, so it's not including everyone who lives in Dublin.

Comment: again, there is no reference to how this could impact on the sample in a systematic way.

The following candidate responses to **part (viii)** are examples of problems that were “not properly stated”.

People selected randomly may be working different hours, therefore effecting their salary this will not depend on time spent in education.

Comment: it is not clear what point the candidate is making. (The statement appears to be suggesting that random selection is a negative aspect of the sampling.)

This is not a true test on people's opinions because it is only receiving information from a sample of the population.

Question 8**Geometry and Trigonometry****(50 marks)****Content area:** Strand 2 – trigonometry**Main assessment objective(s):**

1. In a non-mathematical context, apply routine procedures (*viz.* execute trigonometric calculations).
2. In a non-mathematical context, apply understanding of concepts and connections, interpreting solutions, conditions and implications in the original context (*viz.* “mathematise” the presented problem and show how to solve it).

Question & model solution

Two surveyors want to find the height of an electricity pylon. There is a fence around the pylon that they cannot cross for safety reasons. The ground is inclined at an angle. They have a clinometer and a 100 metre tape measure. They have already used the clinometer to determine that the ground is inclined at 10° to the horizontal.

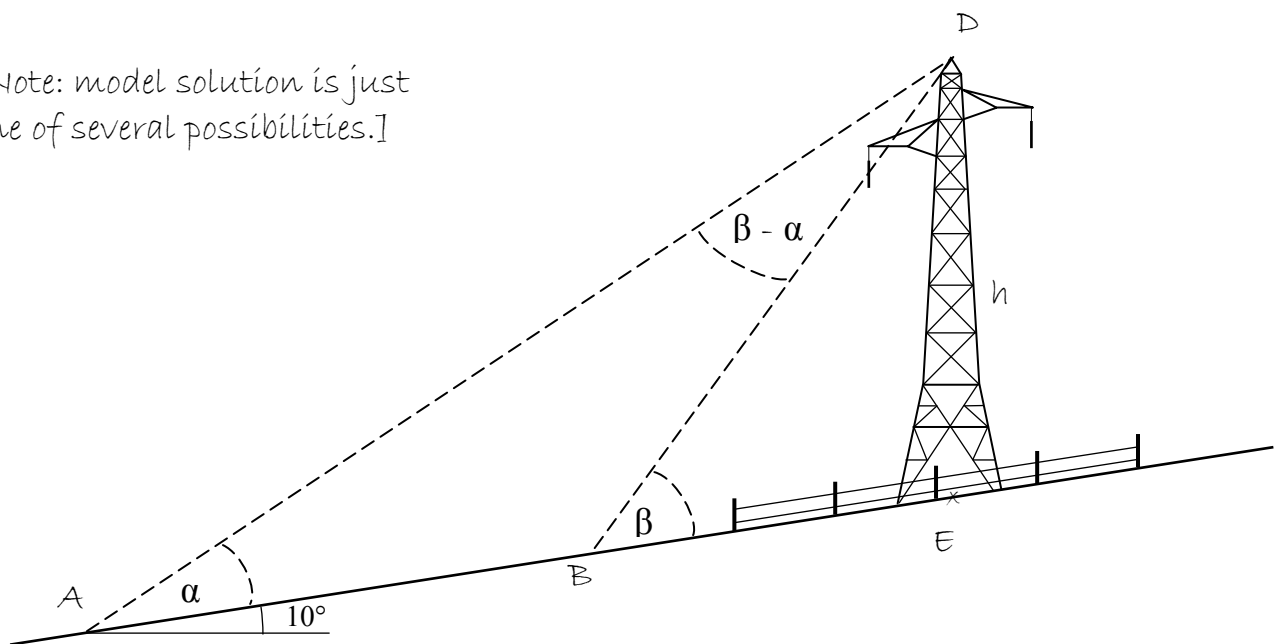
- (a) Explain how they could find the height of the pylon.

Your answer should be illustrated on the diagram below. Show the points where you think they should take measurements, and write down clearly what measurements they should take.



Diagram:

[Note: model solution is just one of several possibilities.]



Measurements to be taken:

- Measure the angle of elevation from two points A and B (and subtract 10 to get)

α and β as shown.

- Measure the distance from A to B.

- use triangle ABD to find $|BD|$, and then use triangle DBE to find $|DE| = h$.

(b) Write down possible values for the measurements taken, and use them to show how to find the height of the pylon. (That is, find the height of the pylon using your measurements, and showing your work.)

Suppose $\alpha = 40^\circ$ and $\beta = 55^\circ$ and $|AB| = 30$ metres.

In triangle ABD:
$$\frac{|BD|}{\sin \alpha} = \frac{|AB|}{\sin(\beta - \alpha)}$$

$$\begin{aligned} |BD| &= \frac{|AB| \sin \alpha}{\sin(\beta - \alpha)} \\ &= \frac{30 \sin 40}{\sin 15} \\ &\approx 74.5 \text{ m} \end{aligned}$$

In triangle BDE:
$$\frac{h}{\sin \beta} = \frac{|BD|}{\sin 100}$$

$$\begin{aligned} h &\approx \frac{(74.5) \sin 55}{\sin 100} \\ h &\approx 62 \text{ metres} \end{aligned}$$

Marking scheme & related notes

Question 8

(a) Scale 20D

Full credit:

two angles of elevation marked **and** text stating that these and the distance between the observation points are to be measured, **and** stating how a relevant intermediate side can be found.

High partial credit:

two angles of elevation marked **and** text stating that these and the distance between the observation points are to be measured

Mid partial credit:

two angles of elevation marked and a distance between the two points indicated or referred to (e.g. writes d between observation points).

Low partial credit:

one or two angles indicated on diagram.

Note: if the candidate's suggested solution involves measuring to the point labeled E in the model solution (or equivalent), then a clear and plausible strategy for making such a measurement must be given. Such a strategy cannot assume that the bottom of the pylon is aligned with the midpoint of the fence, or contravene the constraints of the question.

(b) Taken in two stages, using scale 15C for each:

First stage: Finding an intermediate measurement, such as the distance from one of the observation points to the top of the pylon:

Scale 15C

Full credit: Finding measure of one relevant side of triangle

High partial credit: Finding relevant side with one error

Low partial credit: Proper setup with appropriate data

Second stage: Finding the height of the pylon

Scale 15C

High partial credit: Finds height with one error

Low partial credit: Proper setup for height

Comment on answering

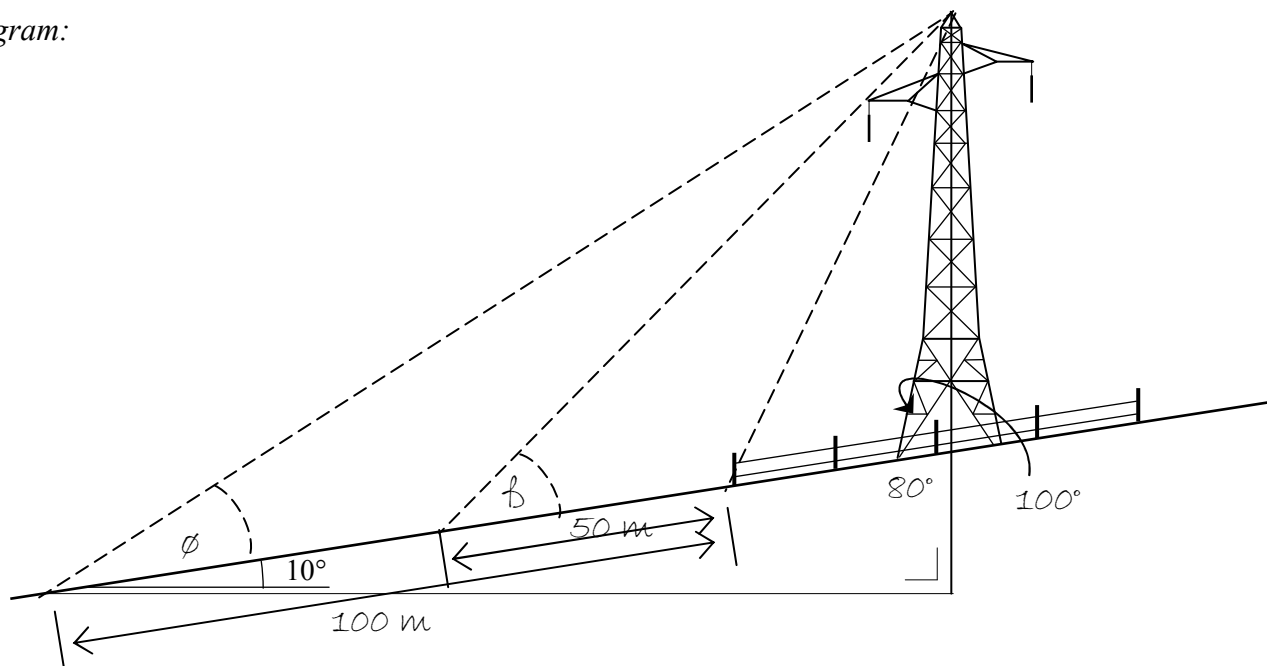
This question was not well answered. It appeared that candidates had little experience of applying their mathematical knowledge in this way. The trigonometry involved was not difficult and one would expect it to be well within the compass of moderately good candidates. However, at this stage, the candidates clearly have had little experience of planning and undertaking field activities or of discussing the practicalities of using trigonometry to solve real problems. This is of concern, as such activities are central to ensuring that several of the key skills referred to in the syllabus are made manifest in its implementation. See the recommendation in Chapter 9 regarding focussing on *key skills*.

Approaches suggested by the candidates to the problem at hand were in many instances impractical, in that they would require the surveyors to take measurements under the ground or from points high in the air.

Candidate Exemplars

The following candidate response to **part (a)** received **high partial credit**:

Diagram:



Measurements to be taken:

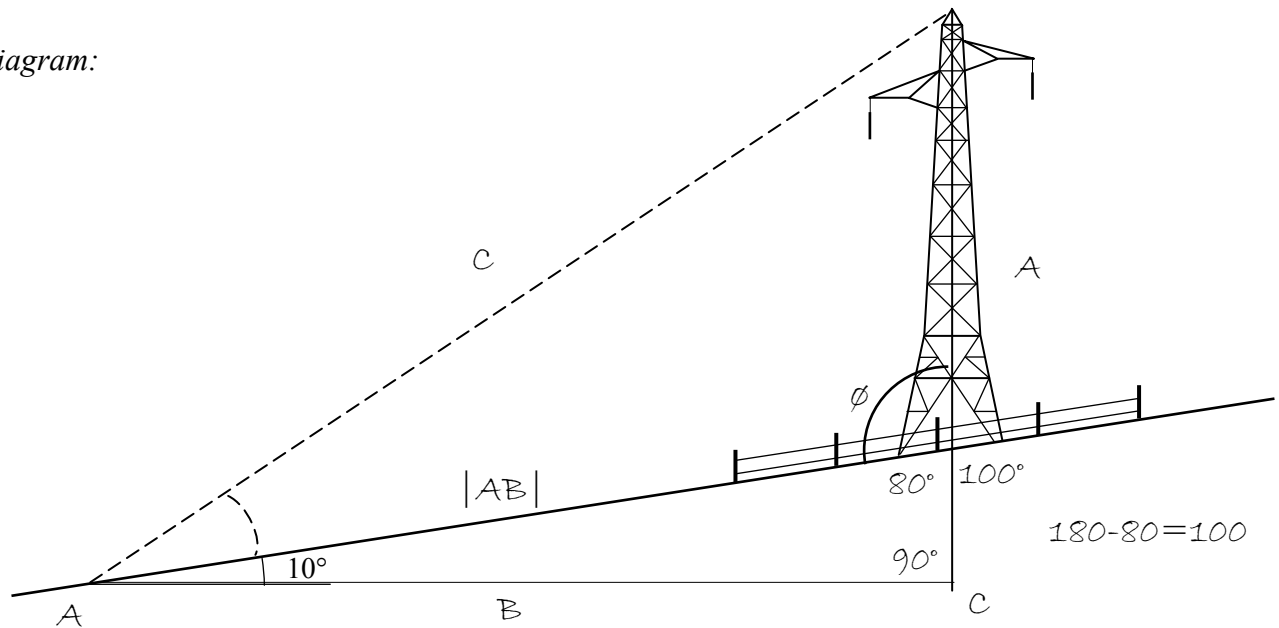
use clinometer to determine angle ϕ which is 100 metres away from fencing.
Angle b , 50 metres from the fencing would also need to be measured.

Comment: The diagram and the measurements to be taken are fully satisfactory. However the candidate did not explain how these measurements were to be used to solve the problem and so did not fully answer the question asked: “Explain how they could find the height of the pylon.” The layout of the answer space was not especially helpful in this regard, as there was no reminder towards the end of the page that such an explanation was required. The final version of the sample paper will be amended in this respect.

Note also that, although the candidate completed a triangle “under the ground”, (s)he did not indicate that any measurements along these lines were required.

The following candidate response to **part (a)** received **low partial credit**:

Diagram:



Measurements to be taken:

Angle from ground at perpendicular point B.

Length from A to C.

Comment: Low partial credit was awarded for the indication on the diagram that a reading should be taken of the angle of elevation of the top of the pylon from the point A. There is no further merit in the remaining work.

Question 9A**Probability and Statistics****(50 marks)****Content area:** Strand 1 – Probability and Statistics**Main assessment objective(s):**

1. In a non-mathematical context, apply routine procedures.
2. In a non-mathematical context, apply understanding of concepts and connections, interpreting solutions, conditions and implications in the original context.

Question & model solution

A car rental company has been using *Evertread* tyres on their fleet of economy cars. All cars in this fleet are identical. The company manages the tyres on each car in such a way that the four tyres all wear out at the same time. The company keeps a record of the lifespan of each set of tyres. The records show that the lifespan of these sets of tyres is normally distributed with mean 45 000 km and standard deviation 8000 km.

- (i) A car from the economy fleet is chosen at random. Find the probability that the tyres on this car will last for at least 40 000 km.

$$P(X \geq 40,000) = P\left(Z \geq \frac{40,000 - 45,000}{8000}\right)$$

$$= P(Z \geq -0.625)$$

$$= P(Z \leq 0.625)$$

$$= 0.734$$

(Accept any value between 0.7324 and 0.7357 inclusive.)

- (ii) Twenty cars from the economy fleet are chosen at random. Find the probability that the tyres on at least eighteen of these cars will last for more than 40 000 km.

At least 18 $\Rightarrow$ 18, 19 or 20.

$${}^{20}C_{18}(0.734)^{18}(0.266)^2 + {}^{20}C_{19}(0.734)^{19}(0.266)^1 + (0.734)^{20}$$

$$= 0.0684$$

(Note: $p = 0.7357 \Rightarrow \text{ans} = 0.0706$; $p = 0.7324 \Rightarrow \text{ans} = 0.0664$)

Accept a solution using the normal approximation.

- (iii) The company is considering switching brands from *Evertread* tyres to *SafeRun* tyres, because they are cheaper. The distributors of *SafeRun* tyres claim that these tyres have the same mean lifespan as *Evertread* tyres. The car rental company wants to check this claim before they switch brands.

The company selects 25 economy cars at random from the fleet and fits them with the new tyres. For these cars, it is found that the mean life span of the tyres is 43 850 km.

Test, at the 5% level of significance, the hypothesis that the mean life span of *SafeRun* tyres is the same as that of *Evertread* tyres. State the conclusion clearly.

$$H_0: \mu (= \text{mean lifespan of new brand}) = 45000$$

$$H_1: \mu \neq 45000$$

$$\sigma_x = \frac{\sigma}{\sqrt{n}} = \frac{8000}{\sqrt{25}} = 1600$$

$$\text{Observed value of } \bar{x} = 43850$$

$$\Rightarrow \text{observed } z = \frac{43850 - 45000}{1600} = -0.71875$$

The critical values for the test at the 95% level are ± 1.96 .

The observed value of z is between -1.96 and 1.96

So, we cannot reject the null hypothesis.

As there is no evidence that the tyres are different, the company should switch to the cheaper tyres.

(See the note on page 42 regarding alternative approaches to the hypothesis test.)

Marking scheme & related notes

Question 9A

(i) Scale 10B

Partial credit: Finds $z = 0.625$ or $z = -0.625$

[Note: tables can be read to two decimal places only. $z = 0.62$ yields 0.7324 and $z = 0.63$ yields 0.7357. Accept either of these or any value in between as a correct reading for full credit.]

(ii) Scale 20D

High partial credit: All three terms written

Mid partial credit: Any one term written correctly

Low partial credit: Calculates (and indicates relevance of) $q = 1 - p$

(iii) **Hypothesis test**

Scale 15C

High partial credit:

Correct calculations but fails to properly explain acceptance (or non-rejection) of null hypothesis (e.g no reference to 1.96) and/or fails to clearly state null hypothesis.

Low partial credit:

Correct statement of both null hypothesis and alternative hypothesis
or

Calculates standard error of mean correctly.

Note: if fails to calculate standard error (e.g. uses σ instead) then award low partial credit if finishes (more or less) correctly, and otherwise no credit.

Contextualizes conclusion

Scale 5A

Accept, for example: “We cannot say that these tyres are any different”, “These tyres are just as good”, “They should switch to the cheaper tyres”.

Note that many statisticians do not say “accept the null hypothesis”, but insist instead on “do not reject the null hypothesis”. The distinction between these two statements is not required here, and either phrasing should be accepted.

Note also that there are variations on how the hypothesis test can be conducted:

1. After calculating $z = -0.71675$, one can calculate the corresponding p -value (one-tailed ≈ 0.24 ; two-tailed ≈ 0.47). Since this is larger than 0.05, do not reject H_0 .
2. One can convert the “acceptance region” for Z into an acceptance region for $\bar{X}$. That is, $z = \pm 1.96$ becomes $\bar{x} = 45,000 \pm 3136$. Hence, the decision rule for this experiment can be established before the experiment is conducted: accept H_0 if the observed value $\bar{x}$ falls between 41864 and 48136 (which it does).
3. Create a confidence interval for the mean of the new brand of tyres:
 $\bar{x} \pm 1.96 \sigma_{\bar{x}} = 43850 \pm 1.96(1600)$ yields $40714 \leq \mu \leq 46986$. Since the confidence interval contains the hypothesized mean, we cannot reject the null hypothesis.

Accept any of the above approaches. In all cases, the conclusion must be properly contextualized to receive the last five marks.

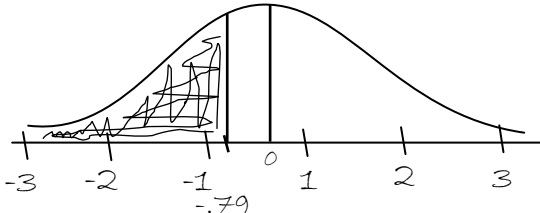
Comment on answering

There were significantly fewer attempts at this question than at 9B. Of these attempts, many were somewhat tentative. It seemed clear that many of these candidates had not covered the material, and the overall standard was therefore low on average. There were, however, a few centres that had a much higher quality of answering, indicating that the material is within the grasp of candidates who have fully covered the relevant topic.

Answering of part (ii) was poorer than the other parts.

Candidate Exemplars

In **part (iii)**, the following candidate response was awarded **high partial credit** for conducting the hypothesis test (but did not receive the marks for giving a properly contextualised conclusion):

$$\frac{43,850 - 45000}{8000 / \sqrt{25}} \Rightarrow \frac{-1150}{1600} = -.79 \quad z = -.79$$


$$= P(Z \leq .79) = P(X \leq .7852)$$

$$1 - .7852 = P(X \leq .2148)$$

As it is between the values $(-1.96 \leq x \leq 1.96)$ we can fully accept the null hypothesis.

Comment: the examiner initially considered that, if the middle portion of the work were ignored, then perhaps the first and last lines together would warrant full credit for the hypothesis test. However, ignoring the middle portion is not justifiable, as it contains notational confusion and also indicates that the candidate is not clear about the circumstances in which the null hypothesis is to be accepted or rejected. (It suggests that the conclusion is based on the fact that 0.2148 rather than -0.79 lies between -1.96 and 1.96.)

Furthermore, there was no initial statement of the null hypothesis, and there is a misreading (presumably from a calculator) of -0.719 as -0.79. Nonetheless, there is a substantially correct effort at the hypothesis test. In retrospect, a “mid-partial” category for such work would have been useful for responses such as this. That is, scale 15D would have been more appropriate than 15C. In the absence of a mid-partial category, high partial credit is the fairest available option.

In **part (iii)**, the following candidate response was awarded **low partial credit**:

$$\frac{\bar{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}} \right)}$$

$\bar{x}$ = normal

μ = mean

σ = standard deviation

$\sqrt{n}$ = population

$$\frac{45000 - 43850}{\sqrt{\frac{8000}{\sqrt{25}}}}$$

$$\frac{1150}{\left(\frac{8000}{5} \right)}$$

$$\frac{1150}{1600} = 0.71$$

Comment: despite the errors throughout, there is sufficient work of merit in the calculations to warrant low partial credit.

Question 9B**Geometry and Trigonometry****(50 marks)****Content area:** Strand 2 – Geometry and Trigonometry**Main assessment objective(s):**

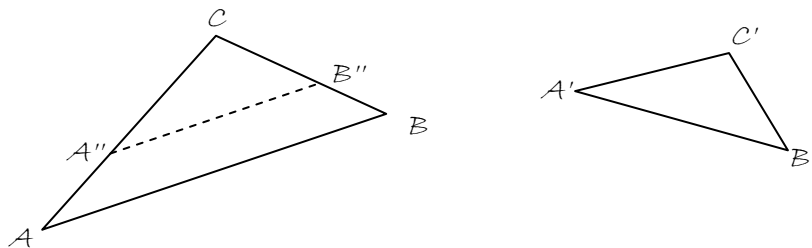
1. In a mathematical context, apply understanding of concepts and connections, including relevant conditions, implications, etc.
2. In a non-mathematical context, apply routine procedures, interpreting the solutions in the original context.

Question & model solution

- (a) Prove that, if two triangles $\triangle ABC$ and $\triangle A'B'C'$ are similar, then their sides are proportional, in order:

$$\frac{|AB|}{|A'B'|} = \frac{|BC|}{|B'C'|} = \frac{|CA|}{|C'A'|}.$$

Diagram:



Given:

Triangles ABC and $A'B'C'$, with $|\angle A| = |\angle A'|$, $|\angle B| = |\angle B'|$, $|\angle C| = |\angle C'|$.

To prove:

$$\frac{|AB|}{|A'B'|} = \frac{|BC|}{|B'C'|} = \frac{|CA|}{|C'A'|}$$

Construction:

Locate A'' on $[CA]$ such that $|A''C| = |A'C'|$

Locate B'' on $[CB]$ such that $|CB''| = |C'B'|$

Join $[A''B'']$

Proof:

$$\triangle A''CB'' \equiv \triangle A'B'C' \quad (\text{by SAS rule})^*$$

$$\therefore |\angle CA''B''| = |\angle CAB| \quad (\text{because both equal } |\angle A'|)$$

$$\therefore A''B'' \parallel AB \quad (\text{because corresponding angles are equal})^*$$

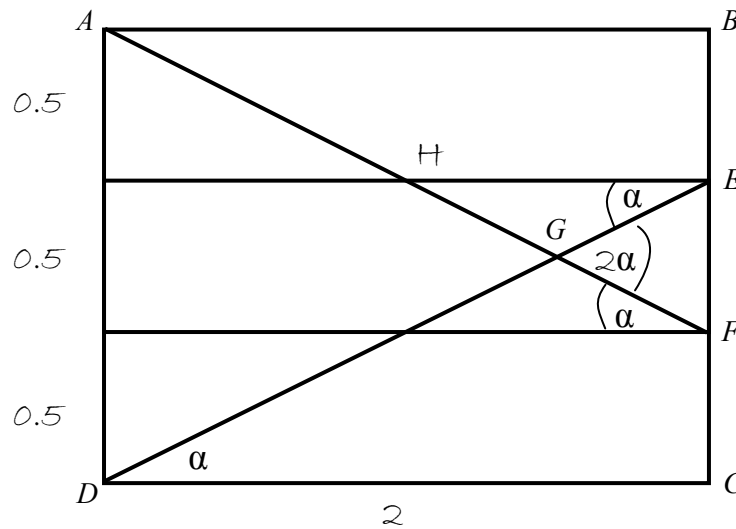
$$\therefore \frac{|CA|}{|CA''|} = \frac{|CB|}{|CB''|} \quad (\text{by a previous theorem})^*$$

(continued overleaf)

$$\therefore \frac{|CA|}{|C'A'|} = \frac{|CB|}{|C'B'|} \quad (\text{replacing equals with equals})$$

Similarly, we can show that these equal $\frac{|AB|}{|A'B'|}$

(b) Anne is having a new front gate made and has decided on the design below.



The gate is 2 metres wide and 1.5 metres high. The horizontal bars are 0.5 metres apart.

(i) Calculate the common length of the bars [AF] and [DE], in metres, correct to three decimal places.

$$|AF|^2 = 1^2 + 2^2 = 5$$

$$\therefore |AF| = \sqrt{5} \approx 2.236 \text{ metres}$$

(ii) In order to secure the bar [AF] to [DE], the manufacturer needs to know:

- the measure of the angle EGF, and
- the common distance $|AG| = |DG|$.

Find these measures. Give the angle correct to the nearest degree and the length correct to three decimal places.

$$\tan \alpha = \frac{1}{2} \Rightarrow \alpha = \tan^{-1} \frac{1}{2} \therefore |\angle EGF| = 2 \tan^{-1} \frac{1}{2} \approx 53^\circ$$

$$|AH| = \frac{1}{2}|AF|, \text{ and } |HG| = \frac{1}{2}|HF|, \text{ so } |AG| = \frac{3}{4}|AF|$$

$$\therefore |AG| = 1.677 \text{ m}$$

Marking scheme & related notes

Question 9B

- (a) Diagram & Given: Scale 5B

Partial Credit: Diagram **or** Given

Construction: Scale 5B

Partial Credit: Construction not explicit

Main body of proof: Scale 15C

High partial credit: All critical steps present but not all steps fully justified (e.g. in model solution, each line is there, but one or more of the explanations labeled * are omitted.)

Low partial credit: Any one critical step omitted (e.g. in the model solution, one or more of the lines is missing, but there is still some substantial work of merit.)

“Similarly,...” Scale 5A

- (b) (i) Scale 5C

Higher partial credit: Correct solution but omits units and/or fails to evaluate $\sqrt{5}$

Low partial credit: Correct use of Pythagoras

- (ii) Angle: Scale 10C

High partial credit:

Correct answer but not in degree format

Low partial credit:

Correctly calculates measure of one relevant angle in degrees

Distance: Scale 5C

High Partial Credit:

Correct solution but omits units and/or makes rounding errors

Low Partial Credit:

Any correct equation that relates $|AG|$ or $|GF|$ or $|DG|$ or $|GE|$ to known quantities.

Comment on answering

Part (a) was not well done in most cases. Whereas the diagram, the “given” and construction were done reasonably well, the main body of the proof was rarely fully correct.

Part (b) was comparatively well answered by those who attempted it. Generally the solutions offered in part (b)(ii) were more complicated than necessary, but executed well nonetheless.

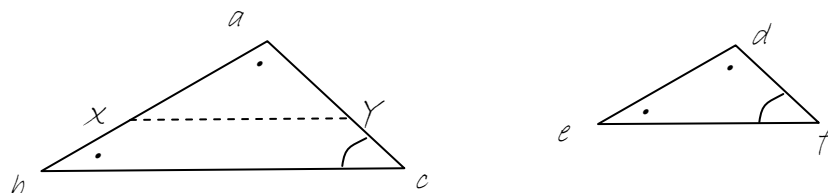
Candidate Exemplars

The following candidate response was awarded **full credit** for “Diagram & Given”, **full credit** for “Construction”, **high partial credit** for the main body of the proof, and **full credit** for the final line:

- (a) Prove that, if two triangles $\triangle ABC$ and $\triangle A'B'C'$ are similar, then their sides are proportional, in order:

$$\frac{|AB|}{|A'B'|} = \frac{|BC|}{|B'C'|} = \frac{|CA|}{|C'A'|}.$$

Diagram:



Given: $\triangle ABC$ and $\triangle DEF$ similar triangles.

$$|\angle bay| = |\angle edf|, |\angle def| = |\angle abc|, |\angle acb| = |\angle dfe|.$$

To prove:

$$\frac{|AB|}{|DE|} = \frac{|BC|}{|EF|} = \frac{|AC|}{|DF|}$$

Construction:

Add in the line xy such that $|Ax| = |DE|$ and $|Ay| = |DF|$

$\triangle Axy = \triangle DEF$ by isometry

Proof:

$$\left. \begin{array}{l} |Ax| = |DE| \\ |Ay| = |DF| \end{array} \right\} \text{isometry}$$

$$|\angle xAy| = |\angle EDF| \quad \} \text{given}$$

$\therefore$ triangles are congruent

$$|\angle axy| = |\angle def|$$

but $|\angle def| = |\angle abc|$ (given)

so $|\angle axy| = |\angle abc|$

$\therefore xy \parallel bc$

using a previous theorem, $\frac{|AB|}{|Ax|} = \frac{|AC|}{|Ay|}$

but $|Ax| = |DE|$ and $|Ay| = |DF|$.

$$\therefore \frac{|AB|}{|DE|} = \frac{|AC|}{|DF|} \quad \text{and similarly it equals } \frac{|BC|}{|EF|}.$$

Comment: a number of issues are worth commenting on in relation to the above example:

1. The inconsistencies in relation to lettering have not been penalised. That is, the candidate has not been penalised for changing the letters from those given in the question, nor for mixing upper and lower case forms.
2. The placing of a common mark on angles that are not equal has not been penalised, since the candidate does not rely on this in the course of the proof. (That is (s)he refers to all angles properly throughout.)
3. The references to isometries are inappropriate. That is, there are no axioms or theorems related to isometries in the formal geometry course and these cannot therefore be used in proving theorems. However, despite these references, the candidate's work does not use the isometries and the work stands without them. (The candidate's second line in the construction is not required, given what has been marked on the diagram. The second reference to isometries can be ignored. The candidate is being penalised in any event for the lack of full justification of steps, as (s)he has failed to reference the SAS rule in asserting congruence.)
4. The reference to an isometry in place of a reference to congruence by the SAS axiom is one of two omissions from an otherwise fully satisfactory proof. The other is that the candidate's assertion that XY is parallel to BC should refer to the relevant theorem (e.g. "since corresponding angles are equal" or "by the corresponding angles theorem" or "by theorem 5").

7. Detailed analysis – Ordinary Level

Summary of the Ordinary Level scheme

– mark allocations and scales applied

Section 0

Question 1: see below

Section A

Question 2

- (i) 5A
- (ii) 5A
- (iii) 5A
- (iv) 5A
- (v) 5A

Question 3

- (a) 15C
- (b) 10C

Question 4

- (a) 5B
- (b) 10C
- (c) 10B

Question 5

- (a) 5B
- (b) intermediate calc.: 5B
find k 15C

Question 6

- (a) 10C
- (b) (i) 5B
(ii) 5A
(iii) 5A

Section B

Question 7

- (a) (i) 5B
(ii) 10C
(iii) 5A
- (b) (i) 5B
(ii) 10B
(iii) 5B

Question 8

- angles 5B
- (a) $|DA|$ 10C
 $|DB|$ 10C
- (b) 10C
- (c) 5A

Question 9A

- (a) 10C
- (b) 5B
- (c) 10C
- (d) 10C
- (e) 10B

Question 9B

- (a) (i) 5A
(ii) 10C
(iii) 10C
- (b) (i) 5B
(ii) 10C
(iii) 5B

Question 1**(50 marks)**

This question consisted of the corresponding question from the Leaving Certificate Ordinary Level examination of 2009. It was reformatted for inclusion in a completion-booklet examination.

This question is not part of the focus of the trialling exercise. Examiners were given copies of the relevant extract from the 2009 marking scheme, which may be accessed on the SEC website. They marked this question whenever it was encountered. However, no detailed information was collected, and no commentary or analysis is offered here.

In the real examination in 2010, this question will be the same question as for candidates in all other schools. It will be reformatted for inclusion in a completion-booklet examination. The same marking scheme will apply to candidates in the initial schools as will apply in all other schools.

Question 2**(25 marks)****Content area:** Strand 1 – Statistics**Main assessment objective(s):**

1. Demonstrate understanding of concepts within a mathematical context.
(viz. understanding of specified summary statistics; application of this understanding)

Question & model solution:

The size, mean, and standard deviation of four different data sets are given in the table below.

	A	B	C	D
size (N)	1000	100	100	10
mean (μ)	10	100	1000	100
standard deviation (σ)	20	30	20	10

Complete the sentences below by inserting the relevant letter in each space:

- (a) The biggest data set is A and the smallest is D.
- (b) In general, the data in set C are the biggest and the data in set A are the smallest.
- (c) The data in set B are more spread out than the data in the other sets.
- (d) Set A must contain some negative numbers.
- (e) If the four sets are combined, the median is most likely to be a value in set A.

Marking scheme & related notes

Question 2

- (a) Scale 5A
Full credit: both answers correct
No credit: any other response (e.g. one correct and one incorrect)
- (b) Scale 5A
Full credit: both answers correct
No credit: any other response (e.g. one correct and one incorrect)
- (c) Scale 5A
- (d) Scale 5A
- (e) Scale 5A

Comment on answering

Answering on this question was somewhat poorer than was anticipated. It is clear that many candidates did not correctly interpret the information as presented. For example, a number of candidates appeared to consider set A to be the set {1000, 10, 20}, and so on for B, C, and D. Apart from these candidates, many others clearly had difficulty distinguishing between the size of a data set and the size of the numbers in that set. The particular wording used in the question parts **(a)** and **(b)** was not helpful in this regard. In finalising the sample paper, the wording of part **(a)** will be rephrased so as to distinguish this task more clearly from that in part **(b)**.

Other points to note

Note that the marking scheme for parts **(a)** and **(b)** is **not** structured according to how many individual letters are correct or incorrect. Credit is awarded for clear evidence that the relevant target skill or concept has been acquired. A candidate who cannot distinguish **both** the largest **and** smallest number in a list of four numbers clearly is not displaying such evidence. One correct and one incorrect letter is more likely to be due to coincidence than to significant partial understanding.

It should also be noted that candidates are expected to be comfortable reading and working with standard statistical terminology and notation.

Question 3

(25 marks)

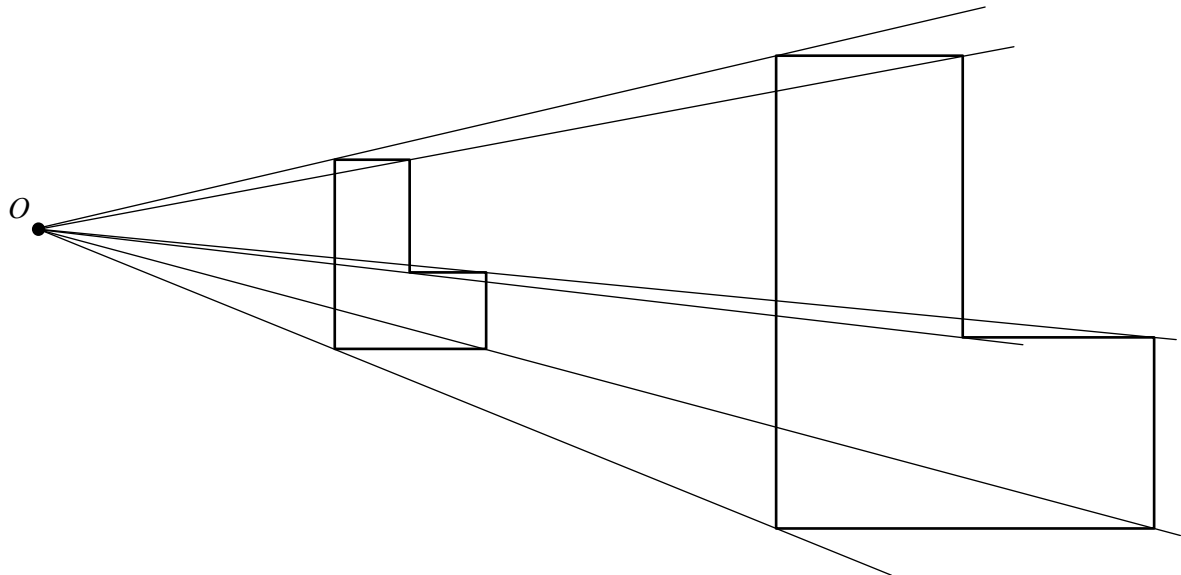
Content area: Strand 2 – Geometry

Main assessment objective(s):

1. Execute routine procedure within a mathematical context.
2. Demonstrate use of geometrical instruments
3. Demonstrate understanding of concepts and connections (*viz.* the effect of enlargement on area.)

Question & model solution:

- (a) Construct the image of the shape under the enlargement with centre O and scale factor 2.5.



- (b) Given that the area of the original shape is 3.5 cm^2 , find the area of the image.

$$\text{Area of image} = (\text{original area}) \times (\text{scale factor})^2$$

$$= 3.5 \times (2.5)^2 = 21.875 \text{ cm}^2.$$

Marking scheme & related notes

Question 3

- (a) Scale 15C
Full credit: Correct solution (Image correct size and location, with at least one correct construction ray shown)
High partial credit: Right shape, size and orientation, but in the wrong place
Low partial credit: Right shape, wrong size (drawn anywhere in correct orientation) or: at least one point correctly found
- (b) Scale 10C
High partial credit: Correct expression not correctly finished
Correct answer without units
Low partial credit: Significant work on finding area (e.g., subdivides image into rectangles and measures or calculates relevant lengths)
No credit: Solution by calculation without squaring the scale factor.

Comment on answering

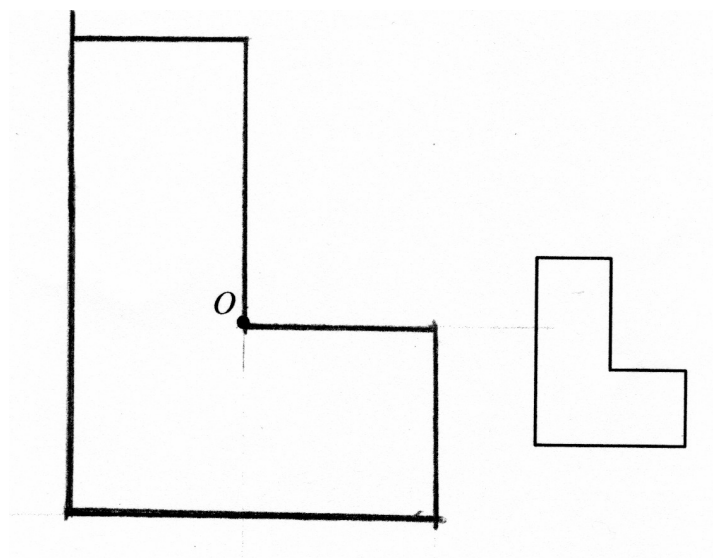
This question was not as well answered as might have been expected, given that the material is similar to that on the previous syllabus. However, it is worth noting that the geometry question is not popular on the old syllabus and is often omitted.

Many candidates used an incorrect scale factor in part (a) (often 2).

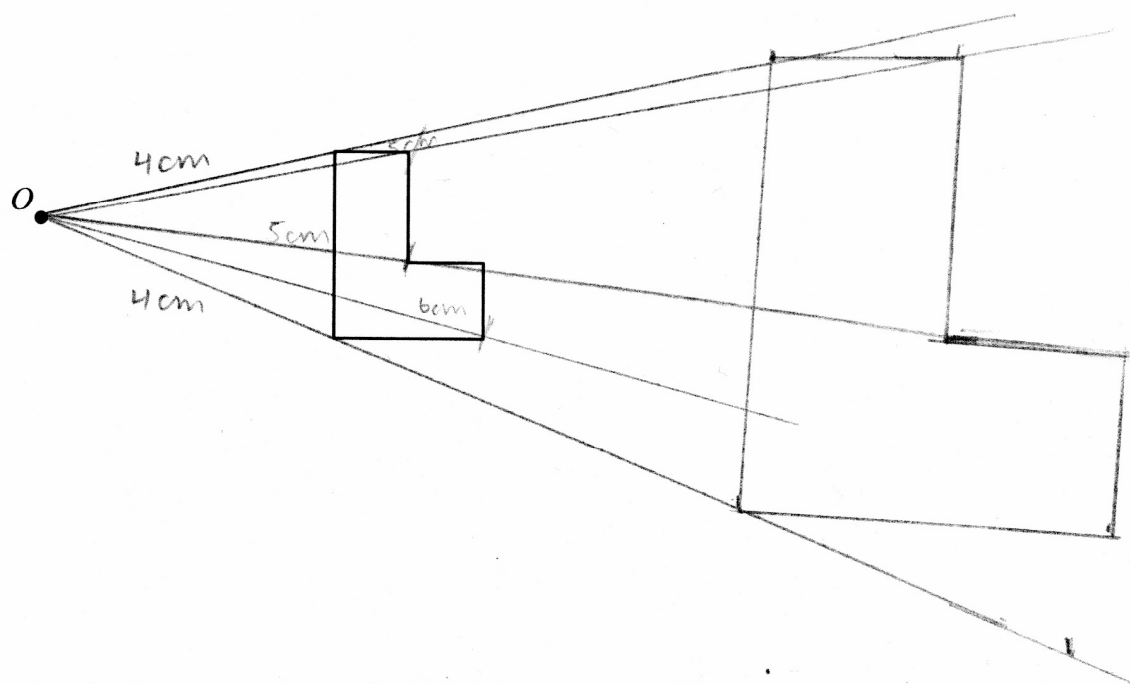
Many candidates were not able to demonstrate understanding of the relationship between the image area, the original area, and the scale factor.

Candidate Exemplars

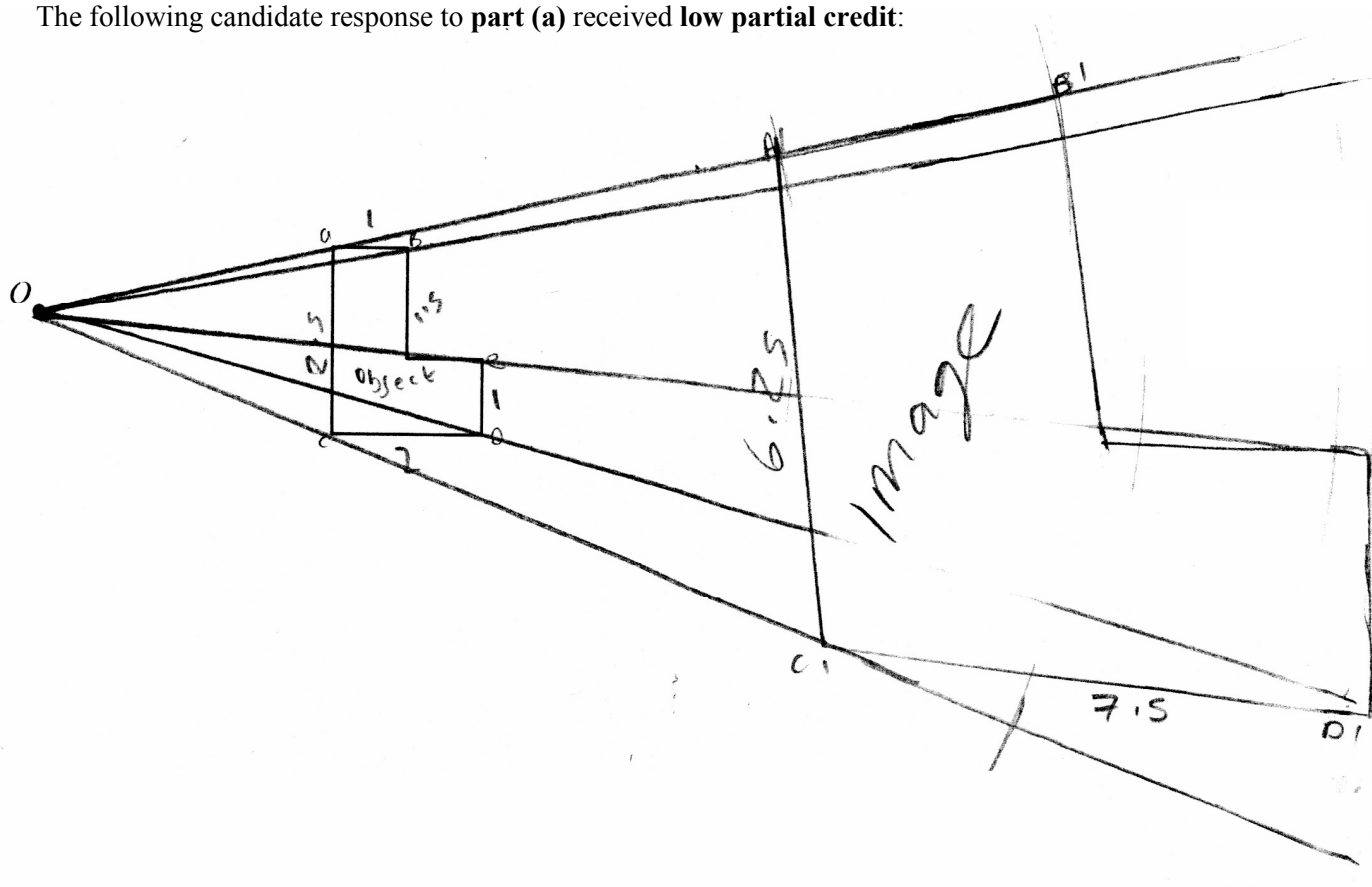
The following candidate response to **part (a)** received **high partial credit**:



The following candidate response to **part (a)** received **high partial credit**:



The following candidate response to **part (a)** received **low partial credit**:



The following candidate response to **part (b)** received **low partial credit**:

$$\begin{aligned} \text{factor } 2.5^2 &= 6.25 \\ \text{image area} &= 6.25 \text{ cm} \end{aligned}$$

The following candidate response to **part (b)** received **no credit**:

$$\text{area} = 2.5 \times 3.5 = 8.75 \text{ cm}^2$$

Other points to note

The marking scheme as applied did not give any explicit direction to examiners regarding the tolerance to apply in accepting a construction as accurate and it transpired that the standards applied by examiners in this respect were not entirely consistent. In the future, examiners will be given a specific tolerance and/or template to apply.

Candidates and teachers should note that, for full credit, a high degree of accuracy may be required – a tolerance in the region 2 or 3 mm may be anticipated.

Question 4**(25 marks)****Content area:** Strand 1 – Probability**Main assessment objective(s):**

1. Demonstrate knowledge of terminology and facts (*viz.* probability as long-run relative frequency).
2. Execute routine procedure within a mathematical context. (*viz.* systematic listing of outcomes; use of result relating to equally likely outcomes)
3. Demonstrate understanding of concepts (*viz.* proper interpretation of statements related to chance.)

Question & model solution:

The 2006 census indicates that the number of males living in Ireland is about the same as the number of females.

- (a) If a person is selected at random, write down the probability that the person is male.

Answer: $\frac{1}{2}$

- (b) Four people are chosen at random. By writing out a set of equally likely outcomes, or otherwise, complete the table of probabilities below.

four males	three males; one female	two males; two females	one male; three females	four females
$\frac{1}{16}$	$\frac{4}{16} = \frac{1}{4}$	$\frac{6}{16} = \frac{3}{8}$	$\frac{4}{16} = \frac{1}{4}$	$\frac{1}{16}$

Show your work.

MMMM	FMMM	FMMF	FFMF	(or tree diagram)
MMMF	MMFF	FMFM	FMFF	
MMFM	MFMF	FFMM	MFFF	
MFMM	MFFM	FFFM	FFFF	

- (c) A person states the following: “If you pick four people at random, it’s more likely than not that you’ll get two males and two females.”

Is this statement correct? Justify your answer using the answer(s) to part (b).

Answer: No

Justification:

The probability that it will happen ($\frac{3}{8}$) is less than the probability that it won't happen, so it's not "more likely than not".

Marking scheme & related notes

Question 4

(a)	Scale 5B	
	<i>Full credit</i>	Correct answer written as fraction, decimal or percentage
	<i>Partial credit</i>	Answer not in form of probability [0-1] e.g. 50:50, “evens”, “1 to 1”
(b)	Scale 10C	
	<i>Full credit</i>	Work + table complete
	<i>High partial credit</i>	List of possibilities written out correctly – no table, or error(s) in table
		Tree diagram correct – no table, or error(s) in table
		Correct table – incomplete or no supporting work
	<i>Low partial credit</i>	List of entries incomplete or with errors
		Tree diagram incomplete or with errors
		At least one correct table entry
	<i>No credit</i>	Nothing correct in table or work
(c)	Scale 10B	
	<i>Full credit</i>	Correct answer with correct justification
	<i>Partial credit</i>	Correct answer with incorrect or omitted justification
	<i>No credit</i>	Incorrect answer (from candidate’s value of probability of two males & two females). Note: no credit available in part (c) if candidate has no value for two males and two females in part (b), or if they have scored no marks in part (b).
	Note: correct justification requires explicit reference to one or other of the following:	
	- the probability of the event being less than the probability of its complement	
	- the probability of the event being less than one half	

Comment on answering

Part (a) was well answered, although many gave the answer in a form other than a probability value (frequently 50:50).

Very few candidates were able to systematically list the possible outcomes in part (b). Accordingly, correct answers in the table were rare. In light of this, it may be necessary to give more structure and guidance in questions like this until such time as this important skill is more embedded. The finalised sample paper will be amended to reflect this.

Part (c) was not well answered even by candidates who had given correct or substantially correct work in part (b). They were generally not able to see and/or articulate the connection between the everyday phrase “more likely than not” and the relevant probability values. Frequently, the responses indicated that the candidates confused “more likely than not” with “more likely than any other single combination”.

Candidate Exemplars

The following candidate response to **part (b)** received **high partial credit**:

four males	three males; one female	two males; two females	one male; three females	four females
$\frac{1}{16}$	$\frac{4}{16}$	$\frac{4}{16}$	$\frac{4}{16}$	$\frac{1}{16}$

Show your work.

male, male, male, male / female, female, female, female / mmmf / mmff /
mfff / fmmm / ffmm / fffm / mffm / mfmf /
fmmf / fmfm / fmff / ffmf / mfmf / mmfm

Comment: the candidate's list of outcomes is fully correct; (s)he has then miscounted the number of outcomes corresponding to "two males; two females".

The following candidate response to **part (c)** received **no credit**:

Answer: It is correct

Justification:

The above table shows that the possibility of getting 2 males and 2 females is $\frac{6}{16}$ which is higher than any other combination.

Comment: the value $\frac{6}{16}$ for the probability was the value in the candidate's table; the fact that this is not the correct value is not relevant when evaluating this response. The relevant point is that the candidate has clearly not correctly analysed the meaning of the phrase "more likely than not".

The following candidate response to **part (c)** received **full credit**, following a considerable discussion:

Answer: No

Justification:

As the probability of getting two males and two females is only $\frac{1}{4}$ (25%) which is not a high chance as it's not even half.

Comment: here again, the value $\frac{1}{4}$ was the value in the candidate's table, so that matter is not relevant to evaluating this response. There were considerable reservations regarding whether this candidate was demonstrating a correct understanding or was simply considering whether the event had "a high probability" or "a low probability". In the end, the reference to the probability being less than a half was considered sufficient in the circumstances.

Question 5

(25 marks)

Content area: Strand 2 – Trigonometry

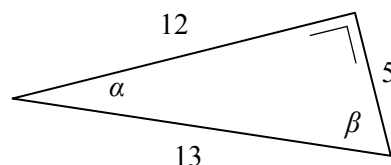
Main assessment objective(s):

1. Demonstrate knowledge of terminology and definitions (*viz.* trigonometric ratios).
2. Execute routine procedure within a mathematical context. (*viz.* solve two-step problems of a familiar type involving triangles)

Question & model solution:

- (a) The lengths of the sides of a right-angled triangle are shown in the diagram, and the angles are labelled.

Complete the tables below, writing the answers as fractions.



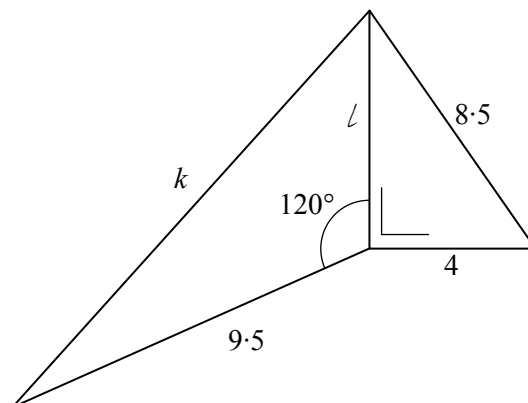
$\sin \alpha$	$\cos \alpha$	$\tan \alpha$
$\frac{5}{13}$	$\frac{12}{13}$	$\frac{5}{12}$

$\sin \beta$	$\cos \beta$	$\tan \beta$
$\frac{12}{13}$	$\frac{5}{13}$	$\frac{12}{5}$

- (b) Find the length k in the diagram shown.

Give your answer correct to one decimal place.

$$\begin{aligned}
 l^2 &= 8.5^2 - 4^2 \\
 &= 56.25 \\
 l &= \sqrt{56.25} \\
 &= 7.5
 \end{aligned}$$



$$\begin{aligned}
 k^2 &= l^2 + (9.5)^2 - 2l(9.5)\cos 120^\circ \\
 &= (7.5)^2 + (9.5)^2 - 2(7.5)(9.5)(-0.5) \\
 &= 217.75 \\
 k &\approx 14.8
 \end{aligned}$$

Marking scheme & related notes

Question 5

- (a) Scale 5B
- | | |
|-----------------------|-----------------------------|
| <i>Fully correct</i> | All entries correct |
| <i>Partial credit</i> | four correct entries |
| <i>No credit</i> | less than 4 correct entries |
- (b) Step 1 (find third side of right-angled triangle): scale 5B
- | | |
|-----------------------|-------------------------------------|
| <i>Partial credit</i> | Correct method; error in completion |
| <i>No credit</i> | Incorrect method |
- Step 2 (find k): scale 15C
- | | |
|----------------------------|----------------------------------|
| <i>Full credit</i> | k correct to one decimal place |
| <i>High partial credit</i> | k^2 found correctly |
| <i>Low partial credit</i> | Fills in cosine rule correctly |
| <i>No credit</i> | Assumes a right angled triangle |

Note Correct method completed with **one** error: award high partial credit, **provided the answer is reasonable.**

Comment on answering

Part (a) was quite well answered, although a higher standard might have been expected in light of the availability of the relevant information in the new *Formulae and Tables* booklet.

Part (b) was well answered. This content area is similar to that on the previous syllabus, and this style of questioning is familiar.

Other points to note

There was some comment on the use of the Greek letters α and β in part (a), which some believed would not be familiar to students at this level. Teachers should note that this notation is quite standard and is specified in the geometry appendix to the syllabus. This notation will continue to be used where relevant on examination papers. Upper case Roman letters may also be used to refer to angles or their measures.

In light of the fact that a full question in Section B was devoted to applications of trigonometry, this question will be replaced by one relating to co-ordinate geometry when the sample paper is finalised. This does not indicate any difficulty with the question itself, but is instead intended to reflect more closely the relative time that teachers indicated they have spent dealing with these two subtopics.

Question 6

(25 marks)

Content area: Strand 2 – Co-ordinate geometry

Main assessment objective(s):

1. Demonstrate understanding of concepts (*viz.* slope of a line).
2. Demonstrate knowledge of results (*viz.* knowledge of forms of equation of circle).
3. Execute routine procedures (*viz.* apply knowledge to simple geometric problem)

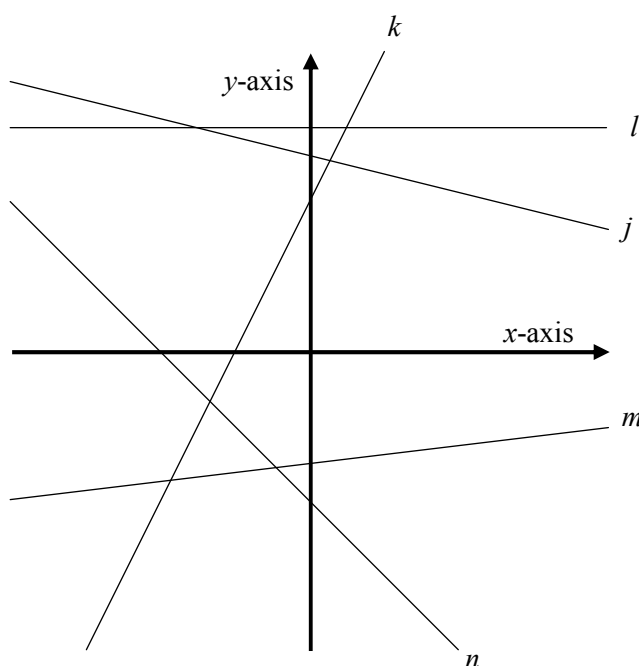
Question & model solution:

- (a) Five lines j , k , l , m , and n in the co-ordinate plane are shown in the diagram.

The slopes of the five lines are in the table below.

Complete the table, matching the lines to their slopes.

slope	line
2	k
$\frac{1}{8}$	m
0	l
$-\frac{1}{4}$	j
-1	n



- (b) The diagram shows four circles of equal radius. The circles are touching as indicated.

The equation of c_1 is $x^2 + y^2 = 9$.

- (i) Write down the radius of c_1 .

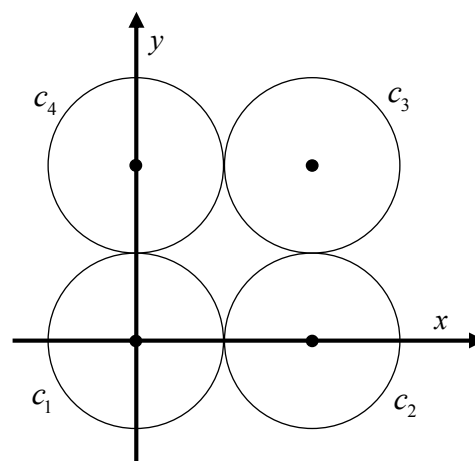
Answer: 3

- (ii) Write down the co-ordinates of the centre of c_3 .

Answer: (6, 6)

- (ii) Write down the equation of c_3 .

Answer: $(x-6)^2 + (y-6)^2 = 9$



Marking scheme & related notes

Question 6

- (a) Scale 10C
- | | |
|----------------------------|---|
| <i>High partial credit</i> | Correctly identifies the sign of each slope (swaps k and m and/or swaps j and n) |
| <i>Low partial credit</i> | Identifies that slope of l is 0 <i>or</i>
Identifies that both k and m have positive slopes <i>or</i>
Identifies that both j and n have negative slopes |
| <i>No credit</i> | Any other response |
- (b) (i) Scale 5B
- | | |
|-----------------------|---|
| <i>Partial credit</i> | Stops at $r = \sqrt{9}$ (or finishes incorrectly) |
|-----------------------|---|
- (ii) Scale 5A
Note: Accept answer consistent with part (i)
- (iii) Scale 5A
Note: Accept answer consistent with parts (i) and (ii); accept 3^2 on right-hand side.

Comment on answering

Answering was not as good as would have been anticipated. Whereas the format of part (a) may not have been familiar, the question should not have posed any difficulty for a candidate with an understanding of the concept of slope. The level of performance may be taken to indicate that concentration has remained on practising routine procedures rather than on the development of conceptual understanding.

Part (b) was of a more familiar type and yet also not as well answered as would be expected. The ability to apply the given information to solve the simple problem in part (ii) was not widespread. Many candidates offered an answer to part (iii) that was in the form of the equation of a line rather than a circle, indicating a lack of understanding of the material.

Other points to note

It may be noted that the marking scheme for part (a) was not based on how many of the particular cells were filled correctly. It was instead based on the level of understanding that is evident when the answer as a whole is considered. This minimises the effects of guessing and concentrates on rewarding evidence of achievement of the target skill.

Question 7

(40 marks)

Content area: Strand 1 – Statistics

Main assessment objective(s):

1. Present and read information in graphical form [parts (a)(i), (a)(ii), (b)(i)]
2. Interpret solutions of routine procedures in context [parts (b)(ii) & (v)].
3. Demonstrate understanding of concepts, connections, conditions and implications [parts (a)(ii), (a)(iii), (b)(iii)]

Question & model solution:

- (a) In a particular Leaving Certificate class, many of the students had had the experience of relatives commenting about how tall they were. They decided to investigate their heights. The height of each student was measured, in centimetres, and the results were as follows:

173	167	180	168	180	175
171	161	164	187	176	160
170	171	167	178	174	149
157	161	176	166	167	172

- (i) Construct a stemplot of the above data.

14 9	OR	14 9
15		15 7
15 7		16 0 1 1 4 6 7 7 7 8
16 0 1 1 4		17 0 1 1 2 3 4 5 6 6 8
16 6 7 7 7 8		18 0 0 7
17 0 1 1 2 3 4		
17 5 6 6 8		
18 0 0		
18 7		

Key: 14|9 = 149 cm

- (ii) Describe the distribution of the data, by making **one** statement about **each** of the three characteristics indicated below.

shape of distribution: (Roughly) symmetric / bell-shaped /
not skewed / mound-shaped

location of data (central tendency): average is about 170 cm /
mostly around the low 170's / etc

spread of data (dispersion): Most are within 10 cm of the average /
inter-quartile range is about 10 cm / etc.

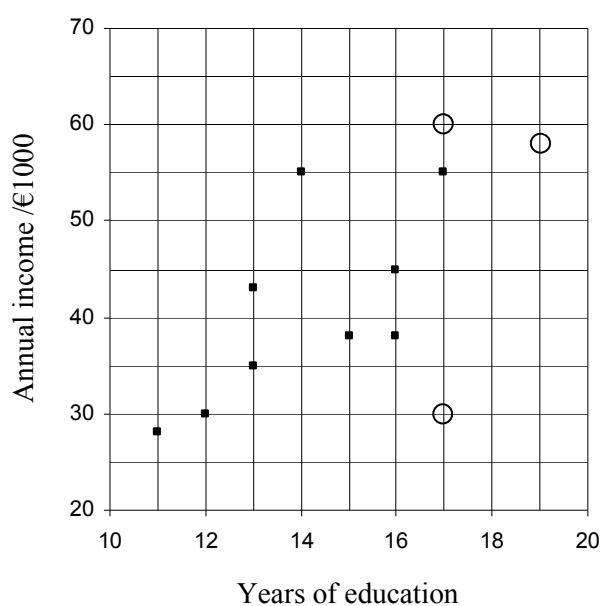
- (iii) This investigation arose from comments that these students were unusually tall. State **one** additional piece of information that you would need in order to decide whether that is true?

What is the usual or average height for students of this age?

(alternatives acceptable – see scheme below)

- (b) An economics student wants to find out whether the length of time people spend in education affects the income they earn. The student carries out a small study. Twelve adults are asked to state their annual income and the number of years they spent in full-time education. The data are given in the table below, and a partially completed scatter plot is given.

Years of education	Income /€1,000
11	28
12	30
13	35
13	43
14	55
15	38
16	45
16	38
17	55
17	60
17	30
19	58



- (i) The last three rows of data have not been included on the scatter plot. Insert them now.

- (ii) What can you conclude from the scatter plot?

It looks as though people with more education tend to have a higher annual income. (etc.)

- (iii) The student collected the data using a telephone survey. Numbers were randomly chosen from the Dublin area telephone directory. The calls were made in the evenings, between 7 and 9 pm. If there was no answer, or if the person who answered did not agree to participate, then another number was chosen at random.

Give **one** possible problem regarding the sample and how it was collected that might make the results of the investigation unreliable. State clearly why the issue you mention could cause a problem.

The sample is very small. Bigger samples give more reliable results.

OR People in Dublin might not be typical of the population as a whole.

(etc. – other potential problems exist)

Marking scheme & related notes

Question 7

- (a) (i) Scale 5B
Full credit Correct plot with proper vertical alignment of digits
 (Leaf values do not need to be ordered; stem values may be in ascending or descending order)
Partial credit Incomplete plot, with at least half of the data plotted
 Plot with error in vertical scale
No credit Incorrect diagram, (e.g. Bar chart)
- (ii) Scale 10C
High partial credit Two satisfactory statements
Low Partial credit One satisfactory statement
- (iii) Scale 5A
Full credit Relevant information, e.g. How old are they? Are they boys or girls?
- (b) (i) Scale 5B
Full credit All three points correctly plotted
Partial credit 1 or 2 points plotted correctly
 (Note: to count as correct, the vertical position of the last point must clearly be between 55 and 60, and must not be closer to 55 than to 60.)
- (ii) Scale 10B
Full credit Correct statement, properly contextualized (explicitly refers to meaning of variables) E.g. “Those with more education tend to have higher incomes”, “There is a moderate positive association between income and education.”
Partial credit Conclusion not contextualised, or not fully correct (e.g. overly deterministic statement)
- (iii) Scale 5B
Full credit Problem correctly identified and properly developed
Partial credit Problem correctly identified but not properly developed.
No credit No correct problem identified
 (Note, for example: “They shouldn’t have used the Dublin phone book only.” is a problem correctly identified but not properly developed. Stating “People living in Dublin may not be representative of the population as a whole” is properly developed. That is, the latter statement has clarified *why* the use of the Dublin phone book might be an issue.)

Comment on answering

Part (a)(i) was very well answered by most candidates. The second of the two versions in the model solution above was by far the more common.

Part (a)(ii) required verbal answers rather than mathematical work. As with other such questions on the paper, for most candidates this proved difficult. Often, very similar or identical answers were offered to describe the three aspects of the data. This presented a difficulty for marking, although not an insurmountable one. A statement such as “the data is mostly between 160 and 170” could be considered an acceptable statement of location or of spread, depending on what statements were offered for the alternative aspects.

It was clear that many candidates had difficulty distinguishing between these three distinct aspects of a data distribution, and were perhaps not used to regular discussion of such matters in class. It is critical to the proper implementation of the syllabus that candidates engage in such description and discussion both verbally and in writing, and that they become familiar with the proper vocabulary associated with the topic.

Part (a)(iii) was generally well answered but the relevance of some of the answers was tentative.

In part (b)(i), the three data points were usually plotted correctly. Part (ii) tested the candidates’ capacity to interpret the information presented in order to draw a conclusion. This again posed difficulties for most candidates. Most candidates failed to display an understanding of the critical underlying statistical ideas.

Part (iii) was answered much better than part (ii) and many candidates clearly appreciated the need to avoid bias when collecting data. This part was handled significantly better than other text-intensive questions on the paper. In exploring the possible reasons for this, it is worth observing that this part did not require connections to be made between the mathematics and the context. That is, the potential difficulties with the sampling process described are identifiable without referring to the actual calculations or data presented. Other text-based answers tended to require interpretation of some mathematics that the candidates had themselves done. This may indicate that candidates are relatively comfortable describing and understanding “real world” issues and concepts, but not as yet able to meaningfully relate these to the mathematics they are engaged in.

Candidate Exemplars

The following candidate response to **part (a)(ii)** received **full credit**:

shape of distribution: low at two extremes – strong around the middle

location of data (central tendency): most students were around the 160 cms
§ 170 cms – quite tall

spread of data (dispersion): Not very dispersed – most around one height. – quite tall on average.

The following candidate response to **part (a)(ii)** received **high partial credit**:

shape of distribution: most of the students are between 160 cm and 178 cm

location of data (central tendency): the average height is 170 cm

spread of data (dispersion): The main spread of the data is between 160 cm and 178 cm

Comment: the statement about the **shape** is not satisfactory, (as distributions of virtually any shape could still lie in this interval); the other two statements are satisfactory.

The following candidate response to **part (a)(ii)** received **high partial credit**:

shape of distribution: a  -shaped distribution

location of data (central tendency): will be at 170 cm

spread of data (dispersion): There will be few points at the start and more in the centre and towards the end.

Comment: the statement about the **spread** is not satisfactory, (as this is a description of the shape and could be true irrespective of the size of the spread); the other two statements are satisfactory.

The following candidate response to **part (a)(ii)** received **low partial credit**:

shape of distribution: most common height is 167 cm

location of data (central tendency): most seem to be around the height.

spread of data (dispersion): smallest is 149 cm and the tallest is 187 cm.

Comment: the statement of the range is a satisfactory statement related to the spread of the data. The other two statements are not satisfactory.

The following candidate response to **part (a)(iii)** received **full credit**:

- (iii)** This investigation arose from comments that these students were unusually tall. State **one** additional piece of information that you would need in order to decide whether that is true?

You would need a control, test it with other students to see if they were tall or small. This will show you if it is true or untrue.

Comment: this is a different kind of response from that expected but was nonetheless regarded as satisfactory. The natural comparison for a statistician to seek here is to compare the sample with a suitable known population. However, since no indication was given that relevant population statistics could be found, the suggestion of comparing this sample to a control group is reasonable.

The following candidate response to **part (b)(ii)** received **full credit**:

- (ii)** What can you conclude from the scatter plot?

There is a weak correlation between the number of years in education and the adults' annual income

The following candidate response to **part (b)(ii)** received **full credit**:

- (ii)** What can you conclude from the scatter plot?

In general, the longer a person stays at school the income they earn will be increased.

Comment: contrast with the partial-credit response below; in this case, the candidate's use of the term "In general" is important. Note that there is a suggestion of causality in the statement, which perhaps should not go unqualified. However, at this level, and given the context, this was viewed as acceptable.

The following candidate response to **part (b)(ii)** received **partial credit**:

(ii) What can you conclude from the scatter plot?

I can conclude that the more years spent in education the higher the annual income.

Comment: contrast with the full-credit response above; in this case, the candidate is stating that there is a deterministic relationship between the variables. (This would be an acceptable statement in the case of perfect correlation.) To render this response acceptable here, the candidate would need to use a phrase such as “in general” “tends to” or “is likely to”.

The following candidate response to **part (b)(ii)** received **partial credit**:

(ii) What can you conclude from the scatter plot?

That it has a strong positive correlation.

Comment: the statement is not properly contextualised, and the reference to “strong” is dubious. Full credit would have been awarded for: “There is a positive correlation between education and income”.

Note: For candidate exemplars relating to part (b)(iii), refer to the exemplars related to the corresponding question on the Higher Level paper: Question 7, part (viii). (See page 33.)

Question 8

(40 marks)

Content area: Strand 2 – Trigonometry

Main assessment objective(s):

1. In a non-mathematical context, apply routine procedures, interpreting the solutions in the original context.
2. In a non-mathematical context, apply understanding of concepts and connections, interpreting solutions, conditions and implications in the original context.

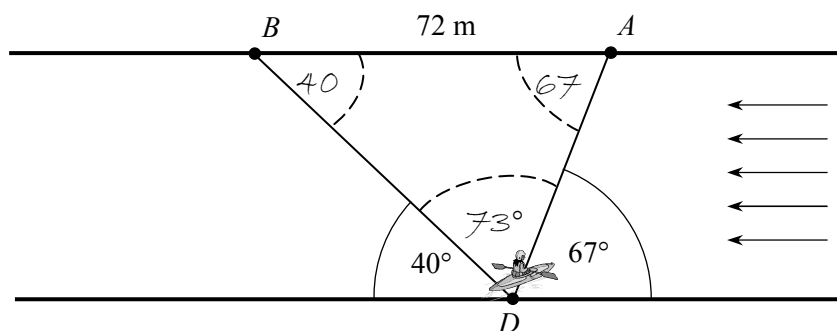
Question & model solution:

Deirdre has been kayaking on a river and has arrived at a point on the southern riverbank. However, she wants to get out on the northern side. There are only two possible landing points that she can see. One is slightly upstream from where she is now, and one is farther downstream. Because of the current, Deirdre can paddle faster towards the downstream landing point than the upstream one.



The situation is shown in the diagram below. The banks of the river are parallel. Deirdre's position is marked D , the upstream landing point is A , and the downstream landing point is B . The angles from D to A and B are as shown. The distance from B to A is 72 metres.

If she travels in a straight line to A , Deirdre can go at 0.9 m/s and if she travels in a straight line to B she can go at 3.2 m/s.



- (a) Find the distances from D to A and from D to B .

Distance from D to A :

$$\frac{|DA|}{\sin 40^\circ} = \frac{72}{\sin 73^\circ} \Rightarrow |DA| = \frac{72 \sin 40^\circ}{\sin 73^\circ}$$

$$|DA| \approx 48.4 \text{ m}$$

Distance from D to B :

$$\frac{|DB|}{\sin 67^\circ} = \frac{72}{\sin 73^\circ} \Rightarrow |DB| = \frac{72 \sin 67^\circ}{\sin 73^\circ}$$

$$|DB| \approx 69.3 \text{ m}$$

- (b) Find the time it will take to cross by each route.

Time from D to A :

$$\frac{48.4}{0.9} = 53.8 \text{ seconds}$$

Time from D to B :

$$\frac{69.3}{3.2} = 21.7 \text{ seconds}$$

- (c) Deirdre is late and wants to get home as fast as possible. Give one possible reason why she might **not** choose the faster of the two routes across the river.

It could be harder to get out at B than A .

She might have left her car at A .

There might be danger on the way to B .

(etc.)

Marking scheme & related notes

Question 8

Angles in triangle: Scale 5B

Accept angles written on the diagram or implied by later work.

Full credit All three interior angles correct

Partial credit One required angle correct

No credit No angle correctly found

(a) $|DA|$ Scale 10C

Full credit Correct answer, including units.

High partial credit Full solution with **one** error, provided transposing is correct

Low partial credit Correctly sets up sine rule

No credit Treats as right angled triangle

Error(s) in setting up sine rule

$|DB|$ Scale 10C

Full credit Correct answer, including units.

High partial credit Full solution with **one** error, provided transposing is correct

Low partial credit Correctly sets up sine rule

No credit Treats as right angled triangle

Error(s) in setting up sine rule

(b) Scale 10C

Full credit Both values correct, with units

High partial credit One value correct, with units

Both values correct, without units

Low Partial credit Sets up correctly

No credit Incorrect relationship between speed, distance, and time.

(c) Scale 5A

Full credit Any plausible reason

Comment on answering

The mathematical content of this question was very similar to material that is, in general, handled reasonably well by candidates studying the previous syllabus. However, the contextualisation and presentation were different, reflecting the changed emphasis in Project Maths. This caused some difficulties for many candidates.

Parts (a) and (b) were answered reasonably well by those who attempted them, although candidates frequently omitted units in their answers. In part (b), some candidates were not able to state or apply the correct relationship between speed, distance and time.

Part (c) was generally well answered with many achieving full marks for reasonable answers. However, some answers did indicate that the context of the question was not fully understood, and in particular the effect of the current.

Candidate Exemplars

The following candidate response to **part (c)** received **no credit**:

- (c) Deirdre is late and wants to get home as fast as possible. Give one possible reason why she might **not** choose the faster of the two routes across the river.

She will be going against the current on the journey back so will take longer

The following candidate response to **part (c)** received **no credit**:

- (c) Deirdre is late and wants to get home as fast as possible. Give one possible reason why she might **not** choose the faster of the two routes across the river.

Because the faster route is uphill and will take more time, but the slower route will be easier and will be done a lot quicker.

Other points to note

Concern was expressed by some commentators regarding the amount of text to be read by candidates at the beginning of this question. This has not typically been a feature of questions on this topic on the previous syllabus. In the past, information that was not strictly necessary to the calculations involved was eliminated. Furthermore, the information relevant to each part of a question was generally presented only as needed, whereas in this case, all of the relevant information was provided at the start of the question. It should be noted that this change is critical to achieving the aims of *Project Maths*. The candidates' ability to handle the mathematics involved is only one aspect of what is to be tested. Equally important is their capacity to extract relevant mathematical information from situations and problems presented in a non-mathematical form, including making decisions about which information is relevant to which task.

It may be noted that the first paragraph of the question does not contain any “mathematisation” of the situation – there is no labelling of points, etc. The second paragraph is where the mathematical model of the situation is set up and the relevant numerical data given. This sequencing is deliberate and is intended to reflect the way in which contextualised problems would be encountered during the learning process. In class, one might expect that only the first paragraph would be presented to students. They might be expected to create the mathematical model for themselves and discuss and identify what information will be necessary in order to solve the problem, before such information would be given to them. Whereas a written examination cannot follow such a procedure, it can nonetheless be structured in the same sequence, so that the candidates who are most familiar with the presentation are the ones who have engaged with the syllabus in the appropriate way. Problems will not necessarily always be presented in this way, but it should be noted that this will be regarded as desirable, where feasible.

In finalising the sample papers, there will be some reworking of the text of the question to improve its readability.

Question 9A

Probability and Statistics

(45 marks)

Content area: Strand 1 – Probability and Statistics

Main assessment objective(s):

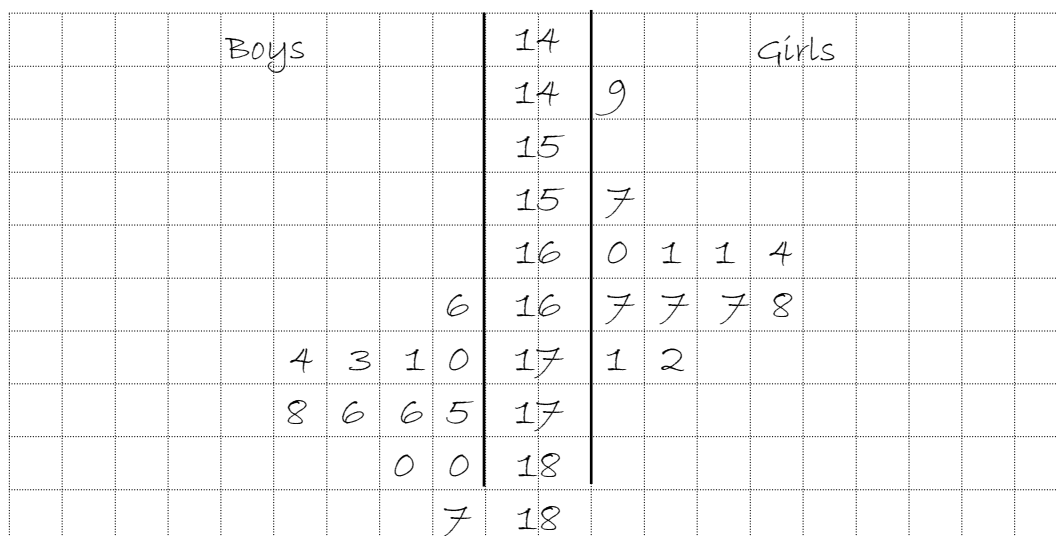
1. Present and read information in graphical form.
2. In a non-mathematical context, apply routine procedures.
3. In a non-mathematical context, apply understanding of concepts and connections, interpreting solutions, conditions and implications in the original context.

Question & model solution

The students described in **Question 7(a)** decide to look at the heights of the boys and girls separately. The heights of the boys and the girls in the class are given below:

Boys			Girls		
173	180	174	167	161	160
175	178	176	157	164	172
180	171	170	168	149	161
187	176	166	167	167	171

- (a) Construct a back-to-back stemplot of the above data.



- (b) State **one difference** and **one similarity** between the distributions of the heights of the boys and the girls in the class.

Difference: The boys are taller on average

Similarity: The spread of the data is about the same for each set

- (c) Assume that this class can be treated as a random sample of Leaving Certificate students. Perform a *Tukey Quick Test* on the data, stating clearly what can be concluded.

lower tail (girls) = 6

upper tail (boys) = 9

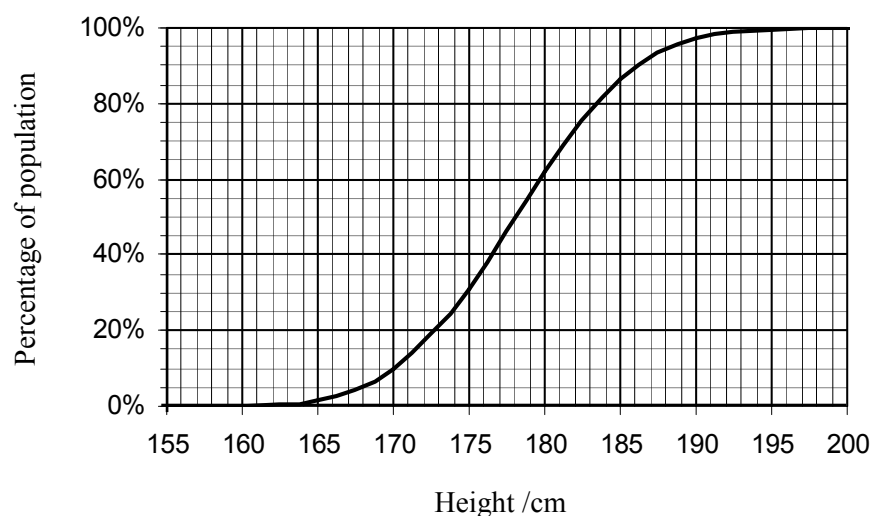
Tail count = 15, which is (much) bigger than 7. So this is (highly)

significant. That is, it is very unlikely that this would happen by

chance. So we conclude that Leaving Certificate boys are indeed taller in

general than Leaving Certificate girls.

- (d) The following cumulative distribution curve (ogive) represents the current heights of Irish males born in 1991.



From the curve, find the median height and the quartiles.

Median: 178 cm

Lower quartile: 173.8 cm

Upper quartile: 182.3 cm

- (e) Compare the heights of the boys in the class to the population above. State whether they are similar or different; state whether that is surprising or not, and say why.

They are similar. This is not surprising. That is about the year (1991)

that you would expect Leaving Certificate students to be born (or may be

1992) and most men would be fully grown by 17 or 18 years old.

OR:

They are a bit different. This is not surprising, because any sample you

take can be a little bit different from the population.

(etc.)

Marking scheme & related notes

Question 9A

- | | | |
|-----|---|---|
| (a) | Scale 10C | |
| | <i>Full credit</i> | Correct plot with proper vertical alignment of digits
(Leaf values do not need to be ordered; stem values may be in ascending or descending order; stem categories need not be in multiples of 5; multiples of 10 are acceptable.) |
| | <i>High partial credit</i> | Incomplete plot with one or two points missing or incorrect |
| | <i>Low Partial credit</i> | Incomplete plot with at least 3 points on each side |
| | | Plot with error in vertical scale/alignment |
| | <i>No credit</i> | Incorrect diagram (e.g. bar chart) |
| (b) | Scale 5B | |
| | <i>Full credit</i> | Two correct statements |
| | <i>Partial credit</i> | One correct statement |
| | <i>No credit</i> | No correct statement |
| (c) | Scale 10C | |
| | <i>Full credit</i> | Correct solution with properly contextualised conclusion (e.g. “boys are taller than girls”, “LC boys are taller than LC girls”.) |
| | <i>High partial credit</i> | Correct numerical solution without properly contextualised conclusion (e.g. stops at “this is significant” or concludes only about the sample and not the population: “the boys in this class are taller than the girls”) |
| | | Solution with error in calculating tail count, with correct conclusion re significance – accept “bigger than 7”. |
| | <i>Low Partial credit</i> | Correctly calculates upper and/or lower tail |
| | | Calculates tail count correctly |
| | <i>No credit</i> | Statement from <i>Formulae and Tables</i> book. |
| (d) | Scale 10C | |
| | Note: for median, accept 177, 178, 179 cm. For lower quartile accept 173, 174, 175 cm. For upper quartile accept 181, 182, 183 cm.) | |
| | <i>Full credit</i> | Three values correct, including units |
| | <i>High partial credit</i> | Three values correct, without including units |
| | | Two values correct (with or without units) |
| | <i>Low Partial credit</i> | One value correct (with or without units) |
| (e) | Scale 10B | |
| | <i>Full credit</i> | Complete statements with proper justification for conclusion. |
| | <i>Partial credit</i> | Calculates mean or median for initial data (boys only) |
| | | For statements without explanation, accept, for partial credit: “Similar and not surprising”, “Different and surprising”, “Different and not surprising”. (Do not accept “Similar and surprising”.) |

Comment on answering

Part (a) was well answered. As with Question 7, the vast majority took 10 rather than 5 as the stem unit. This was entirely acceptable, but it may be noted that, in both questions, the features of the distributions are more readily visible with a stem unit of 5.

Part (b) was also comparatively well answered.

In part (c) many students were capable of performing the relevant counting and calculation for the hypothesis test. However, the majority appeared not to understand the nature of the conclusion that can be drawn. In particular, many candidates clearly indicated a belief that the significance test leads to a conclusion about the set of boys and girls in the class rather than the population in general. As there is little point in executing the mechanics of any hypothesis test in the absence of an understanding of its basic purpose, this is a matter that should be addressed.

Part (d) was quite well answered. It is worth noting that this material is as on the previous syllabus and is also on the Junior Certificate Higher Level course.

While many students scored a partial credit in part (e), it would appear that they did not understand how to use the available information to draw a reasonable conclusion. As with part (c) this may be taken to indicate a lack of appreciation of the relationship between samples and populations and the importance of this relationship in using statistics to make decisions. In light of this, the finalised version of this question will be reworked to give an increased focus to this aspect of the task.

Candidate Exemplars

The following candidate response to **part (b)** received **full credit**:

Difference: boys are taller

Similarity: They are clumped together

Comment: the second statement was interpreted as a reasonable effort at stating that the shape and/or spread of the two sets are similar.

The following candidate response to **part (b)** received **full credit**:

Difference: the boys are much taller with their 170's over
the girls 160's

Similarity: They are all seen to be average height. Again with the boys
170's and the girls 160's and not too small.

Comment: the second statement was interpreted as a reasonable effort at stating that each set is clustered to a similar extent around its mean.

The following candidate response to **part (b)** received **partial credit**:

Difference: On average the boys are taller

Similarity: One boy and one girl are the same size.

Comment: the statement of similarity, although a true statement, is not a reasonable response in the context of being asked for a similarity between the distributions.

The following candidate response to **part (b)** received **partial credit**:

Difference: the girls are more spread out in height

Similarity: Both have 8 people in one group (170-180 for boys, 160-170 for girls)

Comment: the statement about the *difference* is not reasonable.

The following candidate response to **part (c)** received **full credit**:

$$6+9=15 \quad 15>7$$

From these result I can accept that boys are usually taller than girls.

Comment: the details are not as fully described as one would wish. Nonetheless, the correct procedure has been applied, and the conclusion is contextualised and properly refers to the populations.

The following candidate response to **part (c)** received **high partial credit**:

9 boys are taller than the tallest girl, 6 girls are smaller than the smallest boy = $9+6=15$. This is a significant difference.

Comment: properly contextualised conclusion not drawn. Also, an explicit statement that 15 has been compared to the critical value of 7 would be preferable.

The following candidate response to **part (c)** received **high partial credit**:

Boys bigger than the biggest girl = 9

Girls smaller than the smallest boy = 6

$$9+6=17$$

Significant difference

Comment: the numerical slip would not be regarded as important unless the candidate was otherwise in line for full credit. Accordingly, this solution has similar merit to the preceding example.

The following candidate response to **part (c)** received **low partial credit**:

No of boys taller than the tallest girl = 9

No of girls smaller than the smallest boy = 6

$$9+6=13$$

13 is greater than 7

$\Rightarrow$ null hypothesis is accepted

Comment: the numerical slip would not be regarded as important in the context of the other problems with this response. The acceptance of the null hypothesis when it should be rejected is what brought this from high partial to low partial credit. (It is otherwise similar to the previous example.)

The following candidate response to **part (c)** received **low partial credit**:

No of numbers lower in A than the smallest B + number of numbers in A bigger than the biggest B

$$6+0$$

$$=6$$

$\therefore$ this is not a significant difference

Comment: the specification of the test statistic is incorrect, but the correct size of one tail has been found nonetheless.

The following candidate response to **part (e)** received **full credit**:

Born in 1991 would make them 18 yrs old @ median height of 179.
Median height of boys from sample is 174.5. These answers are quite similar. Perhaps the boys from the curve are a little older, possibly close to 19. the boys of the stemplot are Leaving Certificate and I reckon their age is about 16 or 17.

Comment: although the candidate has not explicitly stated whether them being “quite similar” is surprising or not, there is clearly a well thought out comparison being made, with possible reasons for the slight difference being suggested.

The following candidate response to **part (e)** received **full credit**:

The heights of the boys and those of the male population are similar because they both increase significantly between 170 and 190. This is unsurprising as the males born 1991 would be of LC age meaning that the boys in both graphs are of similar age and so, similar height.

The following candidate response to **part (e)** received **partial credit**:

it is similar as the majority of the boys heights is above 170 cm on both graphs/plots

Comment: the candidate has offered no view as to whether the similarity is surprising or not.

The following candidate response to **part (e)** received **partial credit**:

The boys on the ogive are a lot taller. It's surprising because 15% of the population is over 185 cm. Whereas in the class there is only one boy over 185 cm.

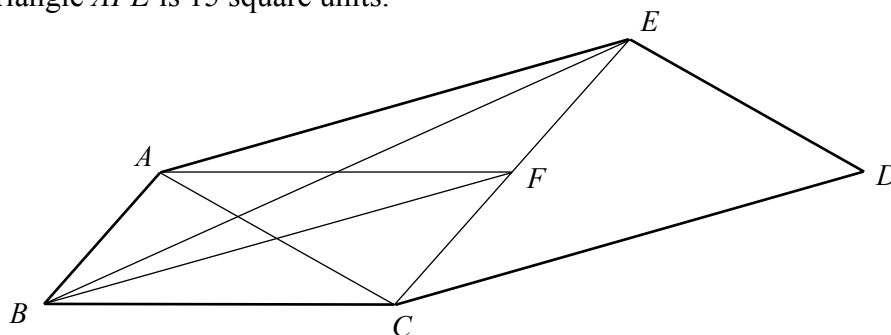
Comment: even though the candidate starts the second sentence with “It’s surprising because...”, they are actually just identifying a feature to support the statement that they are different, rather than giving a reason to be surprised.

Question 9B**Geometry and Trigonometry****(45 marks)****Content area:** Strand 2 – Geometry and Trigonometry**Main assessment objective(s):**

1. In a mathematical context, apply understanding of concepts and connections, including relevant conditions, implications, etc.
2. Within a mathematical context, execute routine procedures.
3. In a non-mathematical context, apply understanding of concepts and connections, interpreting solutions, conditions and implications in the original context.

Question & model solution

- (a) In the diagram below, $ABCF$, $ABFE$, and $ACDE$ are parallelograms. The area of triangle AFE is 15 square units.



- (i) State clearly why the area of triangle AFB must also be 15 square units.

The diagonal $[AF]$ bisects the area of the parallelogram $ABFE$

- (ii) Find the area of the whole figure $ABCDE$.
Show your work.

$$|\Delta AFE| = 15$$

$$|\Delta AFB| = 15$$

$$|\Delta FBC| = 15 \quad (\text{as } [BF] \text{ bisects } ABCF)$$

$$|\Delta ACF| = 15 \quad (\text{as } [AC] \text{ bisects } ABCF)$$

$$|\Delta ECD| = 30 \quad (\text{as } [EC] \text{ bisects } ACDE)$$

$$\text{Answer} = 30 + 15 + 15 + 15 = 75 \text{ square units}$$

- (iii) If the perpendicular distance from D to the line EC is 6, find $|AB|$.

Show your work.

$$\text{Area} = \frac{1}{2} \text{ base} \times \text{height}$$

$$30 = \frac{1}{2} |EC| \times 6$$

$$|EC| = 10$$

$$\therefore |EF| = 5 \text{ (diagonals bisect each other)}$$

$$\therefore |AB| = 5 \text{ (opposite sides of a parallelogram)}$$

- (b) Dónal is making a wooden pull-along toy. He has disks to use as wheels, but the centres are not marked on them. He needs to locate the exact centre of each wheel in order to drill holes for the axles.

He knows that there is a geometrical method for finding the centre of a circle.



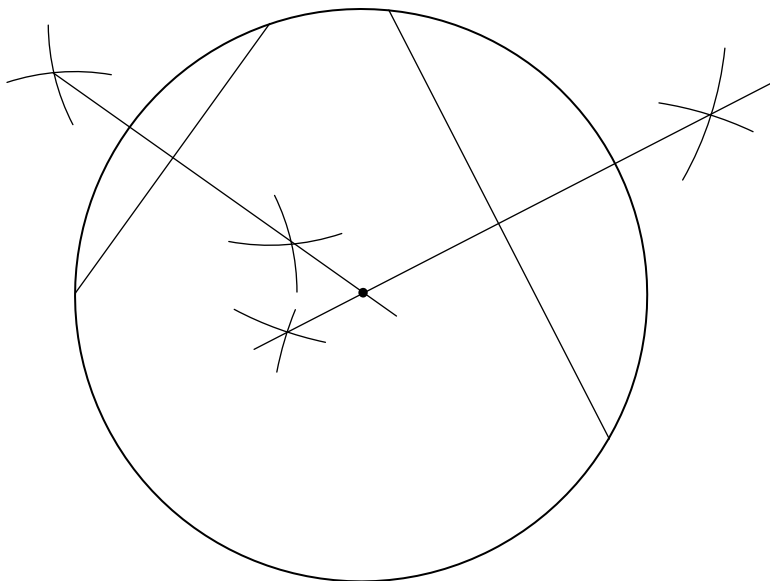
- (i) State a theorem from your geometry course that could be used to locate the centre of a circle with geometrical instruments.

The perpendicular bisector of a chord passes through the centre.

Or

The angle in a semi-circle is a right angle.

- (ii) Locate the centre of the circle below, by applying the theorem you mentioned above. Show your construction lines clearly.



Or:

Inscribe a right-angled triangle by constructing a chord and erecting a perpendicular chord from its endpoint. Then, either bisect the diameter (third side) or repeat the process to get another diameter.

(iii) Describe another way that Dónal could find the centres of the disks.

EG: Trace it onto a page. Fold it exactly in half to get a diameter. Fold it again to get another diameter. This gives the centre. Put it back on the wheel and mark through.

OR: Balance the wheel flat on the edge of a tilted board. When it balances, you know it's resting along a diameter. Mark this line. Then do this again and where they cross is the centre. ETC.

Marking scheme & related notes

Question 9B

(a) (i) Scale 5A

(ii) Scale 10C

Full credit

75 with work, (no units required)

High partial credit

Finds the area of all the triangles but fails to sum correctly (Accept written on the diagram.)

Low Partial credit

Correctly identifies the triangles forming the area. Finds the area of some of the other triangles forming the area (Accept written on the diagram.)

No credit

75 without work
 $AFE = 7.5 + 7.5$

(iii) Scale 10C

Full credit

$|AB| = 5$ with relevant work

High partial credit

Finds $|EF| = 5$ with relevant work

$|AB|$ shown on diagram

Low Partial credit

Finds $|EC|$ correctly

States $|AB| = |FC|$ or $|FC| = \frac{1}{2}|EC|$

(b) (i) Scale 5B

Full credit

Correct theorem stated (one or other of the above two).

Partial credit

Mentions chord or perpendicular bisector

A tangent is perpendicular to radius at point of contact.

Reference to finding circumcentre;

States relevant construction rather than theorem.

(ii)	Scale 10C	
	<i>Full credit</i>	Bisectors of two chords, with construction lines
	<i>High partial credit</i>	Bisectors of two chords without construction lines or bisectors not meeting Completed correct work with <i>tangent</i> theorem Fully correct work but not using theorem stated at part (i)
	<i>Low Partial credit</i>	Finds the perpendicular bisector of one chord Draws two chords
	<i>No credit</i>	Centre marked in with no construction lines One chord drawn
(iii)	5 marks scale B	
	<i>Partial credit</i>	Plausible method but not fully explained

Comment on answering

This question was not popular and candidates did not perform well on it. To those used to the old syllabus, this is perhaps not surprising, as geometry has been very unpopular for some time.

In part (a), candidates found it difficult to articulate the reason for equal areas. This may indicate that they were not used to discussing and writing about such matters. Correct answers to part (ii) were not uncommon, and were often shown on the diagram. Part (iii) was poorly answered, despite being of a relatively traditional style.

Part (b)(i) was not well done. The candidates were either not used to being asked to state theorems on the course, or were unable to identify what theorems might be helpful in the given context. Part (ii) was also poorly done in general. There were a number of exceptions, however, and many of these gave a correct construction despite not having been able to articulate the underlying theorem. These may be candidates who are studying *Design and Communication Graphics* for Leaving Certificate or had studied *Technical Graphics* for Junior Certificate. Some interesting and plausible suggestions were made for part (iii). However, in many cases the suggestions were not fully explained; they frequently involved “drawing a diameter” or “finding the widest part of the wheel” without saying how this could be achieved.

Candidate Exemplars

The following candidate response to **part (b)(iii)** received **full credit**:

fold a circle of paper the same size in 4 and open it up and mark out
where the centre is with the paper.

Comment: While greater clarity would be preferable, the candidate has nonetheless described a method that is close the first one given in the model solutions.

The following candidate response to **part (b)(iii)** received **partial credit**:

Measure the diameter of the circle with a ruler or vernier callipers, and then
half the length of the diameter to find the centre.

8. Detailed analysis – Foundation Level

Summary of the Foundation Level scheme

– mark allocations and scales applied

Section 0

Question 1: see below

Question 2: see below

Section A

Question 3

(a) 10C

(b)(i) 5A

(b)(ii) 10B

Question 4

(a) 5B

(b) 10C

(c) 5A

(d) 5B

Question 5

Construction: 15C

Description: 10B

Question 6

(a) 5B

(b) 5B

(c) 5B

(d) 10C

Section B

Question 7

(a) 10B

(b) 10B

(c) 10B

(d) 10A

(e) 10C

Question 8

(a) 15D

(b) 20D

(c) 15D

Section 0	Area & Volume (old syllabus)	100 marks
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Questions 1 & 2

(50 marks each)

These questions consisted of the corresponding questions from the Leaving Certificate Foundation Level examination of 2009. They were reformatted for inclusion in a completion-booklet examination.

These questions are not part of the focus of the trialling exercise. Examiners were given copies of the relevant extract from the 2009 marking scheme, which may be accessed on the SEC website. They marked these questions whenever they were encountered. However, no detailed information was collected, and no commentary or analysis is offered here.

In the real examination in 2010, these questions will be the same questions as for candidates in all other schools. They will be reformatted for inclusion in a completion-booklet examination. The same marking scheme will apply to candidates in the initial schools as will apply in all other schools.

Question 3

(25 marks)

Content area: Strand 1 – Probability & Statistics

Main assessment objective(s):

1. In a non-mathematical context, apply routine procedures.
2. In a non-mathematical context, apply understanding of concepts and connections.

Question & model solution

- (a) Fiachra is buying a new laptop computer.

The different choices of memory, screen type and colour are shown below

Memory	Screen Type	Colour
1 GB RAM	Regular screen	Black
3 GB RAM	Widescreen	Red
		White



All of the different combinations are possible. For example, Fiachra could order a white 3 GB laptop with regular screen.

How many different versions of the laptop are possible?

$$2 \times 2 \times 3 = 12 \text{ versions}$$

- (b) Séan's French teacher gives tests that are marked out of 10. Séan got the following results in five tests:

7, 5, 6, 10, 7

- (i) Find Séan's mean mark for the five tests.

Answer: 7

- (ii) Áine got the following results in the same five tests. She was absent for the fourth test.

8, 5, 7, -, 7

Is Áine better or worse than Séan at French? Give a reason for your answer.

Answer: Worse or Better

Reason: <u>because her average</u>	<u>because her average mark (6.75) on</u>
<u>mark is lower (6.75)</u>	<u>these four tests is better than Séan's</u>
	<u>average on these four, (6.25).</u>

Marking scheme & related notes

Question 3

(a) Scale 10C

High partial credit: Complete list of possible versions, without stating “12”.
States $2 \times 2 \times 3$, without stating “12”

Low partial credit: Partial list of possible versions (at least three different versions listed).
Gives 2×2 or 2×3

(b) (i) Scale 5A

(ii) Scale 10B

Full credit: Answer with satisfactory reason, such as those listed in the solutions.

Partial credit: Incomplete or unsatisfactory reason, but with some element of reasoning using the information given. e.g. “Better, because she did better in the first test”. “Worse, because she never got a 10 and Seán did.” “Worse, because his total is 35 and hers is only 27”.

No credit: Answer without reason, or with unsatisfactory reason, or one that is not soundly based on the information given. e.g. “Probably better, because girls are usually good at languages.”, “Worse, because if she’s absent more often she won’t learn as much”.

Comment on answering

This question was handled comparatively well by candidates. In part (b), although many candidates had difficulty finding the mean of the five scores in part (i), reasonable answers to part (ii) were more common than might have been expected.

Candidate Exemplars

The following candidate response to **part (b)(ii)** received **full credit**:

Answer: Yes

Reason: Áine always got either the same or higher than Seán apart
from when she was not in. Out of 50 she got 27. Out of 50 he got 35.
But Áine wasn't in when he got 10.

Comment: The “yes” is interpreted as “better”, given the elaboration. This response was different from those listed as models. However, the candidate has given an insightful analysis of the available data to support the conclusion. It recognises that Seán ostensibly scored better overall, but points out that closer examination reveals that Áine matched or outperformed him on all common tests. Accordingly, full credit is deserved.

The following candidate response to **part (b)(ii)** received **full credit**:

Is Áine better or worse than Seán at French? Give a reason for your answer.

Answer: don't know

Reason: don't know because she missed out on one test so
the result would be unfair

Comment: this response was again different from what was expected and was initially awarded partial credit. It is not entirely satisfactory in that it implies that no conclusion at all can be drawn from the data. However, the candidate is displaying at least as much insight into appropriate use of data as someone who compared the means and ignored the issue of absence. Since the latter responses were deemed acceptable, full credit is warranted for this response too. This is a borderline case, and the reference to the comparison being unfair is relevant to the decision to award full credit.

The following candidate response to **part (b)(ii)** received **partial credit**:

Is Áine better or worse than Seán at French? Give a reason for your answer.

Answer: 27

Reason: yes she's better because if she had been in for that test
Seán probably wouldn't have beaten her.

Question 4

(25 marks)

Content area: Strand 1 –Statistics

Main assessment objective(s):

1. Read information in tabular form.
2. Present information in graphical form.
3. In a non-mathematical context, apply understanding of concepts and connections.

Question & model solution

The table below is a record of the amount of money spent by a group of students in one month on credit for their mobile phones.

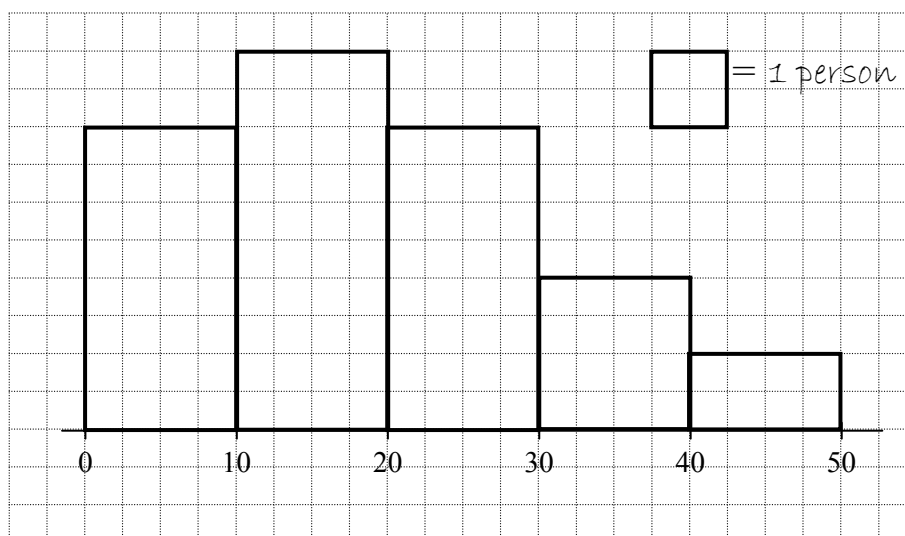
Amount spent in €	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Number of students	8	10	8	4	2

Note: each interval includes the lower boundary but not the upper one.

- (a) How many students were in the group?

Answer: 32

- (b) Illustrate the data on a histogram.



- (c) Using the table and/or the histogram to help you estimate, complete this sentence:

“On average, these students spend about €20 per month on phone credit.”

- (d) Michael is in this group and he spent €27 on phone credit that month. Describe in one sentence Michael’s phone spending by comparison with the others in the group.

He spent a bit more than average, but not unusually so.

Marking scheme & related notes

Question 4

- (a) Scale 5B
Partial credit: Clear indication of attempt to add some or all of the relevant numbers.
- (b) Scale 10C
High partial credit: Correct histogram without scale. (Accept vertical scale or area scale.)
Low partial credit: Histogram with error(s), such as one or more rectangles of incorrect height, or gaps between the rectangles. (There must be at least two rectangles drawn in correct proportion to each other to gain any credit.)
- (c) Scale 5A
Accept any point estimate between 15 and 25 inclusive.
Accept any reasonable interval estimate, such as “€15 to €20” (bearing in mind that the mean and median are a little below 20).
- (d) Scale 5B
Full credit: Sentence that indicates above average spending without implying extreme spending. e.g. “He spends a bit more than most people”. “He’s among the upper half”.
Partial credit: Sentence that indicates average spend or extremely high spend. e.g. “He spends way more than most”. “He’s about average” “It’s a normal amount.”
No credit: Sentence that indicates below average spend.

Comment on answering

Part (a) was well answered, indicating that the majority of candidates were able to interpret the meaning of the frequency table properly. The majority of candidates were not able to draw a correct histogram.

Answering in part (c) was somewhat disappointing in light of the fact that the candidates had shown in part (a) that they understood the meaning of the information in the table. This deficiency followed through into part (d), which was clearly dependent on the candidates having some reasonable idea of what the average spend was.

Candidate Exemplars

The following candidate response to **part (d)** received **partial credit**:

Michael's phone spending is a lot higher than the average teenager would spend.

Question 5

(25 marks)

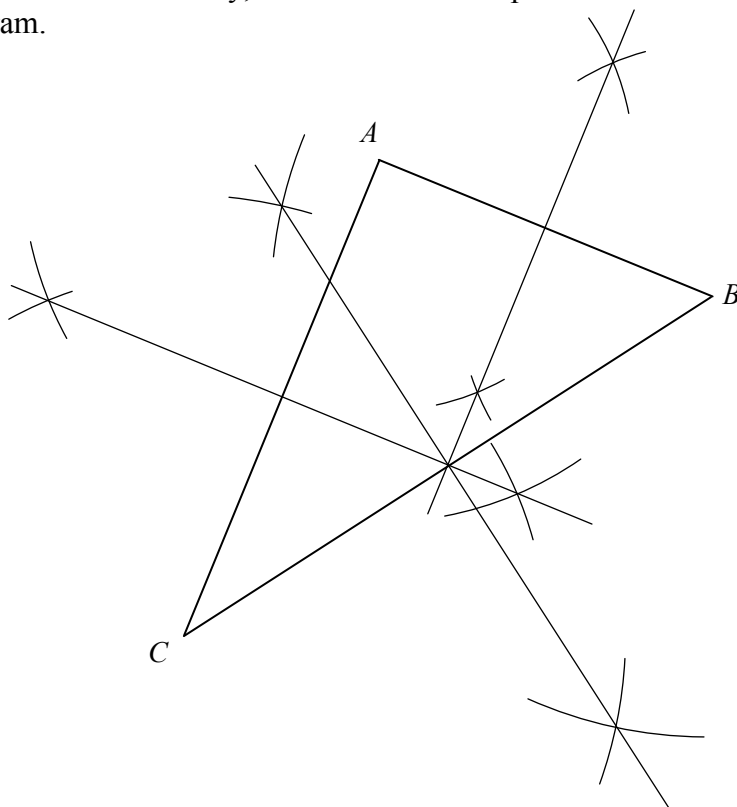
Content area: Strand 2 – Geometry

Main assessment objective(s):

1. Demonstrate use of geometrical instruments.
2. Within a mathematical context, execute routine procedures.
3. Within a mathematical context, demonstrate understanding of concepts and connections.

Question & model solution

Construct the circumcircle of the triangle ABC below using only a compass and straight edge. Show all construction lines clearly, and describe the steps in the construction in the space provided below the diagram.



Steps in construction:

Construct the perpendicular bisector of $[AB]$ as follows:

- put compass at A and draw arcs of fixed radius as shown
- put compass at B and draw arcs of same fixed radius as shown
- join the points of intersection of these arcs.

Similarly, get bisector of $[AC]$

Similarly, get bisector of $[CB]$

All three bisectors should meet at one point. This is the circumcentre.

Marking scheme & related notes

Question 5

Construction: Scale 15C

<i>Full credit:</i>	Accurate construction of any two mediators, including arcs. ("Accurate" means the circumcentre is within about 3 mm of its correct location.)
<i>High partial credit:</i>	Accurate construction, but without all arcs shown. or Inaccurate construction, with arcs shown (more than 3 mm but less than 10 mm from correct location).
<i>Low partial credit:</i>	One bisector constructed, reasonably accurate, with or without arcs shown or Two bisectors drawn by hand (no straight edge used) – with or without arcs. or Complete and reasonably accurate construction of incentre, centroid or orthocentre.
<i>No credit:</i>	Partial or inaccurate construction of incentre, centroid or orthocentre.

Description: Scale 10B

<i>Partial credit:</i>	clear description of how to bisect one side, but no mention of the need to bisect another. or indicates the need to draw bisectors of two (or three) sides, but gives no description (or an unsatisfactory description) of how to do so. or complete and satisfactory description of how to construct the incentre, centroid or orthocentre, if this is what the candidate has done on the diagram.
<i>No credit:</i>	incomplete or unsatisfactory description of how to construct the incentre, centroid or orthocentre.

Comment on answering

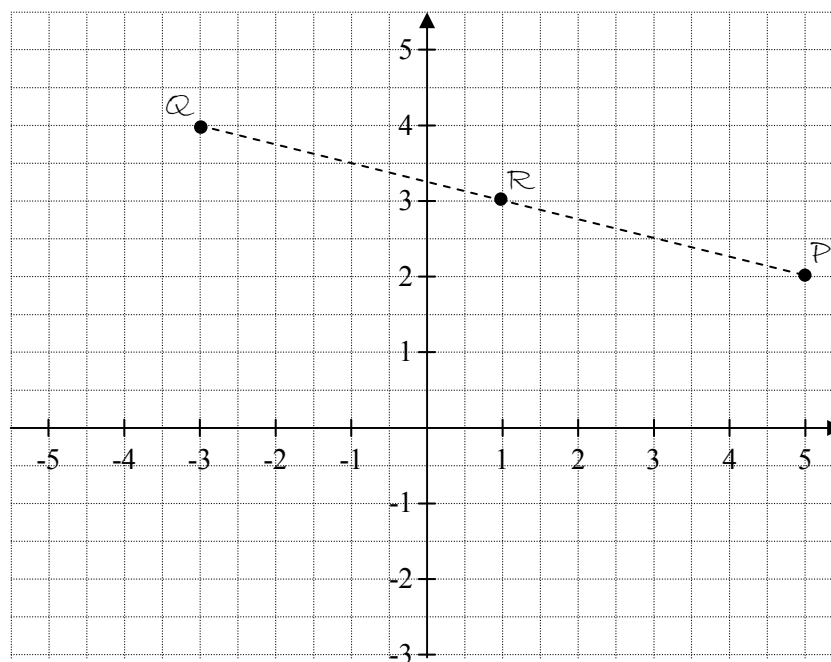
Attempts at this question were very rare. A very small number of candidates completed competent constructions. As with question 9B on the Ordinary Level paper, these may be candidates who are studying *Design and Communication Graphics* for Leaving Certificate or had studied *Technical Graphics* for Junior Certificate, (since the other candidates in the same centre offered no attempt). Of the few candidates who did complete the construction, none offered an attempt to describe it.

Question 6**(25 marks)****Content area:** Strand 2 – Trigonometry**Main assessment objective(s):**

1. Present information in graphical form.
2. Within a mathematical context, execute routine procedures.

Question & model solution

- (a) $P(5, 2)$ and $Q(-3, 4)$ are two points. Plot P and Q on a co-ordinate diagram.



- (b) R is the midpoint of $[PQ]$. Find the co-ordinates of R .

$$R = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{5 + (-3)}{2}, \frac{2 + 4}{2} \right) = (1, 3)$$

- (c) Find the slope of the line PQ .

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 2}{-3 - 5} = \frac{2}{-8} = -\frac{1}{4}$$

- (d) Find the equation of the line PQ .

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{1}{4}(x - 5)$$

$$4y - 8 = -x + 5$$

$$x + 4y = 13$$

Marking scheme & related notes

Question 6

- (a) Scale 5B
Partial credit: One point plotted correctly.
- (b) Scale 5B
Partial credit: Correctly substituted formula, but fails to finish correctly.
or
One error in substitution, and finishes correctly.
No credit: Incorrect formula used, even if finished correctly.
Note: Accept correct answer without work.
- (c) Scale 5B
Partial credit: Correctly substituted formula, but fails to finish correctly.
or
One error in substitution, and finishes correctly.
No credit: Incorrect formula used, even if finished correctly.
Note: Accept correct answer without work.
- (d) Scale 10C
Full credit: Correct answer, tidied up into standard form (like terms gathered and fractions cleared, or else in the form $y = mx + c$).
High partial credit: Correct answer, but not in standard form
or
Correct method, completed and tidied up, with one error
Low partial credit: Correct formula with some correct substitution.
No credit: Incorrect formula used, even if finished correctly.
Correct answer without work.

Comment on answering

There were a number of examination centres in which there were few if any attempts at this question, or attempts involving plotting the points only. This may indicate that the material had not yet been covered. There were a number of other centres where quite competent answering was in evidence.

The plotting of the points was generally done correctly by those who attempted it, although there were a number of instances where a point was plotted just one grid square away from its correct location. These candidates clearly knew where the points were meant to be, but gave insufficient care to plotting them correctly.

There were a number of instances of incorrect formulae being used, indicating that the candidates were not able to find and transcribe the correct formula from the *Formulae and Tables* booklet (or did not consider doing so).

Much difficulty was evident in relation to the arithmetic and algebraic skills required to finish the parts correctly.

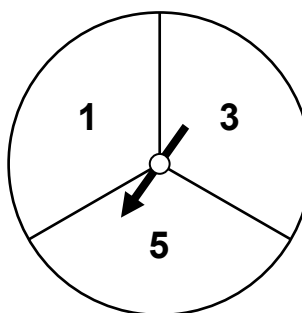
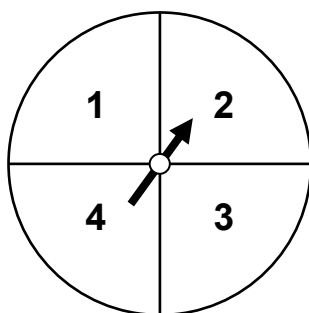
Question 7**(50 marks)****Content area:** Strand 1 – Probability and Statistics**Main assessment objective(s):**

1. Present and read information in graphical form.
2. In a non-mathematical context, execute routine procedures, interpreting the solutions in the original context.
3. In a non-mathematical context, apply understanding of concepts and connections.

Question & model solution:

A game at a festival involves two spinners that are spun at the same time and the scores added. The spinners are fair. (That is, the arrow is just as likely to stop in one sector as in any other.)

Prizes are given when the player spins a total equal to four.



- (a) The table below is partly completed and shows the total scores for the different ways the spinners could land. Complete the table.

	1	2	3	4
1	2	3	4	5
3	4	5	6	7
5	6	7	8	9

- (b) Sue plays the game once. Find the probability that she will get a score of nine.

Answer: $\frac{1}{12}$

- (c) Find the probability that Sue will win a prize, (that is, get a score of four).

Answer: $\frac{2}{12} = \frac{1}{6}$

- (d) Which of the following best describes Sue's chances of winning a prize?

Write the letter corresponding to the correct answer in the box.

- A. Impossible
- B. Not very likely
- C. About 50% likely
- D. Very likely
- E. Certain

B

- (e) From watching other people play the game, Sue thinks that the second spinner is not fair after all. She thinks that it is more likely to point to five than to the other two numbers.

Describe an experiment that would allow her to determine the true probability of getting a five with that spinner.

Spin the spinner lots of times and count how many fives you get.

The fraction of the time you get a five is the best estimate of the probability.

For example, if you do it a hundred times and you get fifty fives, then

you would say that the probability is $\frac{1}{2}$.

Marking scheme & related notes

Question 7

- (a) Scale 10B

Partial credit: At least 4 correct entries in the table.

- (b) Scale 10B:

Partial credit: Correct odds (i.e.: "11 to 1"). Note: no credit for "12 to 1".
 Answer improperly stated, e.g. "1:12" or "12:1" (but accept "1 in 12" for full credit).
 Answer rounded too much: e.g. 0.08 (but accept 0.083 for full credit).
 Gives relevant outcome pair (i.e. "must get 4 & 5" or similar.)

- (c) Scale: 10B

Full credit: Accept answer consistent with candidate's table, if properly simplified.

Partial credit: Correct answer (or consistent with table), not simplified
 Answer stated improperly (e.g. 1:6) or as odds (e.g. "5 to 1").
 Identifies both relevant outcome pairs (e.g. "She needs (3,1) or (1,3)"; or "she needs a 1 and a 3").

(d)	Scale 10A	
	<i>Full credit:</i>	Accept answer consistent with candidate's answer to part (c).
(e)	Scale 10C	
	<i>Full credit:</i>	Fully satisfactory description of suitable experiment, indicating large number of trials, and indicating the need to get the relative frequency.
	<i>High partial credit:</i>	Incomplete description that indicates large number of trials, but doesn't clearly communicate the calculation to be made. (e.g. "Spin it loads and loads of times an see what happens", "Spin it loads and loads of times an see how many fives you get.")
		Or makes an explicit reference to the fraction or relative frequency, without an explicit reference to large number of trials.
	<i>Low partial credit</i>	Refers to repeated spins, but not explicitly to large number of them. (e.g. "Spin it a few times, and see how many fives you get." "Spin it again and again and see how often you get a five", "Spin it three times and if you get one 5 then it's fair." (not explicitly referring to fraction or relative frequency).

Comment on answering

In this case again there were a number of centres in which few if any candidates attempted the question and where the attempts were poor. However, there were other centres where competence was displayed in the first four parts.

However, in many cases, part (e) was not attempted. When it was attempted, answering was generally poor, with candidates achieving either no credit or low partial credit.

Candidate Exemplars

The following candidate response to **part (e)** received **low partial credit**:

She could spin the spinner 10 times and see how many times it lands
on 5.

Question 8

(50 marks)

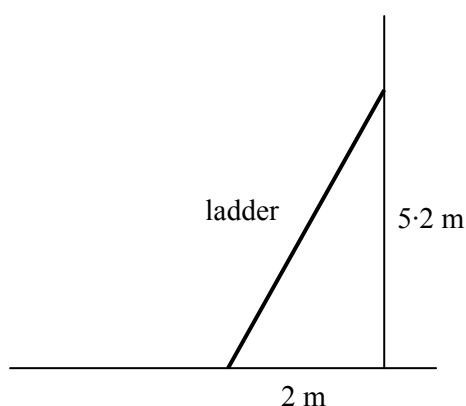
Content area: Strand 2 – Trigonometry

Main assessment objective(s):

1. In a non-mathematical context, apply routine procedures, interpreting the solutions in the original context.
2. In a non-mathematical context, apply understanding of concepts and connections.

Question & model solution:

Michael places a ladder against the side of a building to do some work. The top of the ladder is 5.2 metres from the ground, and the bottom of the ladder is 2 metres out from the wall, as shown in the diagram below.



- (a) Find the length of the ladder, correct to one decimal place.

$$l^2 = 5.2^2 + 2^2$$

$$= 31.04$$

$$l \approx 5.6 \text{ metres}$$

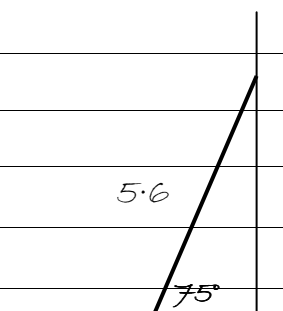
- (b) John, who works with Michael, tells him that the ladder is not safe at that angle. He says that the angle between the ladder and the ground is supposed to be as close as possible to 75° .

If Michael puts the ladder at the correct angle, how far up the wall will it reach?

$$\sin 75^\circ = \frac{x}{5.6}$$

$$x = 5.6 \sin 75^\circ$$

$$\approx 5.4 \text{ metres}$$



- (c) There are reasons why a ladder is supposed to be set up at the correct angle. Name one dangerous thing that could happen if the angle between the ladder and the ground is too small, and one dangerous thing that could happen if the angle is too big.

If the angle is too small, then The ladder might slide down

If the angle is too big, then The ladder might fall backwards when you climb up it.

Marking scheme & related notes

Question 8

- (a) Scale 15D

High partial credit: Correct solution, without units and/or incorrectly rounded

Mid partial credit: Correct method, not completed correctly

Low partial credit: An estimate in the range [5.4, 5.9]
(including correct answer without work)

- (b) Scale 20D

Full credit: Allow reasonable rounding if work is shown, provided answer ends up in the interval [5.35, 5.45].

High partial credit: Correct solution, without units and/or incorrectly rounded

Mid partial credit: Correct method, not completed correctly

Low partial credit: An estimate in the range [5.3, 5.5]
(including correct answer without work).
Incorrect or inverted trig ratio, finished correctly (with or without units).
Correctly labeled diagram showing 5.6, 75° , and letter for unknown.

- (c) Scale 15D

High partial credit: Two correct statements, but reversed.

Mid partial credit: One correct statement, in the right place

Low partial credit: One correct statement, but in the wrong place.

Comment on answering

There were hardly any attempts at parts (a) or (b).

There were many more efforts at part (c). The two problems were correctly identified quite frequently, but candidates commonly mixed them up. It would appear that, despite the explicit reference to the “angle between the ladder and the ground”, candidates were thinking of the angle that the ladder made with the wall.

Candidate Exemplars

The following candidate response to **part (c)** received **high partial credit**:

If the angle is too small, then The ladder and the wall would be un-even and
fall backwards.

If the angle is too big, then The ladder could fall down the wall frontwards.

Comment: these would both be regarded as acceptable had they not been reversed.

9. Conclusions and Recommendations

9.1 Effectiveness of the sample examination papers

The State Examinations Commission, the Department of Education and Science, and the National Council for Curriculum and Assessment have reviewed all of the evidence gathered from the trialling exercise, including the feedback from candidates and teachers and the observations of other interested parties. It has been agreed that the questions are appropriate both in style and standard, and that radical alterations are not required. However, it has also been agreed that further refinement of certain questions is required in order to improve clarity, to make some of the tasks more readily accessible to candidates, and in some cases to provide additional scaffolding or direction. The finalised versions of the sample papers have been amended so as to reflect this agreement.

It has also been decided to replace Question 5 on the Ordinary Level paper with a question focused on co-ordinate geometry rather than trigonometry. This does not indicate any difficulty with the question itself, but is instead a response to comments from teachers that the relative time that they had spent on these two subtopics was not well reflected by their representation on the examination paper, (noting that a full question in Section B was devoted to trigonometry.)

9.2 Effectiveness of the marking scheme

No examination paper can be considered properly in isolation from its marking scheme. It is the combination of the two that determines what competencies are measured and in what proportion. The marking scheme is the means by which the examining team identifies the relevant characteristics of candidate responses and awards credit appropriately to correspond with evidence of relevant achievement. In an examination like the Leaving Certificate, it is also critical to ensure that this can be achieved consistently and efficiently. The trialling exercise was therefore a test of the effectiveness of the marking scheme as much as of the papers themselves.

The marking schemes used in the trialling exercise were designed *ab initio* and were not intended to be structurally similar to those used in the recent past by the SEC for its mathematics examinations. This was appropriate in light of the significant broadening of the nature and style of questions that may arise in the examination, and also serves to create a more direct and explicit link between the marks awarded and the qualitative characteristics of student responses. As well as facilitating the Chief Examiner and the examining teams in carrying out their work effectively, it is hoped that these schemes will assist all interested parties in having a clearer picture of what constitutes a response of a specified level of quality, and a clearer understanding of how the examiners evaluate the relevant evidence when making their assessment judgments.

The feedback from the examining teams and the judgment of the Chief Examiner is that the schemes as they applied were effective in achieving their objectives. It is therefore anticipated that the overall structure of the scheme applied to the real examination will be along the same lines as was applied in the trialling, and that

the notes, commentary and exemplars detailed in the relevant chapters of this report are a good guide as to how candidates' work will be assessed in the real examination. Certain details will need to be refined to address some specific and general issues that arose.

With regard to the above statement regarding the anticipated structure of the marking schemes to be used, it should be noted that all aspects of the marking schemes – both structural and detailed – are at the discretion of the Chief Examiner from year to year.

9.3 Candidate Response to the Trialling Exercise

As previously noted, the focus of the trialling exercise was on testing the effectiveness of the sample papers and the marking scheme. The focus of the trialling was not on candidate achievement; indeed there were a number of significant contextual factors that would serve to make such a focus inappropriate at this time. These factors included, for example, the fact that at the time of the trialling, not all of the relevant material had been completed in class. In many cases, the material had been done during the previous school year, but there had been little, if any, opportunity for revision. The format and style of questioning was very different from those that are used on the existing syllabus, and no official examination paper with questions in this format had previously been issued. It is also noteworthy that performance at an early stage in the year on any examination designed for terminal assessment could not be expected to be of a similar standard to that expected at the end of the programme. Furthermore, candidates and teachers were aware that the purpose of the trialling was to test the examination questions and marking scheme, rather than to test the candidates, and this may have affected candidates' attitudes and their performance.

However, notwithstanding all of the above, analysis of individual questions and individual parts of questions indicates that there is clear evidence that candidates, particularly at both Higher and Ordinary levels, are capable of engaging meaningfully with the types of tasks that are on the examination, and of performing to the anticipated standards.

There is less evidence of this at Foundation level. However, it must be noted that the sample in this case is very small, in that there were only 31 candidates in total. Many of these candidates appear to have had some language difficulties and every effort is being made to address these issues in the context of the sample papers and real examination papers in June.

9.4 Recommendations

Arising from the trialling process, consideration of the feedback received, and the subsequent discussions, the following recommendations are made.

9.4.1 The sample papers

- The structure and format of the sample papers are satisfactory and should remain as they were on the trialled draft, with some adjustment to the formatting of answer space.

- The content and standards of the tasks are appropriate and these tasks should remain largely unchanged.
- Question 5 on the Ordinary Level paper should be amended, as described in section 9.1 above.
- Where possible, the language used in the questions should be further simplified, subject to not compromising the assessment of the candidates' understanding of the proper vocabulary of the subject, and their capacity to apply their mathematical skills to realistic problems set in non-mathematical contexts.

9.4.2 Syllabus development

- Consideration should continue to be given to the difficulties that students have traditionally had with achieving and displaying conceptual understanding, with solving non-routine problems, and with contextualised mathematics in general. Syllabus expectations should remain reasonable, and progress in schools should be monitored closely.

9.4.3 Teaching and learning

- Attention should be focussed on the objectives and learning outcomes laid down in the syllabus documents. It is important to emphasise that, even with respect to subject content that is similar to before, *Project Maths* involves a significant shift in emphasis regarding the skill set being developed. The learning activities undertaken should reflect this shift appropriately.
- The assessment activities undertaken in schools should reflect the *key skills* of the senior cycle curriculum³ and the aims of *Project Maths*. Whereas the purpose of the Leaving Certificate examination is different from that of the many forms of assessment that teachers engage in, they nonetheless share some common features. Teachers can therefore use these sample papers and this report as a basis for reflection and discussion. In particular, when designing a particular assessment task, teachers may wish to consider the following:
 - What learning outcome of the syllabus is it that I am trying to assess?
 - What kinds of evidence will allow me to distinguish between a student who has the knowledge, skill, or competence that I am interested in and one who hasn't (or will allow me to distinguish between different levels of the competence)?
 - What task can I give the students to allow them to display this evidence?

It is also valuable to explicitly address with students what the purposes of their learning activities and of the associated assessment activities are. In this regard, teachers should refer to the work of the NCCA and others in the area of "Assessment for Learning".

³ There are five skills identified as central to teaching and learning across the curriculum at senior cycle: information processing, critical and creative thinking, communicating, working with others, and being personally effective. Refer to the NCCA website for full details.

10: Additional information regarding the 2010 examination in the initial schools

The issuing of sample papers frequently gives rise to follow-on questions from teachers and candidates regarding the extent to which the real examination can be expected to mirror these in various respects. Some such questions are addressed below.

1. Will the number of questions and/or the number of marks allocated to each content area be the same as on the sample papers, and will the same content appear in the same place?

The allocation of marks and questions **between the two strands** will be as on the sample papers.

Furthermore, within each section (Section A and Section B), the proportion of content on each strand will be as on the sample papers. In particular:

- At Higher Level, Section A will contain two 25-mark questions on Strand 1 and four 25-mark questions on Strand 2. These six questions may appear in any order. In Section B, Question 7 will be a 50-mark question on Strand 1, Question 8 will be a 50-mark question on Strand 2, and Question 9 will be a 50-mark question consisting of a choice between 9A on Strand 1 and 9B on Strand 2. Please refer to Information Note #1 issued by NCCA to the initial schools in October 2009 for full information about the restrictions regarding the content that is ring-fenced to Question 9.
- At Ordinary Level, Section 0 will consist of the same 50-mark question as appears as Question 1 on Paper 2 for all other Leaving Certificate candidates. Section A will contain two 25-mark questions on Strand 1 and three 25-mark questions on Strand 2. These five questions may appear in any order. In Section B, Question 7 will be a 40-mark question on Strand 1, Question 8 will be a 40-mark question on Strand 2, and Question 9 will be a 45-mark question consisting of a choice between 9A on Strand 1 and 9B on Strand 2. Please refer to Information Note #1 issued by NCCA to the initial schools in October 2009 for full information about the restrictions regarding the content that is ring-fenced to Question 9.
- At Foundation Level, Section 0 will consist of the same two 50-mark questions as appear as Questions 1 and 2 on Paper 2 for all other Leaving Certificate candidates. Section A will contain two 25-mark questions on Strand 1 and two 25-mark questions on Strand 2. These four questions may appear in any order. In Section B, Question 7 will be a 50-mark question on Strand 1, and Question 8 will be a 50-mark question on Strand 2. As stated in Information Note #1 issued by NCCA to the initial schools in October 2009, there will not be any question on *standard deviation*.

Note that the allocation of marks and questions between subtopics of the strands will not necessarily be the same as on the sample papers. For example, on the Higher Level paper, Question 8 in Section B is an application of trigonometry. It is possible that on the real examination, this question would deal with an application of a different subtopic of Strand 2. In such a case, the Strand 2 material in Section A would be selected in such a way as to ensure that trigonometry retained a reasonable representation.

2. Could the same or similar questions appear at two different levels?

Teachers will have noted a partial overlap on the sample papers between parts of Question 7 on the Higher Level paper and Question 7(a) on the Ordinary Level paper. There is a closer alignment of content between the different levels in the *Project Maths* syllabuses than was the case before. The content at each level is strictly a subset of the level above. The structure of the examination papers is similar, as is the overall style of questions. Candidates are frequently unsure during their studies as to which level they should take. In order to encourage such candidates to reach their full potential and strive for success at the highest possible level for as long as possible, this alignment between the papers is intended to minimise the negative effects on candidates who switch to a lower level towards the end of their studies.

Any of the questions or question parts on the Foundation Level sample paper could equally well appear on the Ordinary or Higher Level papers, and the questions on the Ordinary Level paper are similarly suitable at Higher Level. Such questions could remain as they stand, depending on what other content or skills were being tested elsewhere on the paper, or they could be extended or adjusted to reflect the higher standard of achievement expected at a higher level.

Similarly, where the content is shared, a question on one paper might be suitable for the level below, either with or without adjustment, depending on the circumstances.

Question 7 on the Higher Level sample paper and Question 7(a) on the Ordinary Level sample paper give an indication as to how a question may be adjusted to suit different levels.

3. Will the format of individual questions be the same as on the sample papers?

The sample papers at all levels included questions in a variety of formats: completion of tables, completion of sentences, multiple choice, short answer, longer problems, and so on. This will continue to be the case. The structure of each question will be selected so as to most effectively test the objective(s) intended. As well as ensuring that the examination is as effective and efficient as possible, maintaining this variety of question styles should encourage teachers and candidates to focus on developing the competences indicated in the syllabus, rather than superficial aspects of particular question formats. In accordance with the aims of Project Maths, the format of examination questions will vary from time to time.

4. Will the marking schemes be similar to the ones used for the trialling?

Marking schemes in all subjects are designed so as to identify and appropriately reward the various levels of achievement with respect to the competencies that have been identified for assessment in accordance with the aims and objectives of the syllabus concerned. All aspects of the marking schemes – both structural and detailed – are at the discretion of the Chief Examiner from year to year.

While recognising the above, it may be noted that the trialling exercise has indicated that the scheme as implemented was effective in achieving its purpose, subject to certain aspects requiring refinement. It is therefore anticipated that the overall structure of the scheme applied to the real examination will be along the same lines as was applied in the trialling, and that the notes, commentary and exemplars detailed in the relevant chapters of this report are a good guide as to how candidates' work will be assessed in the real examination.

Appendix 1: Assessment Grid – Project Maths, Strands 1 & 2

 Coimisiún na Scrúduithe Stáit State Examinations Commission		Subject: Mathematics (project)	Exam: LC	Level: Higher	Component: Paper 2		Code:	Year:
Analysis of questions by content area and objective								
Content area / topic / element Assessment objective	Strand 1		Strand 2				Total	
	Statistics	Probability	Synthetic geometry	Co-ordinate geometry	Trigonometry	Vectors		
Demonstrate knowledge of terminology, definitions, facts, and results.								
Demonstrate use of tools such as calculators, geometrical instruments, formula book, etc.								
Present, or read, information in tabular, graphical or pictorial form.								
Within a mathematical context, execute routine procedures.								
Within a mathematical context, demonstrate understanding of concepts and connections, including relevant conditions, implications, etc.								
In a non-mathematical context, apply routine procedures, interpreting the solutions in the original context.								
In a non-mathematical context, apply understanding of concepts and connections, interpreting solutions, conditions, and implications in the original context.								
Explore patterns, formulate conjectures, explain findings and justify conclusions.								
Total:								

