Interactive I.T. Student Activity Sheets
Leaving Certificate Strand 5

- **Student Activities written to match the I.T. interactive modules on the Project Maths Leaving Certificate Student's CD Strand 5**
- **Interactive Activity Sheets included to enhance students' understanding of mathematical concepts**
- **Simple and clear guidelines are provided to facilitate learning**
- **Interesting questions are provided to lead students to explore, construct and consolidate their learning**

Preface

The NCCA have pointed out particular Key Skills in their Draft Syllabus. "While particular emphasis is placed in mathematics on the development and use of information processing, logical thinking and problem-solving skills, the new approach being adopted in the teaching and learning of mathematics will also give prominence to students being able to develop their skills in communicating and working with others. By adopting a variety of approaches and strategies for solving problems in mathematics, students will develop their self-confidence and personal effectiveness." To help our students to adapt to and take advantage of this new spirit of the syllabus, we have produced Interactive I.T. Student Activity Sheets which incorporate an innovative and diversified learning environment for mathematics.

As we all know, the advancement in technology has changed the way we can learn mathematics. Therefore we have developed a number of interactive modules on our student's CD to match this new development. With the help of these interactive modules, students can not only enhance their understanding in mathematics, but they can also enjoy learning it.

In order to help our students use the I.T. tools more effectively, ***Interactive I.T. Student Activity Sheets Leaving Certificate Strand 5*** are produced in this booklet. A student activity sheet is designed for the majority of the interactive modules on the CD. All student activity sheets provide simple and clear guidelines including:

1. Reference to the related topics in ***Project Maths Student's Leaving Certificate Strand 5 section***
2. Purpose of the I.T. tools
3. Instructions for using the I.T. tools.

These Student Activity Sheets, which include many interesting questions, will lead students to explore, construct, and consolidate their knowledge of mathematics on their own with ease. We believe that with the help of these activities, students' knowledge and understanding of mathematics will grow.

Table of Contents

Corresponding Position on Student's CD	**Name of Student Activity Sheet**	**Page**
Student Activity: Derivative of a Cubic Function	Student Activity: To investigate the Derivative of a Cubic Function	49
Student Activity Log Graph	Student Activity: To investigate the graph of $\log_n x$	52
Student Activity on Exponential	Student Activity: To investigate a^n	56
Student Activity on e^x and lnx	Student Activity: To investigate $f(x) = e^x$ and $g(x) = \ln(x)$	60
Student Activity Integration and Area 1	Student Activity: To investigate the relationship between integration of a function and the area enclosed by the curve that represents the function and the x-axis	62
Student Activity Integration and Area 2	Student Activity: To investigate the relationship between integration of a function and the area enclosed by the curve representing the function and the x-axis or the y-axis	66
Student Activity Integration and Area 3	Student Activity: To investigate the relationship between integration of a function and the area enclosed by the curve representing the function and the x axis	71
Student Activity Integration and Area 4	Student Activity: To investigate the relationship between integration of a function and the area enclosed by the curve representing the function and a line that intersects the curve	74

Corresponding Position on Student's CD	**Name of Student Activity Sheet**	**Page**
Student Activity Average Value 1	Student Activity: To investigate the Average Value of a Constant Function	77
Student Activity Average Value 2	Student Activity: To investigate the Average Value of a Linear Function	79
Student Activity Average Value 3	Student Activity: To investigate the Average Value of a Function 3	83

Instructions for use

This booklet contains student activities to accompany the majority of the interactive files on the Junior Certificate Strand 4 section of the student disk. The specific section of the course that the activity relates to is specified in the name of the activity. At the top of each student activity the students are told what interactive file on the student disk is to accompany the student activity.

Technical Problems

The student disk has a link situated on the left hand side of its front page called "Troubleshooting" this section gives instructions, if any of the following problems are encountered:

- Problems opening Office 2007 documents
- You do not have Java on your machine
- You do not have a PDF reader on your machine.

Student Activity 1a

Project Maths Tionscadal Mata Development Team

Tables for each of the functions below are drawn on the next page of this document for $x \in \{-3,-2,-1,0,1,2,3\}$. Fill out all the tables first so that you can decide on a scale which will suit all the functions when plotting a graph. Plot all the graphs **using the same axes and scales** using the grid given on the next page. Verify the shape of each graph by calculating y values of points, in between those plotted, and comparing the answers with the y values of the same points given by your graph.

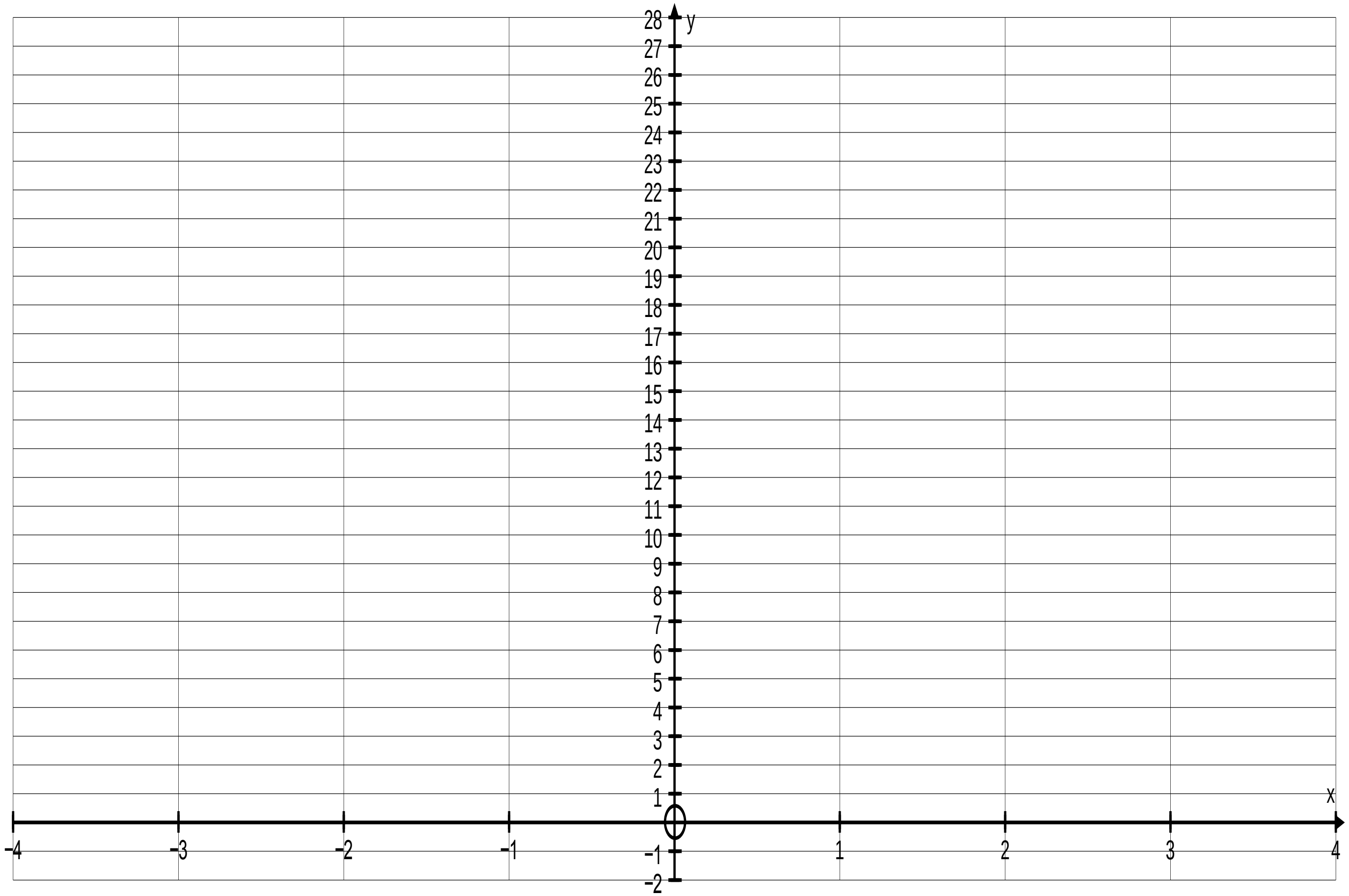
Polynomial in the form $f(x)=ax^2$ $a \in N$	State the shape of the graph and whether it opens upwards or downwards	x – intercepts (algebraic method and using the graph)	y – intercept (algebraic method and using the graph)	Maximum/ minimum point as an ordered pair and labelled as max or min	Real root(s) of f(x)=0	Equation of the axis of symmetry	f (2.7)	Solve f(x) = 8	For what x values is f(x) positive i.e. $f(x)>0$?	For what x values is f(x) negative i.e. $f(x)<0$?	For what x values is f(x) increasing?	For what x values is f(x) decreasing?
$y=x^2$												
$y=2x^2$												
$y=3x^2$												
$y=\frac{1}{2}x^2$												

1. What is the effect of the coefficient "a" on the graph of the function $f(x)=ax^2$?
2. Which of the above functions has the greatest rate of change of y with respect to x? How can you check this?
3. Which of the above functions has the smallest rate of change of y with respect to x? How can you check this?
4. What point on the graph does the axis of symmetry pass through?

Student Activity 1a

Project Maths Tionscadal Mata Development Team

Draw all graphs in pencil first and then outline the graph of $y = x^2$ using a black marker and use different coloured markers to draw the other curves. Label all graphs clearly.



x	$y = x^2$	(x, y)

x	$y = 2x^2$	(x, y)

x	$y = 3x^2$	(x, y)

x	$y = 0.5x^2$	(x, y)

Student Activity 1b

Project Maths
Tionscadal Mata
Development Team

Tables for each of the functions below are drawn on the next page of this document for $x \in \{-3, -2, -1, 0, 1, 2, 3\}$.

Fill out the tables for each function first so that you can decide on a scale which will suit all the functions when plotting a graph.
Plot all the graphs **using the same axes and scales** using the grid given on the next page.
Verify the shape of each graph by calculating y values of points, in between those plotted, and comparing the answers with the y values of the same points given by your graph.

Polynomial in the form $f(x)=ax^2$ $a \in Z$	State the shape of the graph and whether it opens upwards or downwards	x – intercepts (algebraic method and using the graph)	y – intercept (algebraic method and using the graph)	Maximum/ minimum point as an ordered pair and labelled as max or min	Real root(s) of f(x)=0	Equation of the axis of symmetry	f (2.7)	Solve f(x) = 8	For what x values is f(x) positive i.e. $f(x)>0$?	For what x values is f(x) negative i.e. $f(x)<0$?	For what x values is f(x) increasing?	For what x values is f(x) decreasing?
$y=x^2$												
$y=-x^2$												
$y=2x^2$												
$y=-2x^2$												

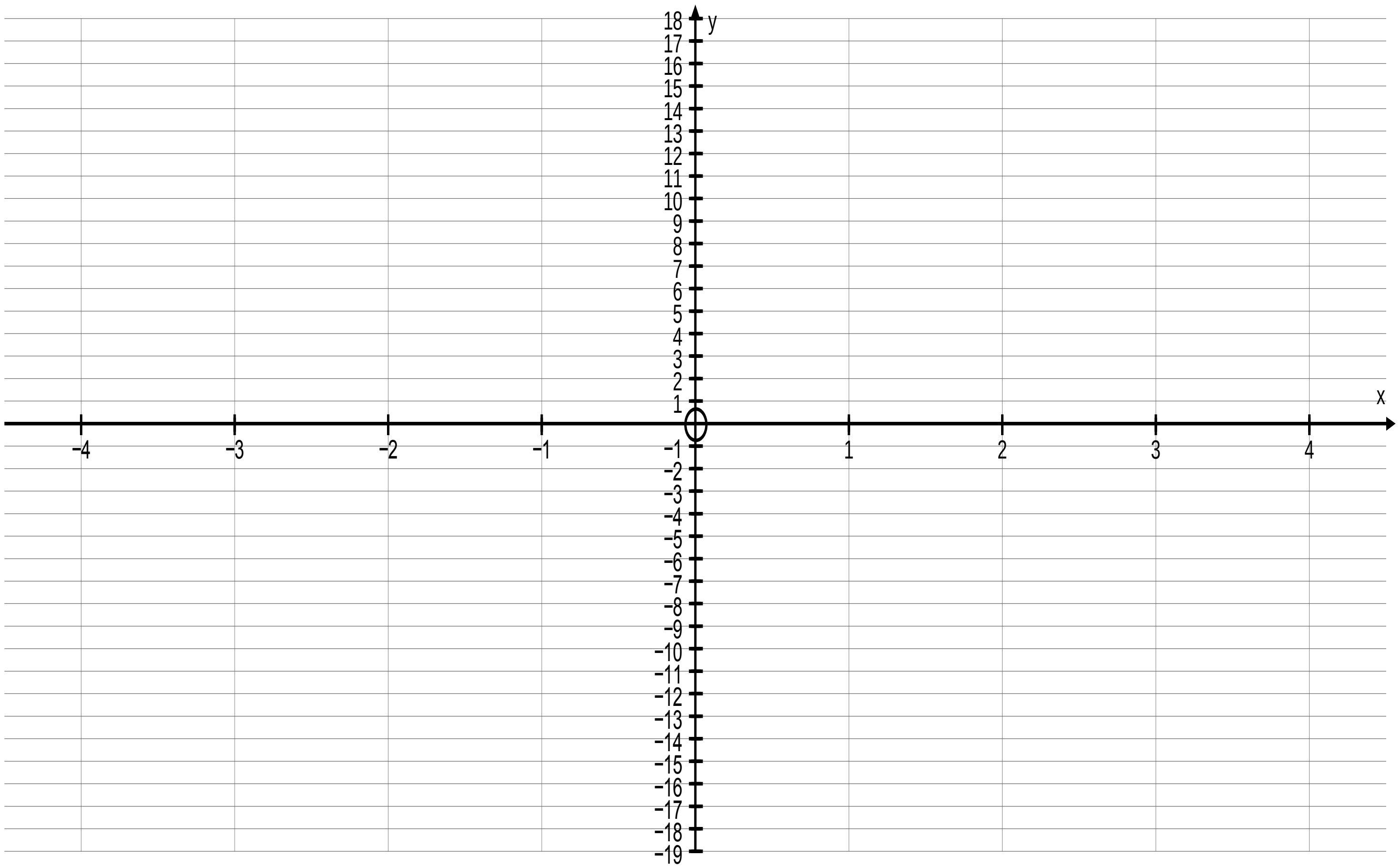
1. What is the effect of the sign of the coefficient "a" on the graph of function $f(x)=ax^2$? Explain

Student Activity 1b

Project Maths
Tionscadal Mata
Development Team

Draw the graph of $y = x^2$ using a black marker and use different coloured markers to draw the other curves.

Label all the graphs clearly.



x	$y = x^2$	(x, y)

x	$y = -x^2$	(x, y)

x	$y = 2x^2$	(x, y)

x	$y = -2x^2$	(x, y)

Student Activity 1c

Tables for each of the functions below are drawn on the next page of this document for $x \in \{-3, -2, -1, 0, 1, 2, 3\}$.

Fill out the tables for each function first so that you can decide on a scale which will suit all the functions when plotting a graph.
Plot all the graphs **using the same axes and scales** using the grid given on the next page.

Verify the shape of each graph by calculating y values of points, between those plotted, and comparing the answers with the y values of the same points given by your graph.

Polynomial in the form $f(x) = x^2 \pm c$	State the shape of the graph and whether it opens upwards or downwards	x – intercepts (algebraic method and using the graph)	y – intercept (algebraic method and using the graph)	Maximum/ minimum point as an ordered pair and labelled as max or min	Real root(s) of f(x)=0	Equation of the axis of symmetry	f (2.7)	Solve f(x) = 8	For what x values is f(x) positive i.e. $f(x) > 0$?	For what x values is f(x) negative i.e. $f(x) < 0$?	For what x values is f(x) increasing?	For what x values is f(x) decreasing?
$y = x^2$												
$y = x^2 + 8$												
$y = x^2 - 8$												
$y = x^2 + 2$												

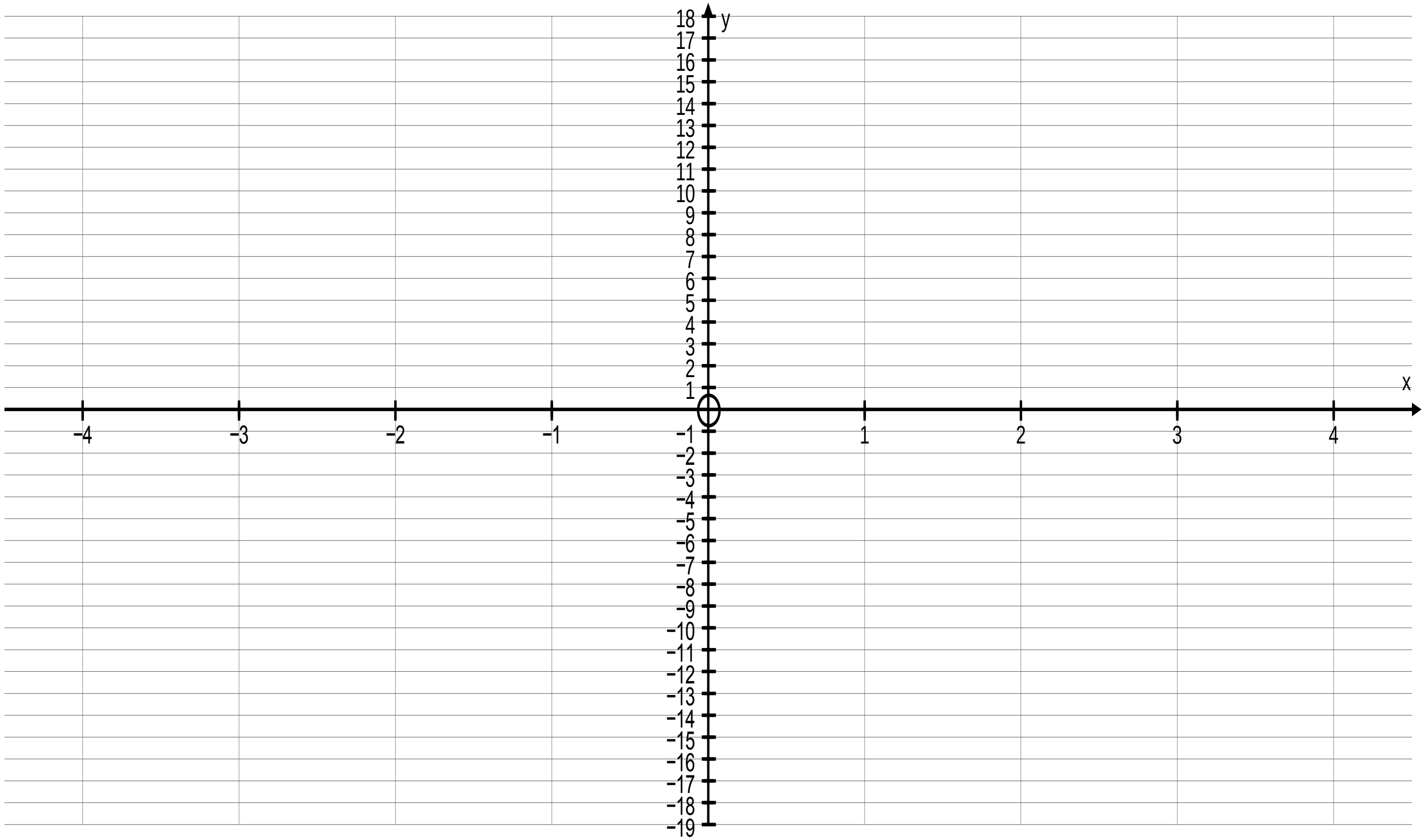
1. What is the effect of the constant *c* on the graph of the function $f(x) = x^2 \pm c$? Explain.

Project Maths
Tionscadal Mata
Development Team

Student Activity 1c

Draw the graph of $y = x^2$ using a black marker and use different coloured markers to draw the other curves.

Label all the graphs clearly.



x	$y = x^2$	(x, y)

x	$y = x^2 + 8$	(x, y)

x	$y = x^2 - 8$	(x, y)

x	$y = x^2 + 2$	(x, y)

Student Activity 1d

Project Maths Tionscadal Mata Development Team

Tables for each of the functions below are drawn on the next page of this document for $x \in \{-3, -2, -1, 0, 1, 2, 3\}$.

Fill out the tables for each function first so that you can decide on a scale which will suit all the functions when plotting a graph.
Plot all the graphs **using the same axes and scales** on the grid given below. Verify the shape of each graph by calculating y values of points, between those plotted, and comparing the answers with the y values of the same points given by your graph.

Polynomial in the form $f(x) = ax^2 \pm c$	State the shape of the graph and whether it opens upwards or downwards	x – intercepts (algebraic method and using the graph)	y – intercept (algebraic method and using the graph)	Maximum/ minimum point as an ordered pair and labelled as max or min	Real root(s) of f(x) =0	Equation of the axis of symmetry	f (2.7)	Solve f(x) = 8	For what x values is f(x) positive i.e. $f(x) > 0$?	For what x values is f(x) negative i.e. $f(x) < 0$?	For what x values is f(x) increasing?	For what x values is f(x) decreasing?
$y = x^2$												
$y = 3x^2$												
$y = 3x^2 - 4$												
Your own example												

1. What is the effect of the constant a on the graph of the function $f(x) = ax^2 \pm c$? Explain

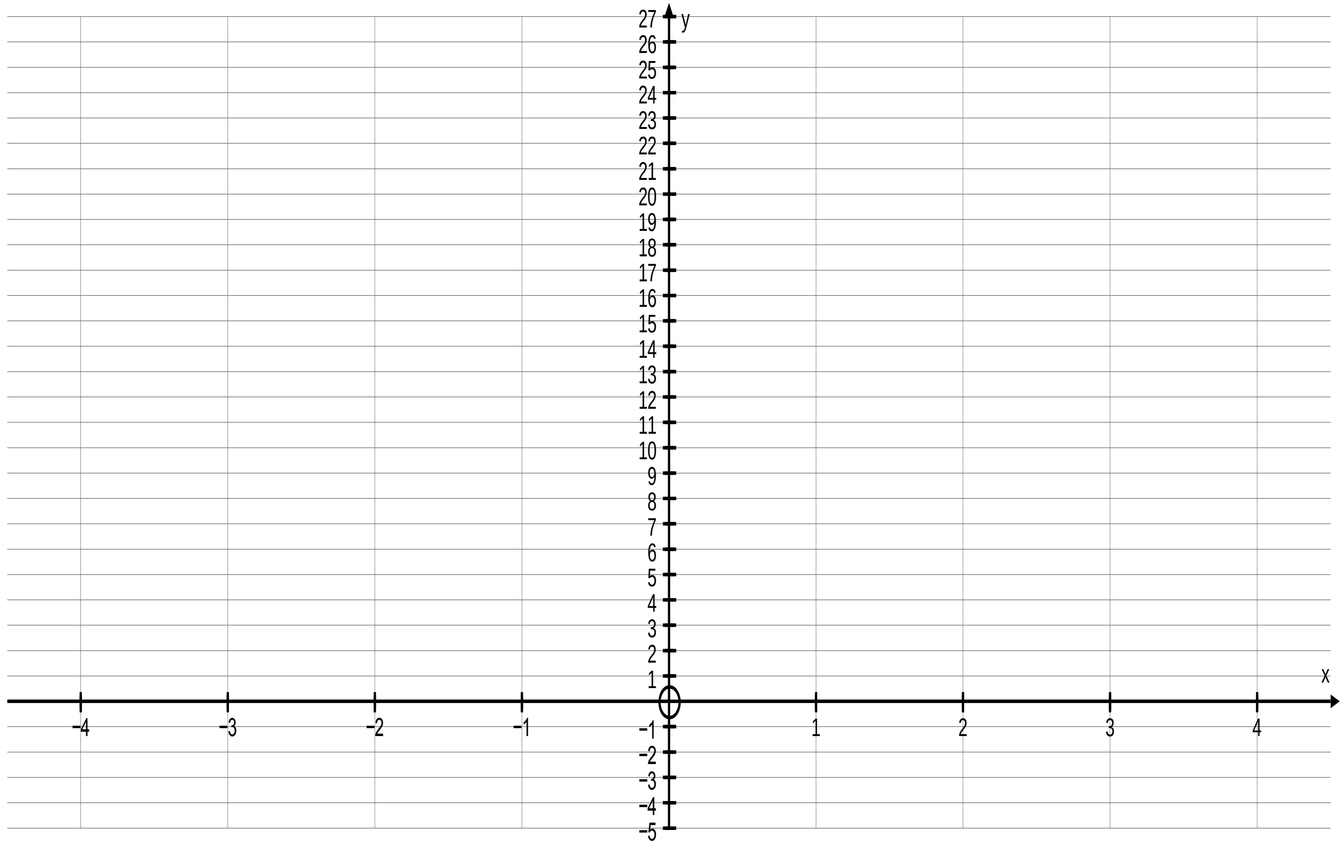
2. What is the effect of the constant c on the graph of the function $f(x) = ax^2 \pm c$? Explain

Student Activity 1d

Project Maths
Tionscadal Mata
Development Team

Draw the graph of $y = x^2$ using a black marker and use different coloured markers to draw the other curves.

Label all the graphs clearly.



x	$y = x^2$	(x, y)
x		

x	$y = 3x^2$	(x, y)

x	$y = 3x^2 - 4$	(x, y)

x	$y =$	(x, y)

Student Activity 2a

Project Maths
Tionscadal Mata
Development Team

Tables for each of the functions below are drawn on the next page of this document.

Fill out the tables for each function first so that you can decide on a scale which will suit all the functions when plotting a graph.

Plot all the graphs **using the same axes and scales** using the grid given on the next page. Verify the shape of each graph by calculating y values of points, between those plotted, and comparing the answers with the y values of the same points given by your graph.

Polynomial in the form $f(x)=(x+h)^2$	State the shape of the graph and whether it opens upwards or downwards	x – intercepts(algebraic method and using the graph)	y – intercept (algebraic method and using the graph)	Maximum/ minimum point as an ordered pair and labelled as max or min	Real root(s) of f(x)=0	Equation of the axis of symmetry	f (2.7)	Solve f(x) = 8	For what x values is f(x) positive i.e. $f(x)>0$?	For what x values is f(x) negative i.e. $f(x)<0$?	For what x values is f(x) increasing?	For what x values is f(x) decreasing?
$y=x^2$												
$y=(x+2)^2$												
$y=(x-2)^2$												

1. If h is positive how does the graph of $y=(x+h)^2$ compare to the graph of $y=x^2$? What transformation of the plane will transform $y=x^2$ onto $y=(x+h)^2$?

2. If h is negative how does the graph of $y=(x+h)^2$ compare to the graph of $y=x^2$? What transformation of the plane will transform $y=x^2$ onto $y=(x+h)^2$?

3. Solve $x^2=(x+2)^2$ using tables, graphs and algebraically.

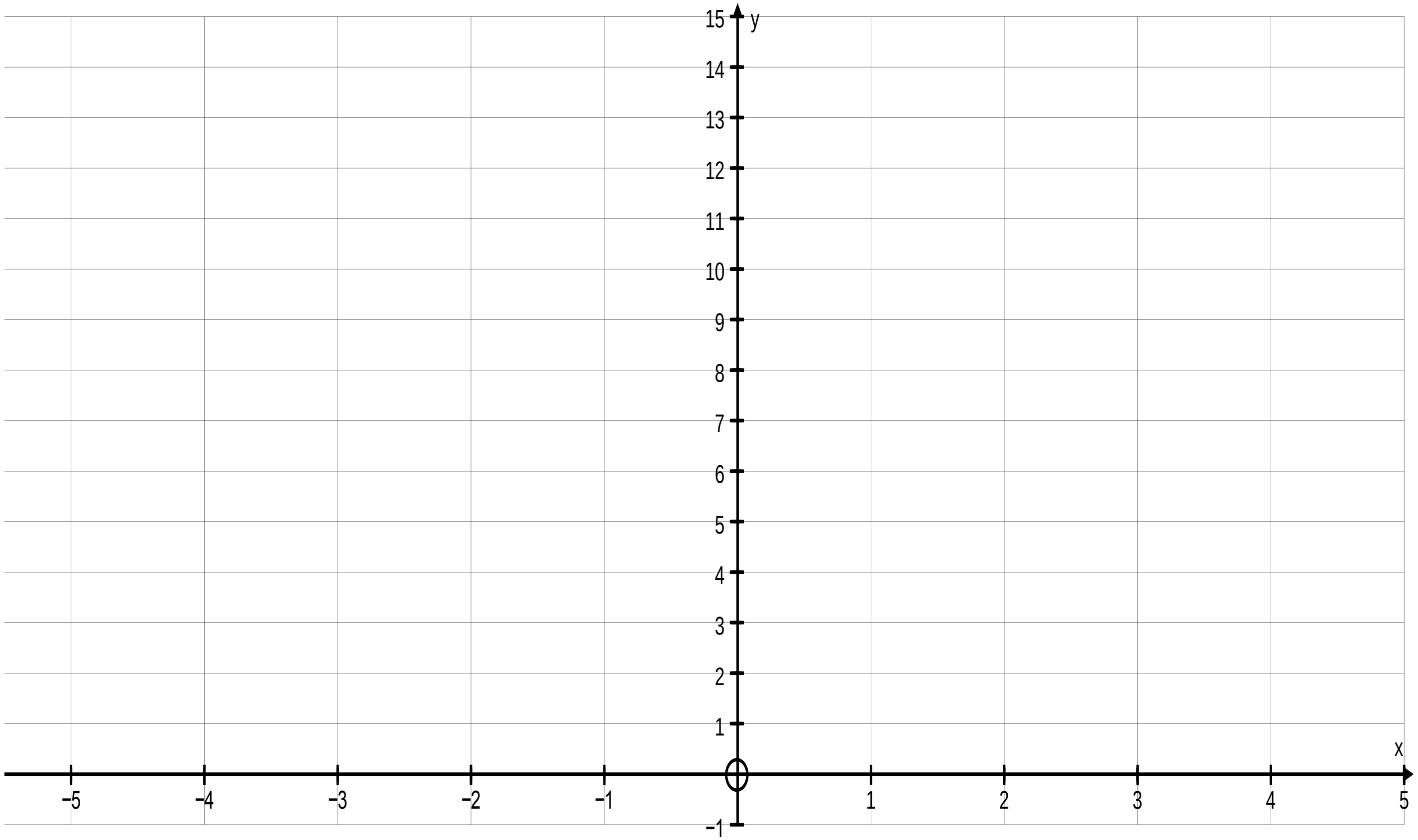
4. Solve $x^2=(x-2)^2$ using tables, graphs and algebraically.

Student Activity 2a

Project Maths Development Team

Draw the graph of $y = x^2$ using a black marker and use different coloured markers to draw the other curves.

Label all the graphs clearly.



	$y = x^2$	(x, y)
-3		
x -2		
x -1		
0		
1		
2		
3		
x	$y = (x+2)^2$	(x, y)
-5		
-4		
-3		
-2		
-1		
0		
1		
x	$y = (x-2)^2$	(x, y)
-1		
0		
1		
2		
3		
4		
5		

1. Can you write $y = (x+2)^2$ in a different way? Verify using the graph.

2. Can you write $y = (x-2)^2$ in a different way? Verify using the graph.

Student Activity 2a

Project Maths Tionscadal Mata Development Team

Tables for each of the functions below are drawn on the next page of this document. Fill out the tables for each function first so that you can decide on a scale which will suit all the functions when plotting a graph.
Plot all the graphs **using the same axes and scales** using the grid given on the next page. Verify the shape of each graph by calculating y values of points, between those plotted, and comparing the answers with the y values of the same points given by your graph.

Polynomial in the form $f(x)=(x+h)^2$	State the shape of the graph and whether it opens upwards or downwards	x – intercepts(algebraic method and using the graph)	y – intercept (algebraic method and using the graph)	Maximum / minimum point as an ordered pair and labelled as max or min	Real root(s) of f(x)=0	Equation of the axis of symmetry	f (2.7)	Solve f(x) = 8	For what x values is f(x) positive i.e. $f(x)>0$?	For what x values is f(x) negative i.e. $f(x)<0$?	For what x values is f(x) increasing?	For what x values is f(x) decreasing?
$f(x)=x^2$												
$y=(x+1)^2$												
$y=(x-1)^2$												

1. If h is positive how does the graph of $y=(x+h)^2$ compare to the graph of $y=x^2$? What transformation of the plane will transform $y=x^2$ onto $y=(x+h)^2$?

2. If h is negative how does the graph of $y=(x+h)^2$ compare to the graph of $y=x^2$? What transformation of the plane will transform $y=x^2$ onto $y=(x+h)^2$?

3. Solve $x^2=(x+1)^2$ using tables, graphs and algebraically.

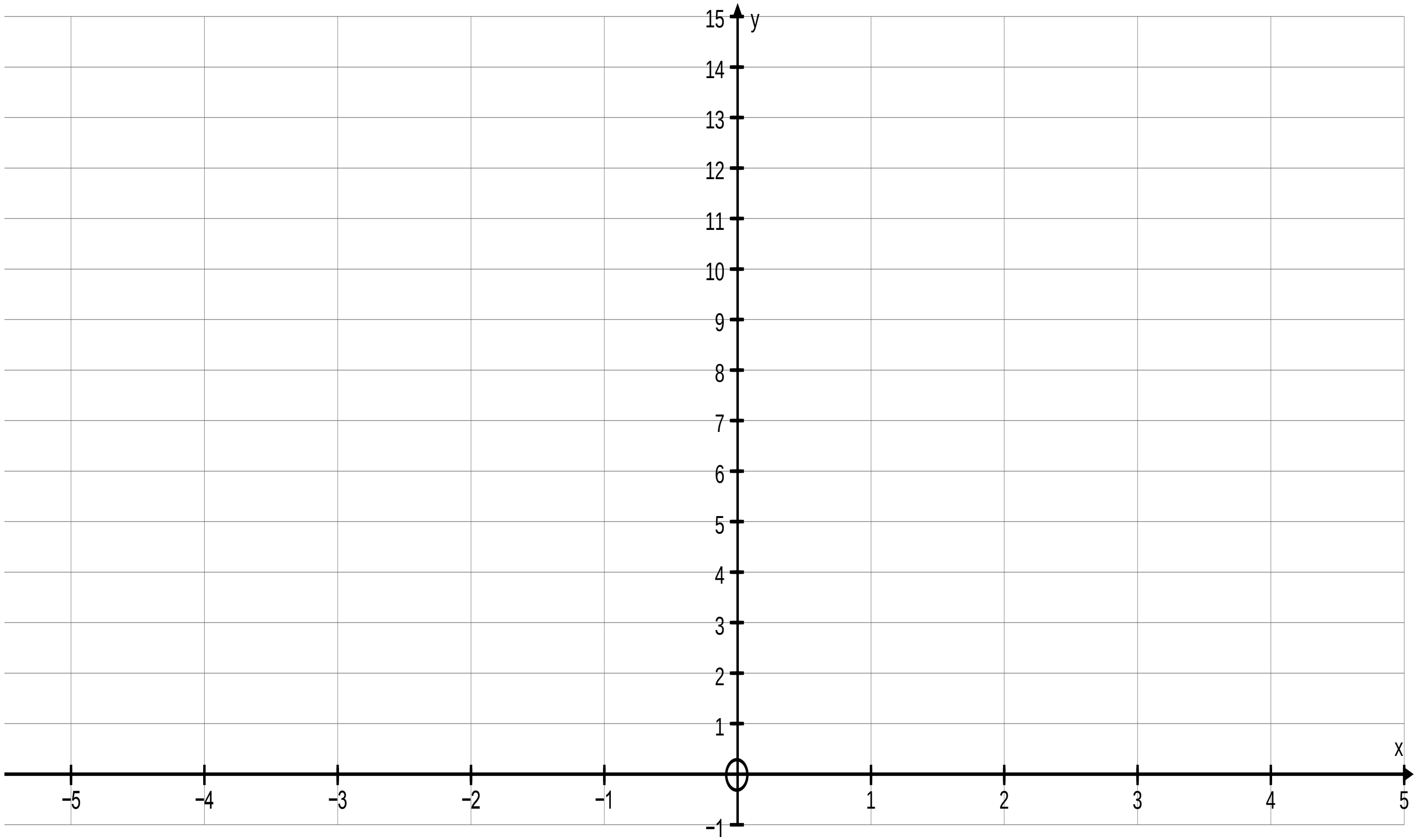
4. Solve $x^2=(x-1)^2$ using tables, graphs and algebraically.

Student Activity 2a

Project Maths
Tionscadal Mata
Development Team

Draw the graph of $y = x^2$ using a black marker and use different coloured markers to draw the other curves.

Label all the graphs clearly.



x	$y = x^2$	(x, y)
-3		
-2		
-1		
0		
1		
2		
3		

x	$y = (x+1)^2$	(x, y)
-4		
-3		
-2		
-1		
0		
1		
2		

x	$y = (x-1)^2$	(x, y)
-2		
-1		
0		
1		
2		
3		
4		

x		(x, y)

1. Can you write $y = (x+1)^2$ in a different way? Verify using the graph.

2. Can you write $y = (x-1)^2$ in a different way? Verify using the graph.

Student Activity 2a

Project Maths Tionscadal Mata Development Team

Tables for each of the functions below are drawn on the next page of this document. Fill out the tables for each function first so that you can decide on a scale which will suit all the functions when plotting a graph.
Plot all the graphs **using the same axes and scales** using the grid given on the next page. Verify the shape of each graph by calculating y values of points, between those plotted, and comparing the answers with the y values of the same points given by your graph.

Polynomial in the form $f(x)=(x+h)^2$	State the shape of the graph and whether it opens upwards or downwards	x – intercepts(algebraic method and using the graph)	y – intercept (algebraic method and using the graph)	Maximum / minimum point as an ordered pair and labelled as max or min	Real root(s) of f(x)=0	Equation of the axis of symmetry	f (2.7)	Solve f(x) = 8	For what x values is f(x) positive i.e. $f(x)>0$?	For what x values is f(x) negative i.e. $f(x)<0$?	For what x values is f(x) increasing ?	For what x values is f(x) decreasing ?
$f(x)=x^2$												
$y=(x+3)^2$												
$y=(x-3)^2$												

1. If h is positive how does the graph of $y=(x+h)^2$ compare to the graph of $y=x^2$? What transformation of the plane will transform $y=x^2$ onto $y=(x+h)^2$?

2. If h is negative how does the graph of $y=(x+h)^2$ compare to the graph of $y=x^2$? What transformation of the plane will transform $y=x^2$ onto $y=(x+h)^2$?

3. Solve $x^2=(x+3)^2$ using tables, graphs and algebraically.

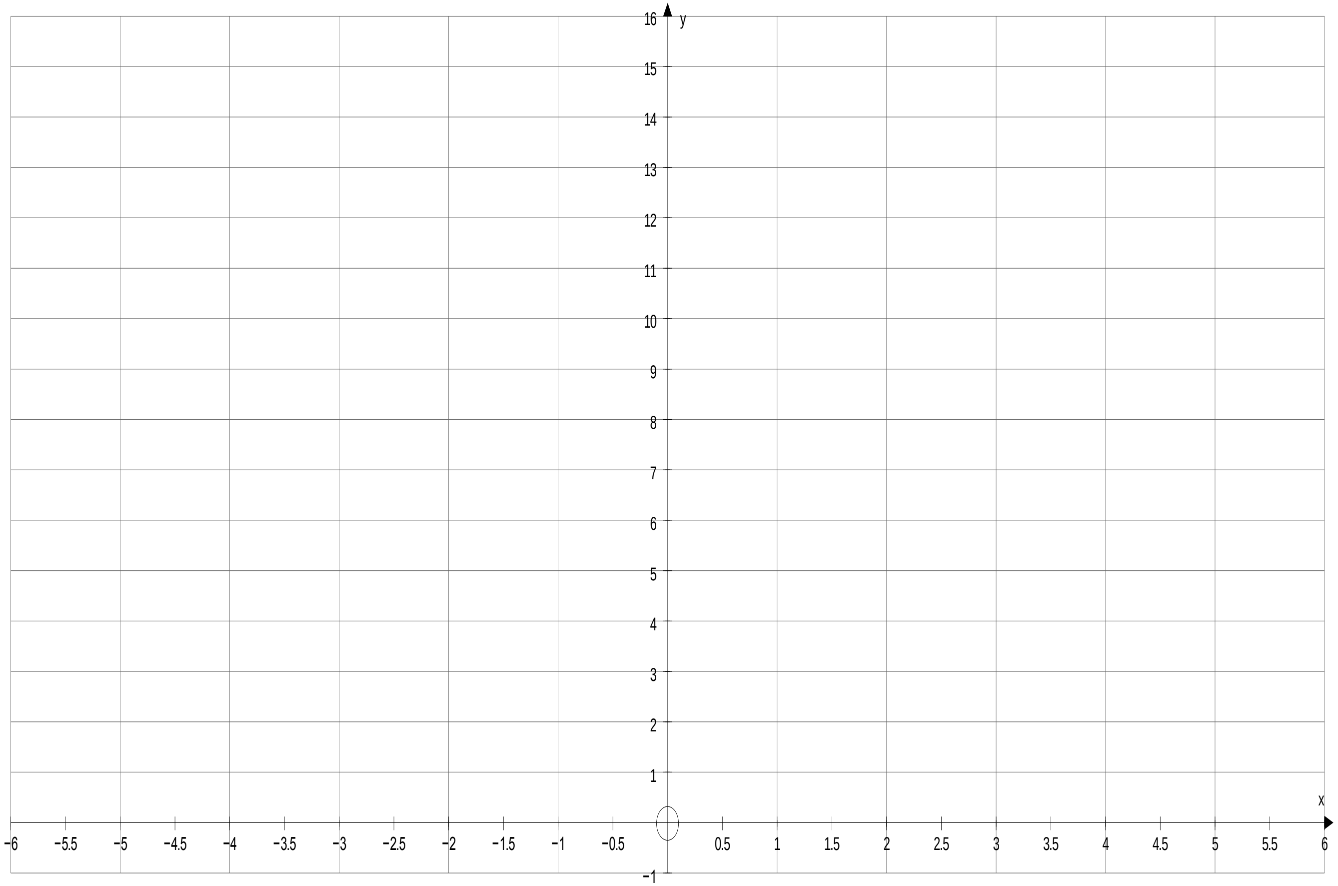
4. Solve $x^2=(x-3)^2$ using tables, graphs and algebraically.

Student Activity 2a

Project Maths
Tionscadal Mata
Development Team

Draw the graph of $y = x^2$ using a black marker and use different coloured markers to draw the other curves.

Label all the graphs clearly.



x	$y = x^2$	(x, y)
-3		
-2		
-1		
0		
1		
2		
3		
x	$y = (x+3)^2$	(x, y)
-6		
-5		
-4		
-3		
-2		
-1		
0		
x	$y = (x-3)^2$	(x, y)
0		
1		
2		
3		
4		
5		
6		
x		(x, y)

. Can you write $y = (x+3)^2$ in a different way? Verify using the graph.

2. Can you write $y = (x-3)^2$ in a different way? Verify using the graph.

Student Activity 2b

Project Maths
Tionscadal Mata
Development Team

Tables for each of the functions below are drawn on the next page of this document for $x \in \{-3, -2, -1, 0, 1, 2, 3\}$.

Fill out the tables for each function first so that you can decide on a scale which will suit all the functions when plotting a graph.
Plot all the graphs **using the same axes and scales** using the grid given on the next page. Verify the shape of each graph by calculating y values of points, between those plotted, and comparing the answers with the y values of the same points given by your graph.

Polynomial in the form $f(x) = a(x+h)^2 + k$	State the shape of the graph and whether it opens upwards or downwards	x – intercepts (algebraic method and using the graph)	y – intercept (algebraic method and using the graph)	Maximum/ minimum point as an ordered pair and labelled as max or min	Real root(s) of f(x) =0	Equation of the axis of symmetry	f (2.7)	Solve f(x) = 8	For what x values is f(x) positive i.e. $f(x) > 0$?	For what x values is f(x) negative i.e. $f(x) < 0$?	For what x values is f(x) increasing?	For what x values is f(x) decreasing?
$f(x) = x^2$												
$y = (x+2)^2$												
$y = (x+2)^2 - 3$												
$y = 2(x+2)^2$												
$y = 2(x+2)^2 - 3$												

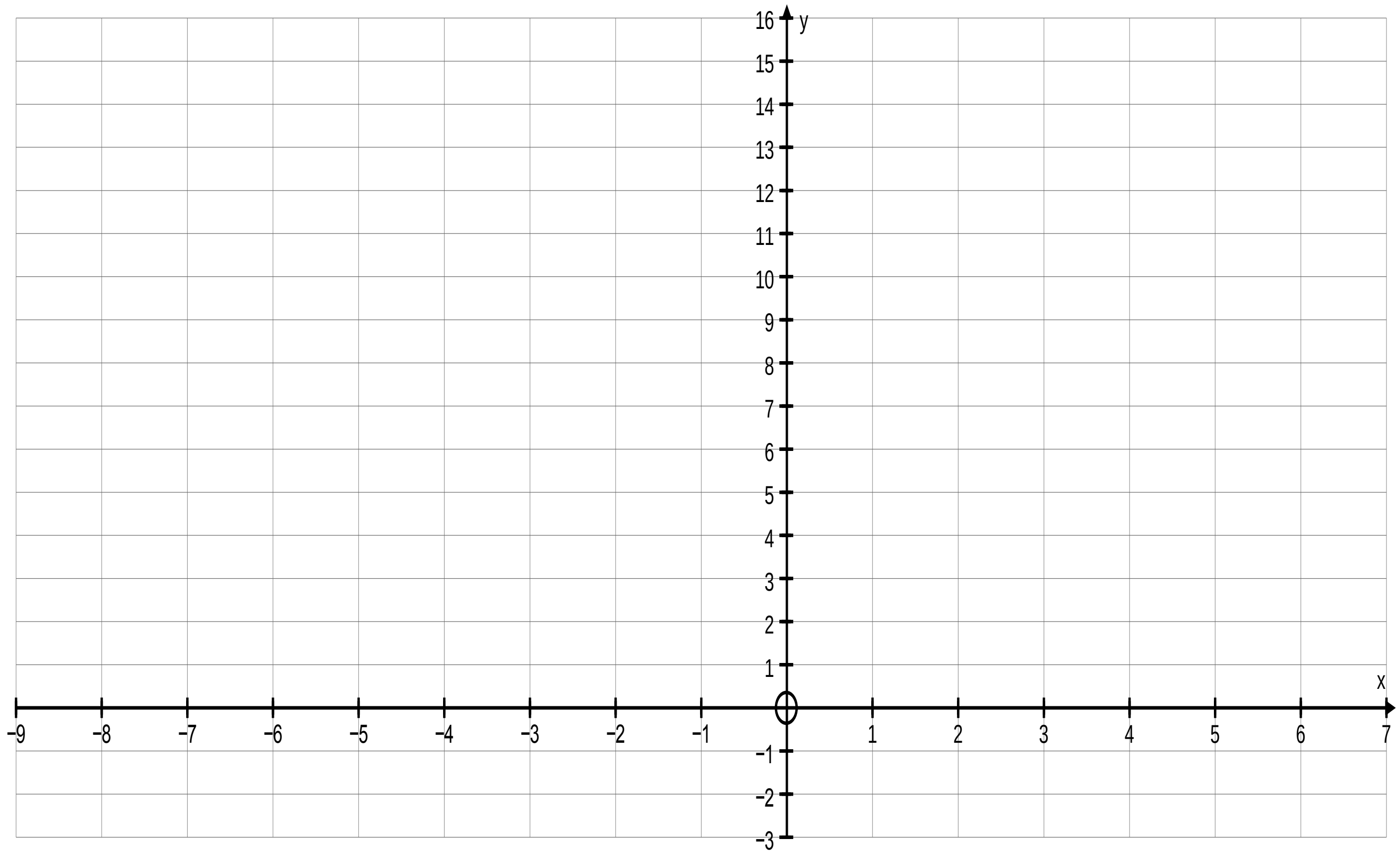
1. How does the graph of $y = (x+2)^2 + 3$ compare to the graph of $y = x^2$? What transformation of the plane will transform y = x^2 onto $y = (x+2)^2 + 3$?

2. How does the graph of $y = 2(x+2)^2 - 3$ compare to $y = x^2$?

3. Compare and contrast the graphs of $y = (x+2)^2 - 3$ and $y = 2(x+2)^2 - 3$.

Student Activity 2b

Draw the graph of $y = x^2$ using a black marker and use different coloured markers to draw the other curves. Label all the graphs clearly.



x	$f(x) = x^2$	(x, y)
x		

x	$y = (x+2)^2$	(x, y)

x	$y = (x+2)^2 - 3$	(x, y)

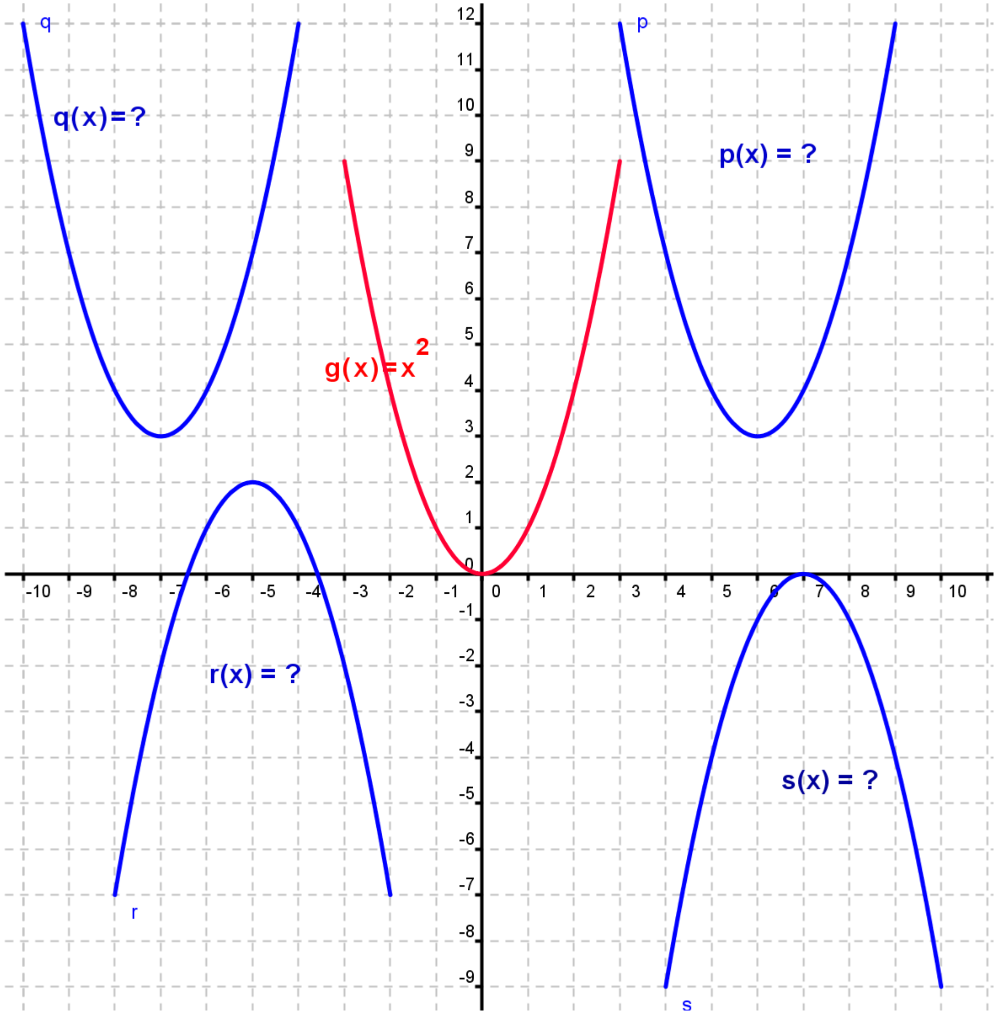
x	$y = 2(x+2)^2 - 3$	(x, y)

Student Activity 2b

Project Maths
Tionscadal Mata
Development Team

On the axes below g is the graph of the function $g(x) = x^2$

Write the equations for the graphs the functions $p, q, r,$ and s in the form $y = (x+h)^2 + k$



$p(x) =$
$s(x) =$
$r(x) =$
$q(x) =$

Student Activity 3a

Project Maths Tionscadal Mata Development Team

Tables for each of the functions below are drawn on the next page of this document for $x \in \{-2,-1,0,1,2,3,4,5,6\}$.

What do you notice about all the tables?

Using the same axes and scales plot the points for each function and join up the points to form an appropriate curve.

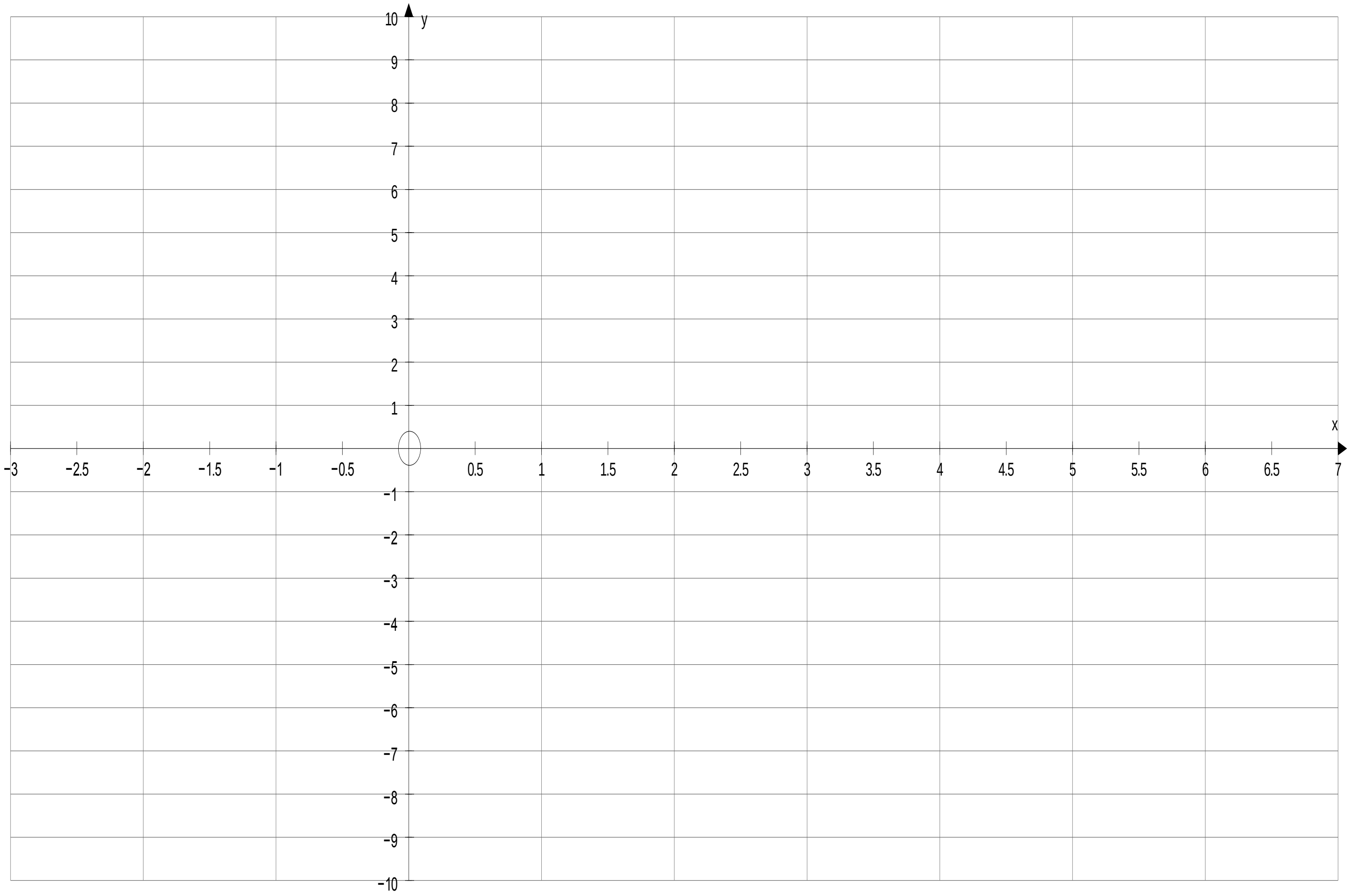
Polynomial in the form $f(x) = ax^2 + bx + c$ $f(x) = (x+r)(x+s)$ $f(x) = (x+h)^2 + k$	State the shape of the graph and whether it opens upwards or downwards	x – intercepts (algebraic method and using the graph)	y – intercept (algebraic method and using the graph)	Maximum/ minimum point as an ordered pair and labelled as max or min	Real root(s) of f(x) =0	Equation of the axis of symmetry	f (2.7)	Solve f(x) = 8	For what x values is f(x) positive? $f(x) > 0$	For what x values is f(x) negative? $f(x) < 0$	For what x values is f(x) increasing?	For what x values is f(x) decreasing?
$y = x^2 - 4x - 5$												
$y = (x-5)(x+1)$												
$y = (x-2)^2 - 9$												

1. What do you notice about all of the graphs and all of the three functions you have plotted in this activity?
2. What items of information about the graph can you read from the equation $y = x^2 - 4x - 5$ before you plot its graph?
3. What extra items of information can you tell about the graph in this factored form $y = (x-5)(x+1)$?
4. What are the roots of $y = (x-5)(x+1)$?
5. What are the roots of $y = (x+r)(x+s)$
6. What extra item of information can you tell about the graph when f(x) is in the form $y = (x-2)^2 - 9$?
7. How does knowing the x- intercepts (roots) help us to find the axis of symmetry?

Project Maths
Tionscadal Mata
Development Team

Student Activity 3a

Plot the points and draw the graph for each of the functions in the tables on this page.



x	$y = x^2 - 4x - 5$	(x, y)

x	$y = (x-5)(x+1)$	(x, y)

x	$y = (x-2)^2 - 9$	(x, y)

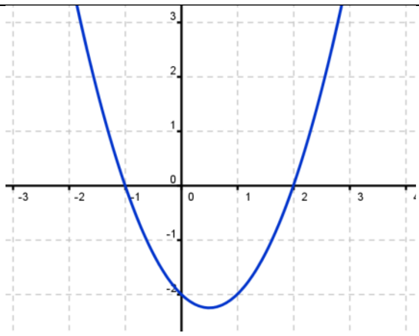
Student Activity 3a

Write the equation for each graph below in factored form i.e. $y=(x+r)(x+s)$ and also in the general form $y=ax^2+bx+c$.

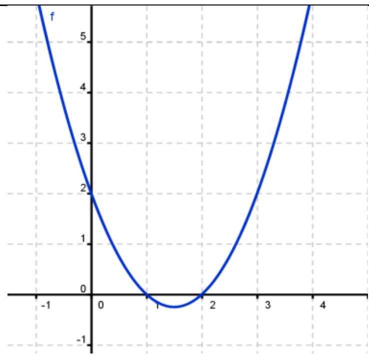
1. How are the roots linked to the factored form? Explain your answer.

2. How is the y - intercept linked to the general form? Explain your answer.

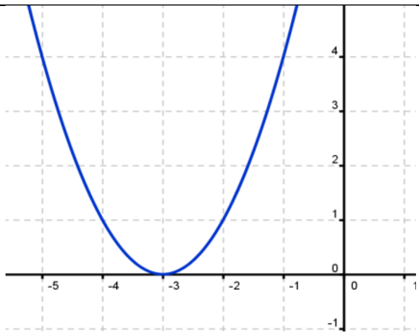
1.



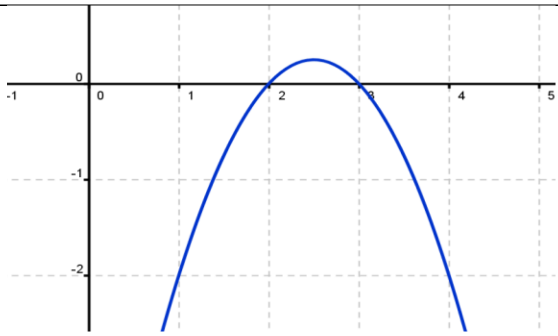
2.



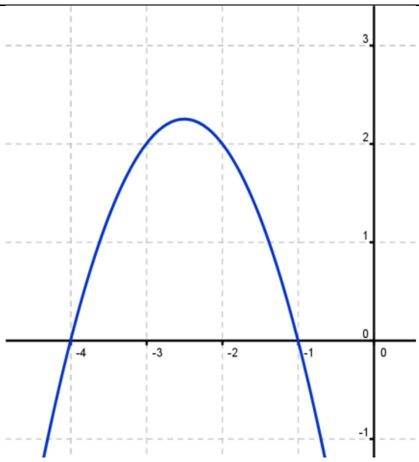
3.



4.



5.

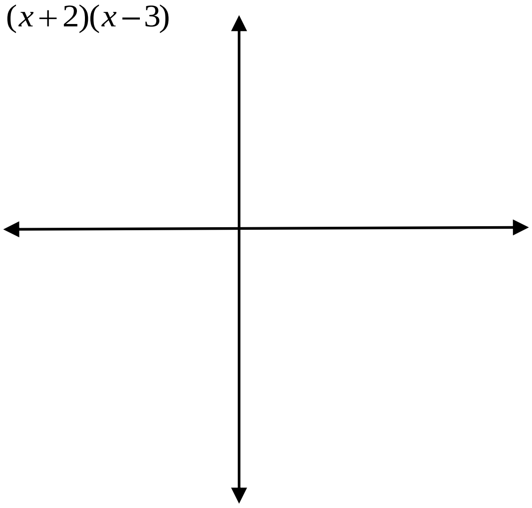


Draw x and y-axis and draw the graph of a quadratic function below giving its equation in the form $y=(x+r)(x+s)$.

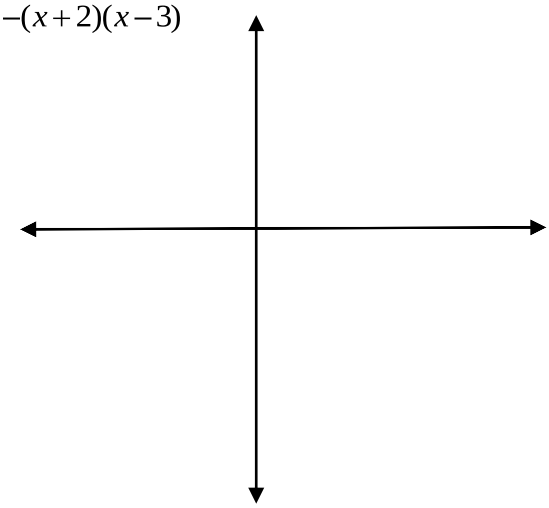
Student Activity 3a

Working in pairs, **sketch** the following graphs on the axes below.
Note particularly the intercepts on the axes and whether the graph has a local maximum or local minimum. (Check the sign of y values for x values between the roots.)
Verify that you are correct by using a graphing calculator or graphing software such as GeoGebra if you have access to these. Alternatively use the “Table” mode on your calculator to verify points.

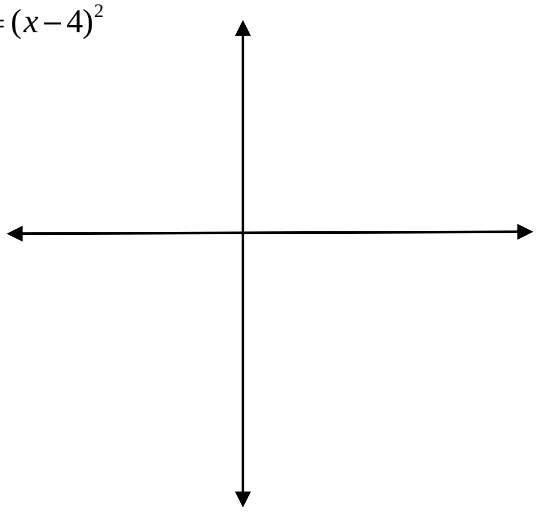
$y=(x+2)(x-3)$



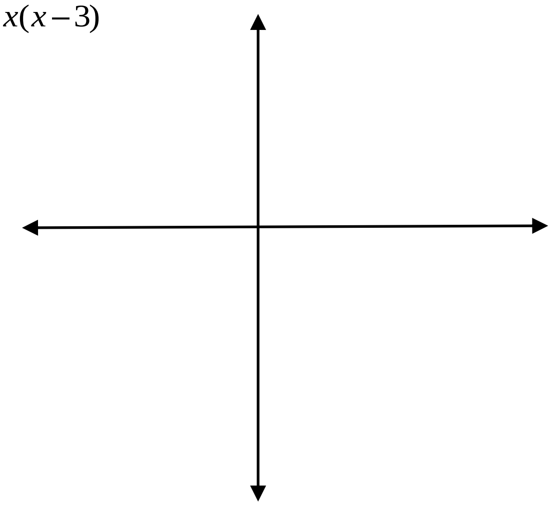
$y=-(x+2)(x-3)$



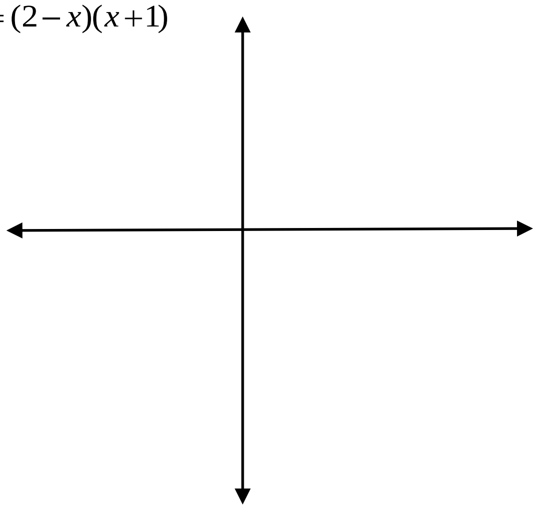
$y=(x-4)^2$



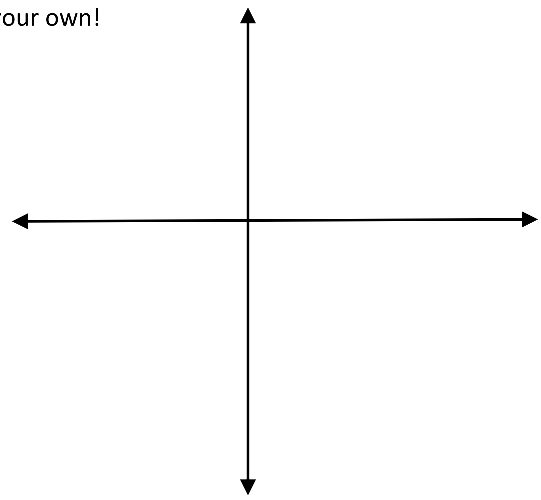
$y=x(x-3)$



$y=(2-x)(x+1)$



Add your own!



Student Activity 4

Matching cards activity: Match the cards into 7 sets taking one card from each group. Discuss reasoning and be able to explain decisions.

Set A

1. $y=(x-3)(x-3)$	2. $y=(x+2)(x+4)$	3. $y=(x+1)(3-x)$
4. $y=(x-2)(6-x)$	5. $y=(x-4)(x+2)$	6. $y=(x-4)(x-6)$

Set B

7. $y=-x^2+2x+3$	8. $y=x^2+6x+8$	9. $y=x^2-6x+9$
10. $y=x^2-10x+24$	11. $y=x^2-2x-8$	12. $y=-x^2+8x-12$

Set C

13. $y=(x-5)^2-1$	14. $y=-(x-4)^2+4$	15. $y=(x-1)^2-9$
16. $y=-(x-1)^2+4$	17. $y=(x+3)^2-1$	18. $y=(x-3)^2$

Set D

19. $x=0, y=9$	20. $x=0, y=8$	21 $x=0, y=-8$
22. $x=0, y=-12$	23. $x=0, y=3$	24. $x=0, y=24$

Set E

25. $y=0,$ $x=-1$ or 3	26. $y=0,$ $x=-2$ or $+4$	27. $y=0,$ $x=2$ or 6

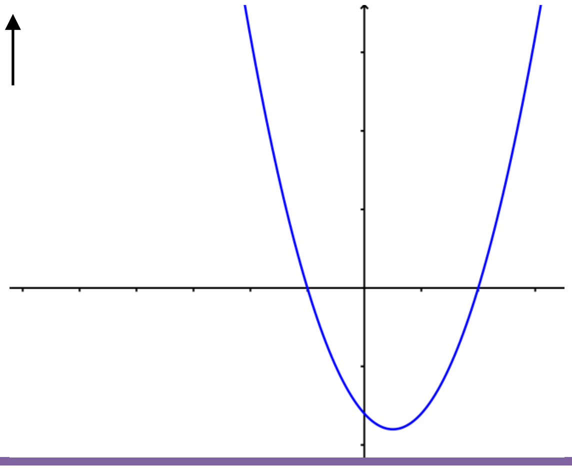
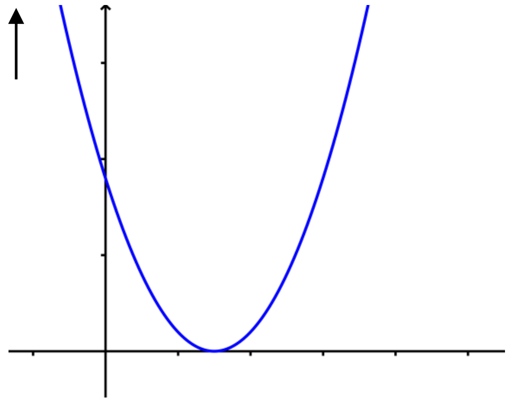
Student Activity 4

28. $y=0$, $x=4$ or 6	29. $y=0$, $x=3$	30. $y=0$, $x=-2$ or -4

Set F

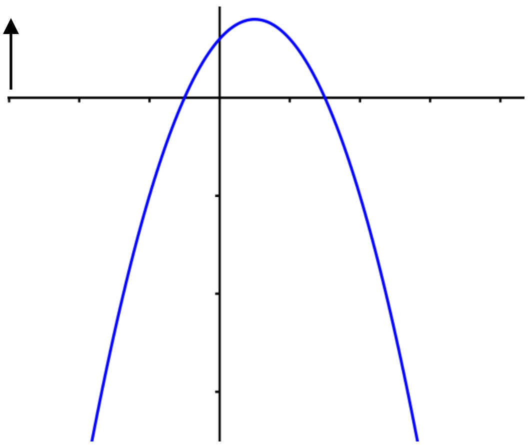
31. local maximum at (1,4)	32. local minimum at (5,-1)	33. local minimum at (1,-9)
34. local minimum at (-3,-1)	35. local maximum at (4,4)	36. local minimum at (3,0)

Set 7

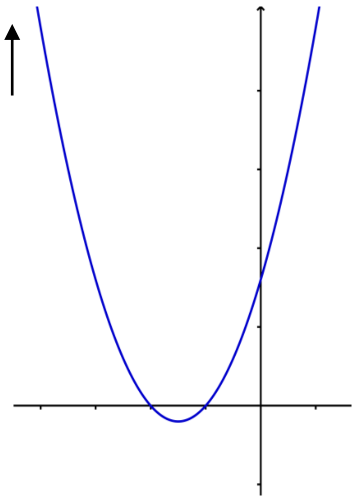
37. 	38. 

Student Activity 4

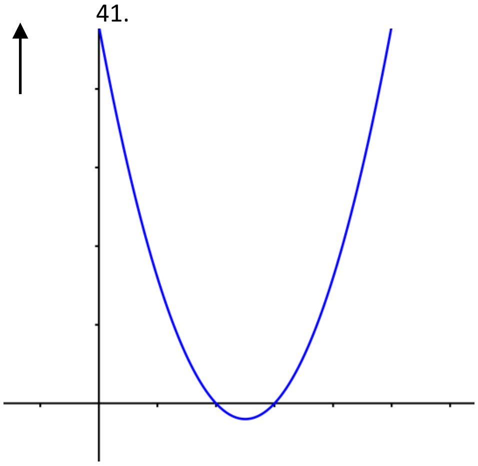
39.



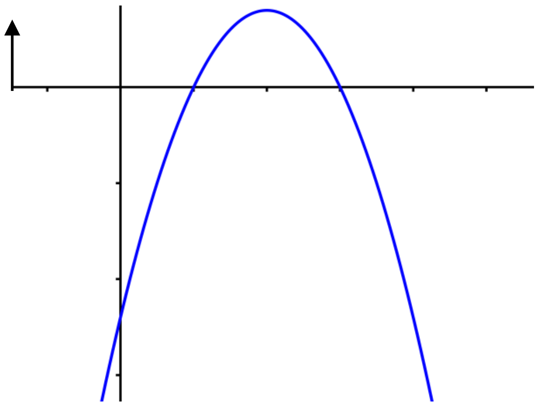
40.



41.



42.



Student Activity 5(i)

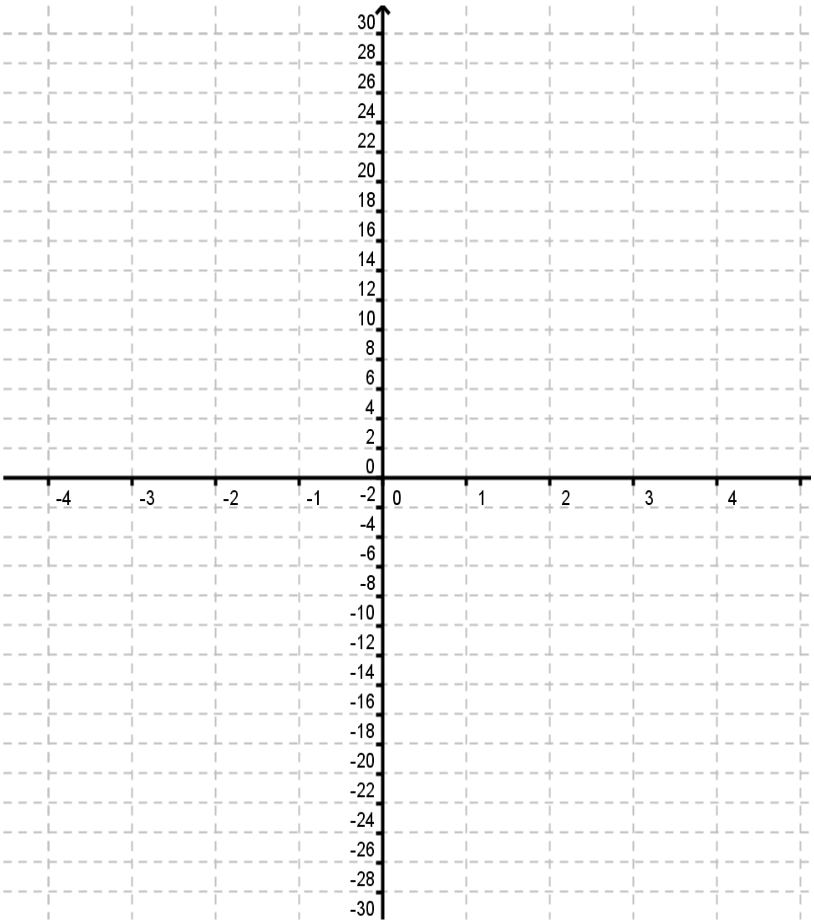
Plot the following graphs using the same axes and scales where $x \in \{-3, -2, -1, 0, 1, 2, 3\}$

(Use the "Table" mode on the calculator and verify the y values you calculate - optional)

(i) How does the graph of $y = x^3$ compare with the graph of $y = x^2$?

1. $y = x^3$	3. $y = 2x^3$
2. $y = -x^3$	4. $y = -2x^3$

x	$y = x^3$	$y = 2x^3$	$y = -x^3$	$y = -2x^3$	
-3					
-2					
-1					
0					
1					
2					
3					



(ii) How many real roots has $f(x) = x^3$? What are they?

(iii) What is the effect of the coefficient a on the graph of $y = ax^3$?

(iv) What is the effect of the sign of a on the graph of $y = ax^3$?

(vi) What transformation maps the graph of $y = x^3$ onto the graph of $y = -x^3$?

(v) For what values of x is the graph of $y = ax^3$ increasing?

(vii) What are the turning points i.e. local max and local min of $y = x^3$?

Student Activity 5(ii)

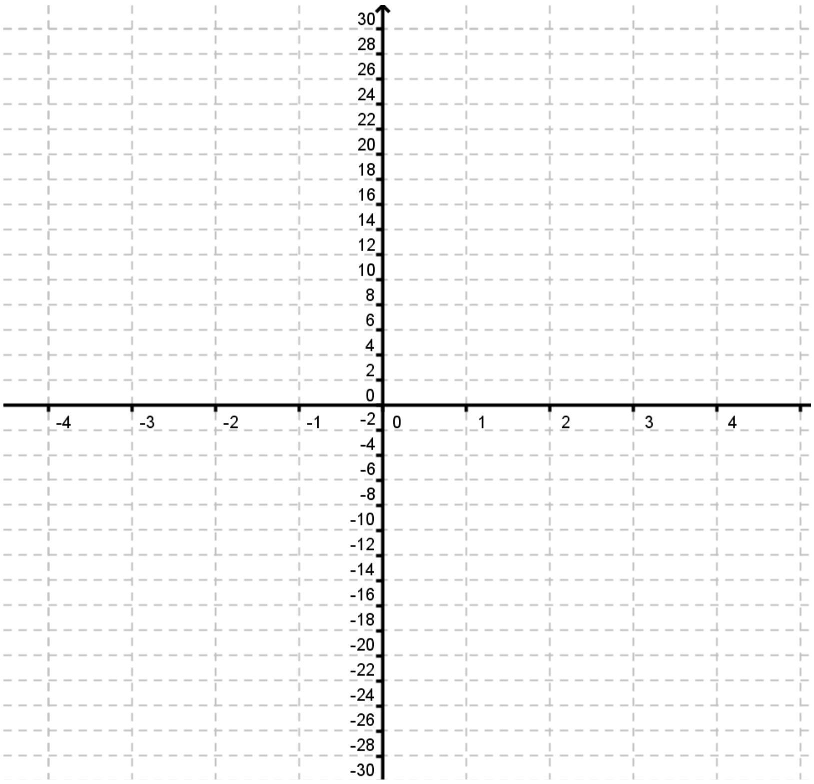
Plot the following graphs using the same axes and scales where $x \in \{-3,-2,-1,0,1,2,3\}$

(Use the "Table" mode on the calculator and verify the y values you calculate - optional)

How does the graph of $y = x^3$ compare with the graph of $y = x^2$? Use a dynamic geometry software package to check your graph.

(i) $y = x^3$	(ii) $y = x^3 - 2$
(iii) $y = x^3 + 2$	Investigate the graph of a similar cubic function

x	$y = x^3$	$y = x^3 + 2$	$y = x^3 - 2$	
-3				
-2				
-1				
0				
1				
2				
3				



(i) What is the effect of c on the graph of $y = x^3 + c$?

(ii) How many real roots has $y = x^3 + 2$?

(Link to complex numbers - find all the roots)

(iii) For what values of x is the graph of $y = x^3 + 2$ increasing?

(iv) For what values of x is the graph of $y = x^3 + 2$ positive?

Student Activity 5(iii)

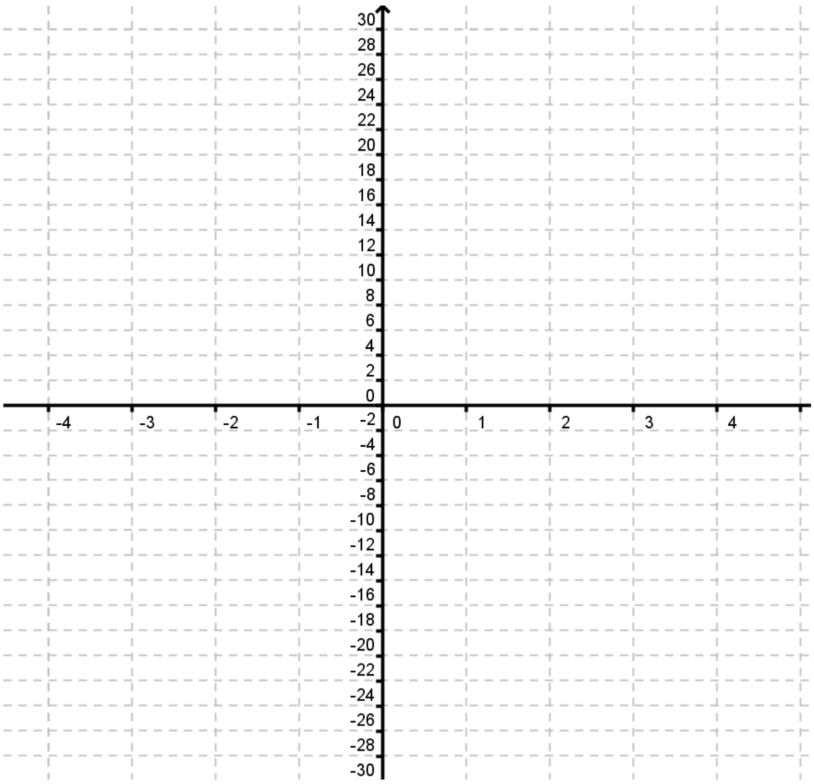
Plot the following graphs using the same axes and scales where $x \in \{-3, -2, -1, 0, 1, 2, 3\}$

(Use the "Table" mode on the calculator and verify the y values you calculate - optional)

How does the graph of $y = x^3$ compare with the graph of $y = x^2$?

(i) $y = x^3$	(iii) $y = (x-2)^3$
(ii) $y = (x+2)^3$	Investigate the graph of a similar cubic function

x	$y = x^3$	$y = (x+2)^3$	$y = (x-2)^3$	
-3				
-2				
-1				
0				
1				
2				
3				



(iv) What is the effect of p on the graph of $y = (x+p)^3$?

Student Activity 6

Plot the following graphs using the same axes and scales where $x \in \{-3, -2, -1, 0, 1, 2, 3\}$ For

(Use the "Table" mode on the calculator and verify the y values you calculate - optional)

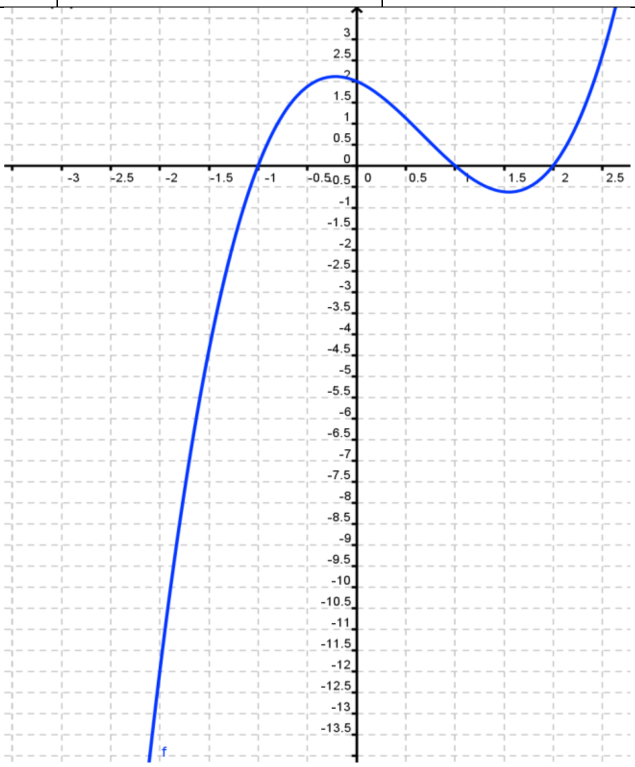
the cubic functions $f(x) = x^3 - 2x^2 - x + 2$ and $g(x) = (x+1)(x-1)(x-2)$ fill in the table below.

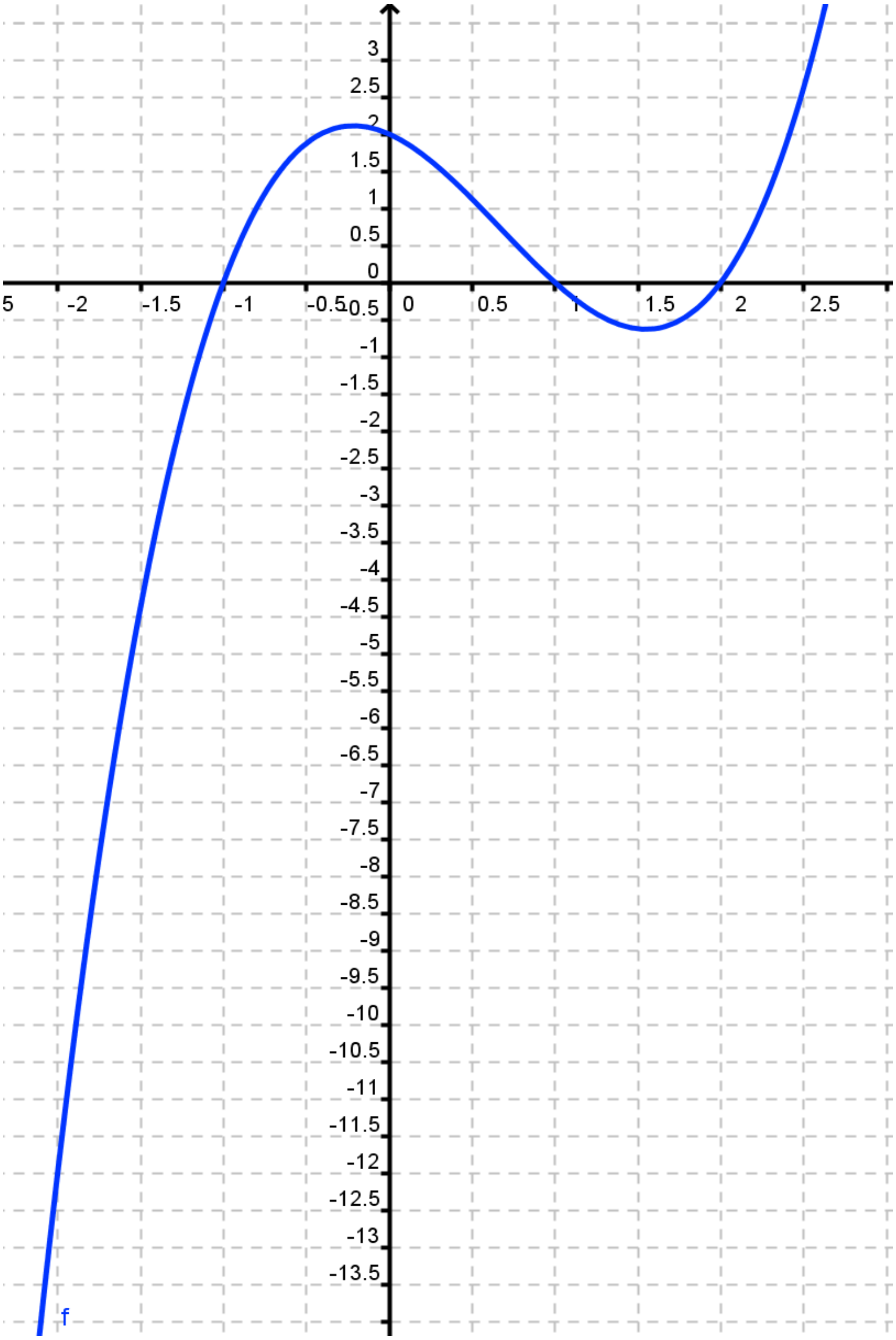
What do you notice?

Multiply out the factors of g(x) to verify your conclusion. Plot the points on the graph below or on the next page.

x	$f(x) = x^3 - 2x^2 - x + 2$	$g(x) = (x+1)(x-1)(x-2)$
-2		
-1.5		
-1		
-0.5		
0		
1		
1.5		
2		
2.5		

What is another way of writing $f(x) = x^3 - 2x^2 - x + 2$?





Fill in the table below for
$y = x^3 - 2x^2 - x + 2$

$y=0$ (roots)	
Local maximum point (approx)	
Local minimum point (approx)	

Sketch the graph of $h(x) = -f(x)$ using the axes and scales above. Fill in the table below for $h(x)$.

Which form of a cubic equation allows us to identify to identify the roots by inspection of the equation?

What transformation of the plane maps $h(x)$ onto f(x)?

Student Activity 7a

Fill in the table for the cubic function $f(x) = x^3 - 12x^2 + 36x - 7$. Mark the points on the graph.

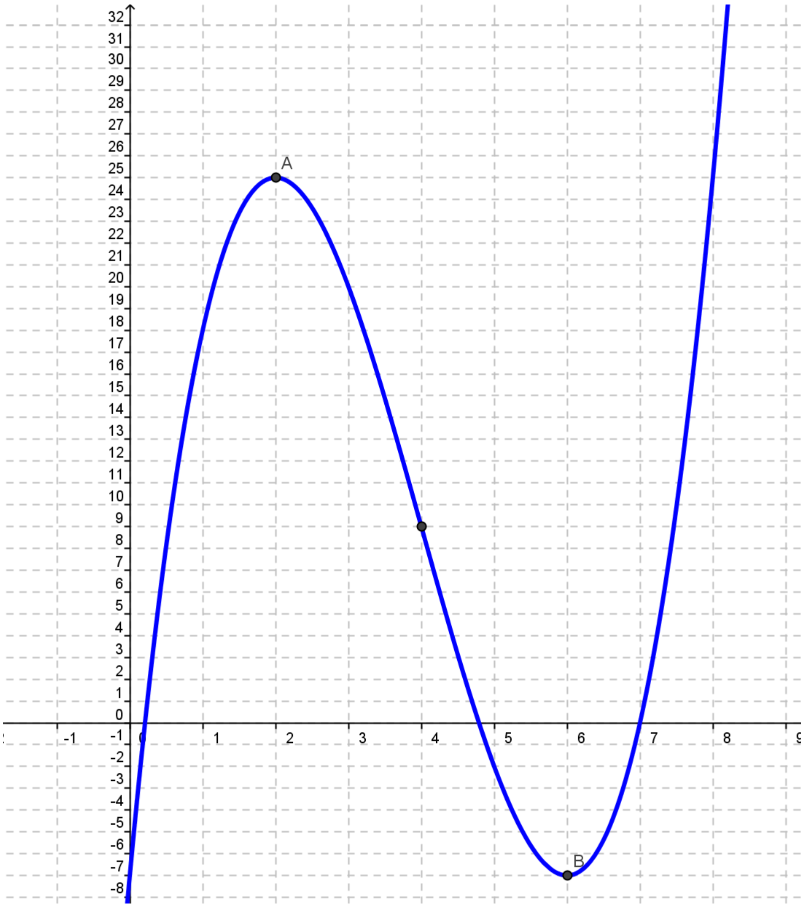
$k(x) = f(x) + 7$ **Write $k(x)$ in the form $ax^3 + bx^2 + cx + d$.**

Fill in the y values for $k(x)$ in the table below using the fact that $k(x) = f(x) + 7$.

Plot the points for function $k(x)$ and draw the graph of the function $k(x)$, using the same axes and scales as for the graph of $f(x)$.

x	$f(x) = x^3 - 12x^2 + 36x - 7$	$k(x) =$
0		
2		
4		
6		
8		

$f(x) = x^3 - 12x^2 + 36x - 7$



How many real roots has the function $f(x)$?

Estimate the real roots of $f(x) = 0$ from the graph of function $f(x)$.

How many real roots has the function $k(x)$?

Use the roots of $k(x)$ to form its equation $k(x) = x^3 - 12x^2 + 36x$

Student Activity 7b

For the cubic function $f(x) = x^3 - 2x^2 - x + 2$ fill in the table below using the graph of the function. Mark the points on the graph.

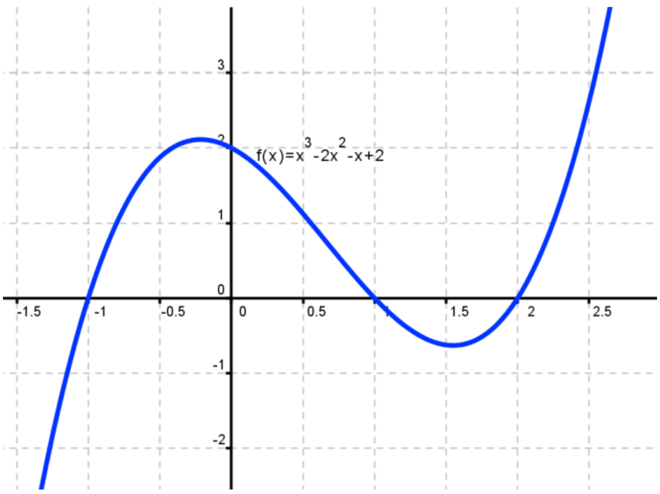
$h(x) = f(x) + 1$ **Write $h(x)$ in the form $h(x) = ax^3 + bx^2 + cx + d$.**

Fill in the y values for $h(x)$ using the fact that $h(x) = f(x) + 1$.

Plot the points for function $h(x)$ and draw the graph of the function $h(x)$, on the same axes and scales as the graph of $f(x)$.

x	$f(x) = x^3 - 2x^2 - x + 2$	$h(x) =$
-1.5		
-1		
-0.5		
0		
1.5		
2		
2.5		

$f(x) = x^3 - 2x^2 - x + 2$ $\qquad h(x) = f(x) + 1$



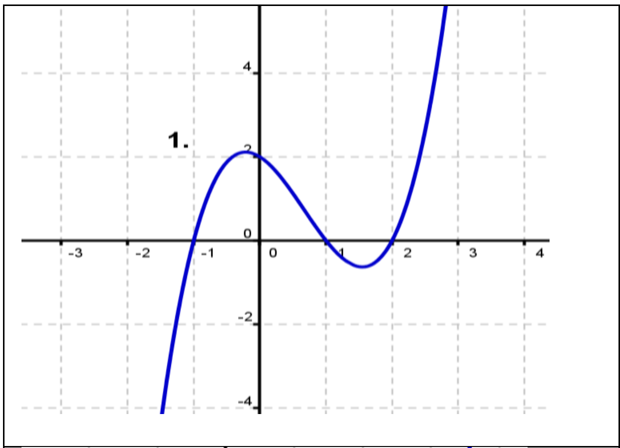
	Real Roots of f(x) =0	Turning points	Local Max. point	Local Min. point
$y = f(x)$				
$y = h(x)$				

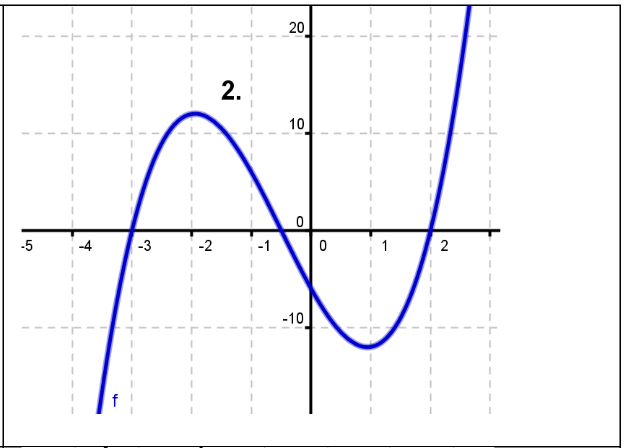
How many real roots has $y = h(x)$?

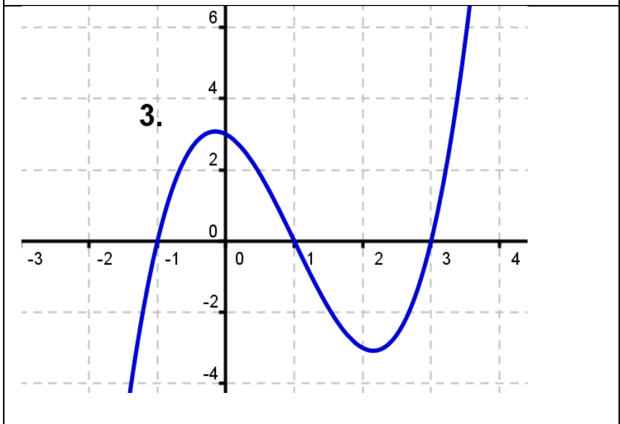
Explain your answer.

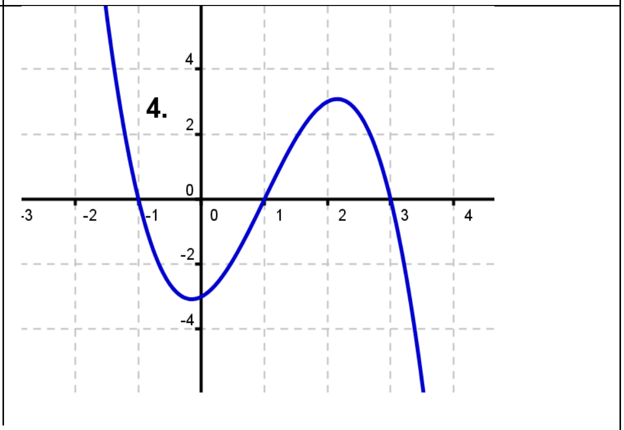
Student Activity 7c

Identify the roots of the cubic functions whose graphs are plotted below.
Hence write the equation of each function in factored form and also in the form
$y = ax^3 + bx^2 + cx + d$.

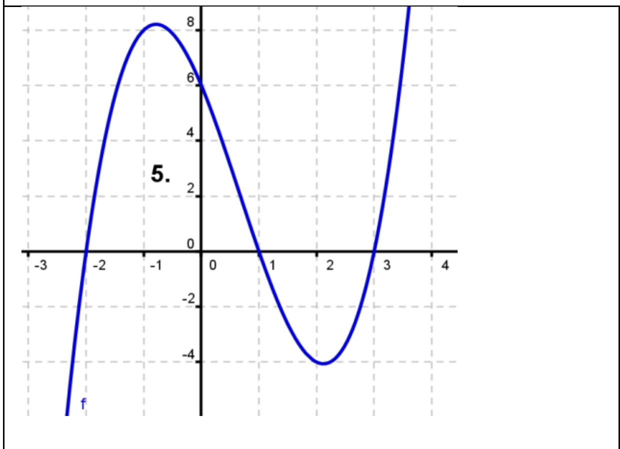


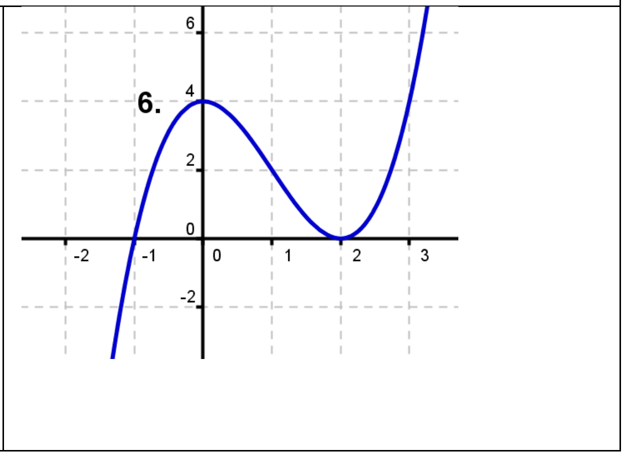






What is the relationship between the graphs of the functions displayed in 3 and 4 above?

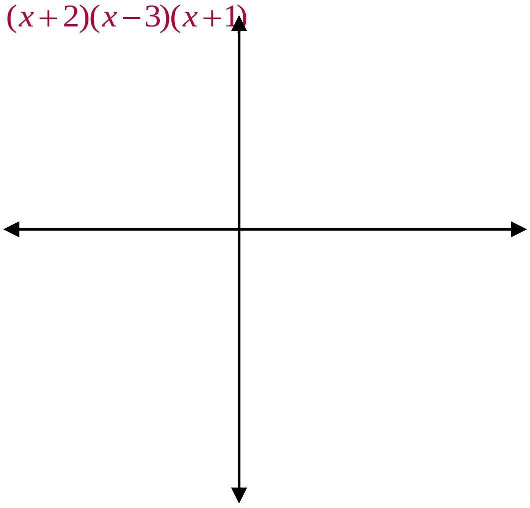




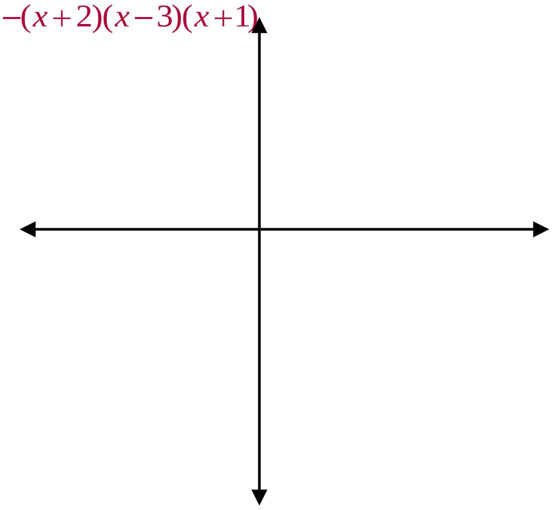
Student Activity 7d

Working in pairs, **sketch** the following graphs on the axes below.
Note particularly the intercepts and whether or not the vertex of the graph is a local maximum or local minimum.
Verify that you are correct by using a graphing calculator or graphing software such as GeoGebra if you have access to these. Alternatively use the "Table" mode on your calculator to verify points.

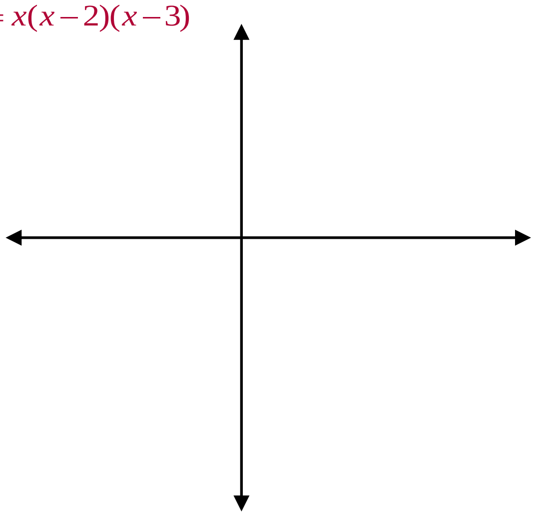
$y=(x+2)(x-3)(x+1)$



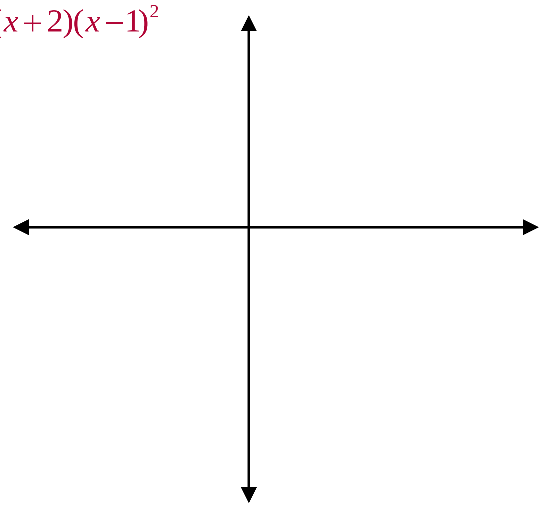
$y=-(x+2)(x-3)(x+1)$



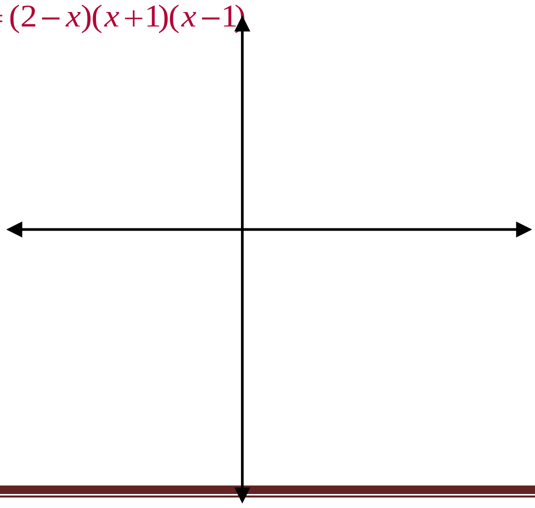
$y=x(x-2)(x-3)$



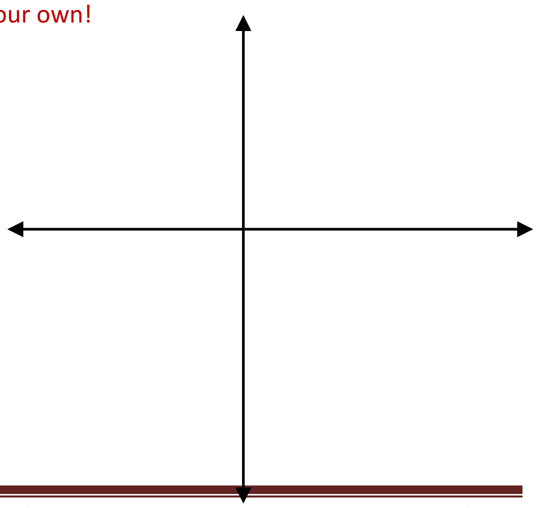
$y=(x+2)(x-1)^2$



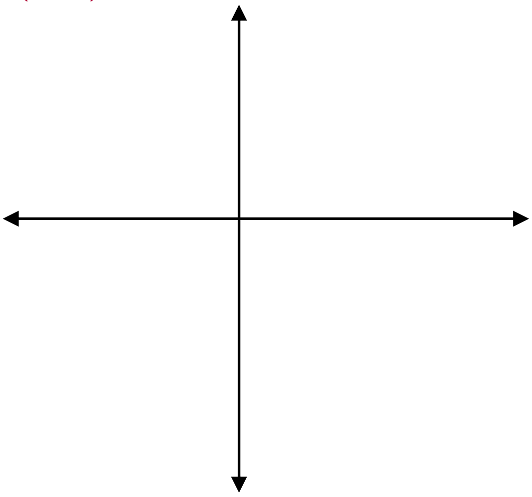
$y=(2-x)(x+1)(x-1)$



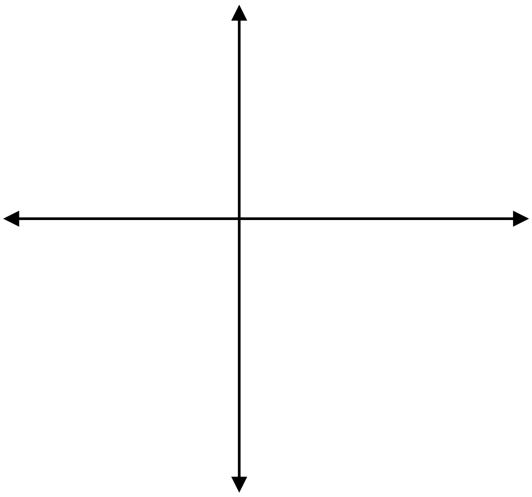
Add your own!



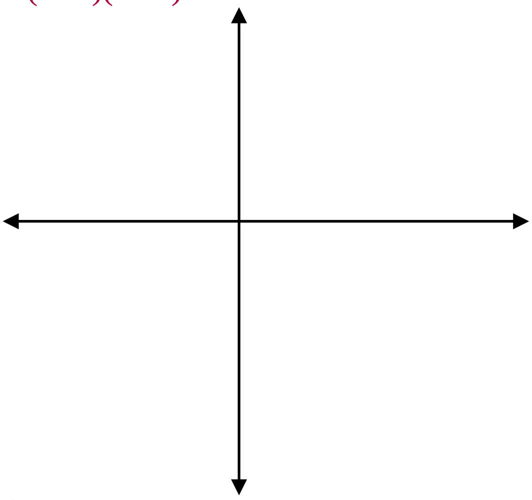
$y = (x-4)^3$



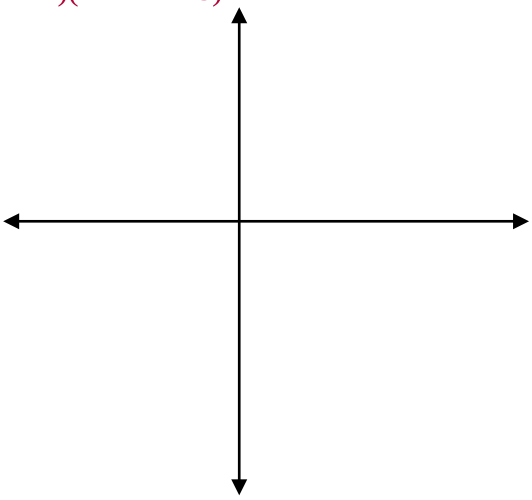
$y = -(x-4)^3$



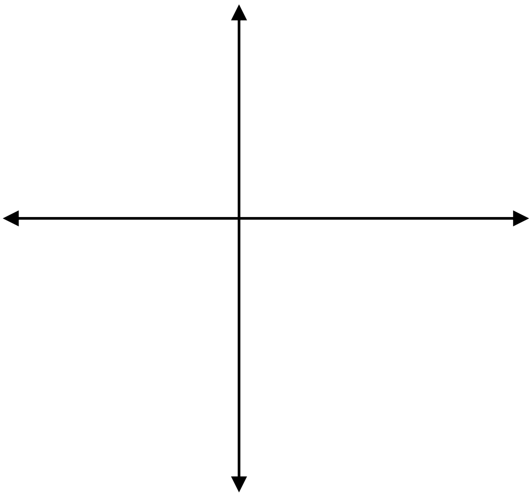
$y = (1-x)(x-3)^2$



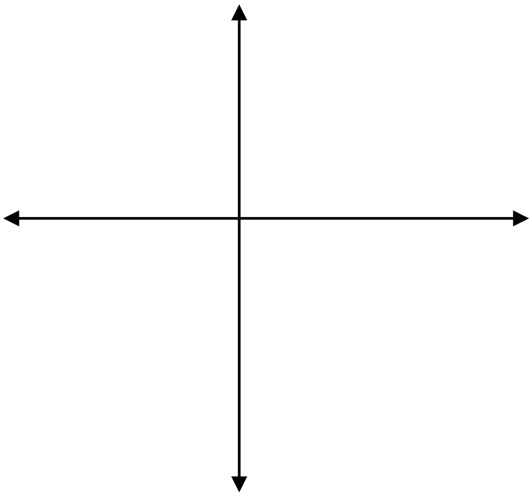
$y = (x+2)(x^2-4x+5)$



Add your own!

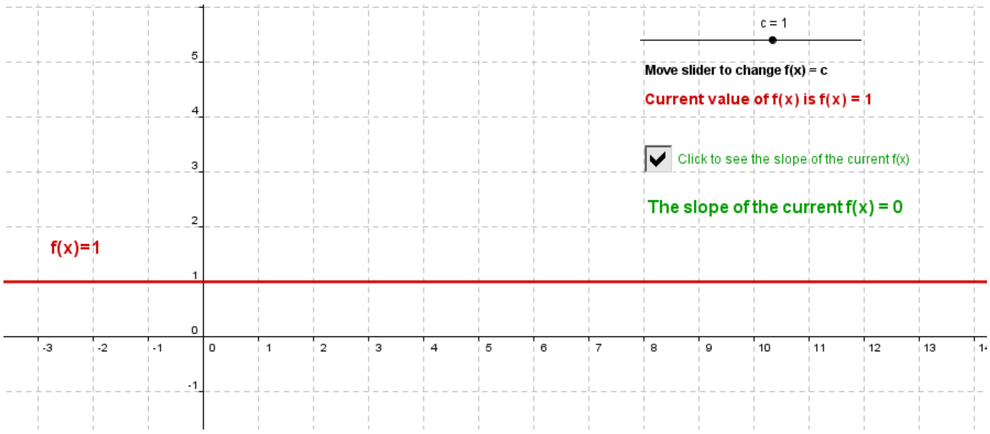


Add your own!



Student Activity: To investigate the Derivative of a Constant Function

Use in connection with the interactive file, 'Derivative of a Constant Function', on the Student's CD.

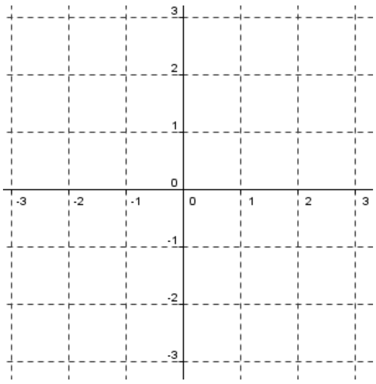


1. What is the slope of the line f(x) = 1? Is it the same at all point on the line?

   ____

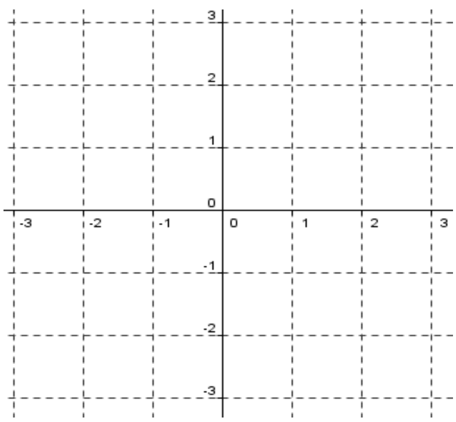
2. Draw the line f(x) = 3. What is its slope? Explain your reasoning.

   ____



3. Draw the line f(x) = −2. What is the slope of this line? Can you give the equation of another line having this slope?
   Complete the statement: All lines parallel to the x-axis have slope________________

   ____



4. Write the equation of the x axis in the form f(x) = c. What is the slope of the x axis?

____

5. What is the slope of any line that takes the form f(x) = c, where $c \in R$?

____

6. Given that the derivative of a function at a particular point on the graph is equal to the slope of the function at that point, what is the derivative of f(x) = c for all points on f(x), where $c \in R$?

____

7. Find the derivative of the following functions for all values of $x \in R$:
   a. f(x) = 2

      ____

   b. f(x) = 10

      ____

   c. f(x) = −5

      ____

   d. f(x) = −¾

      ____

8. Given that $\frac{dy}{dx}$ is the derivative of y with respect to x, find $\frac{dy}{dx}$ when y = 12.

____

9. Given that f′(x) is the derivative of f(x) with respect to x, find f′(x) when f(x) = −4.

____

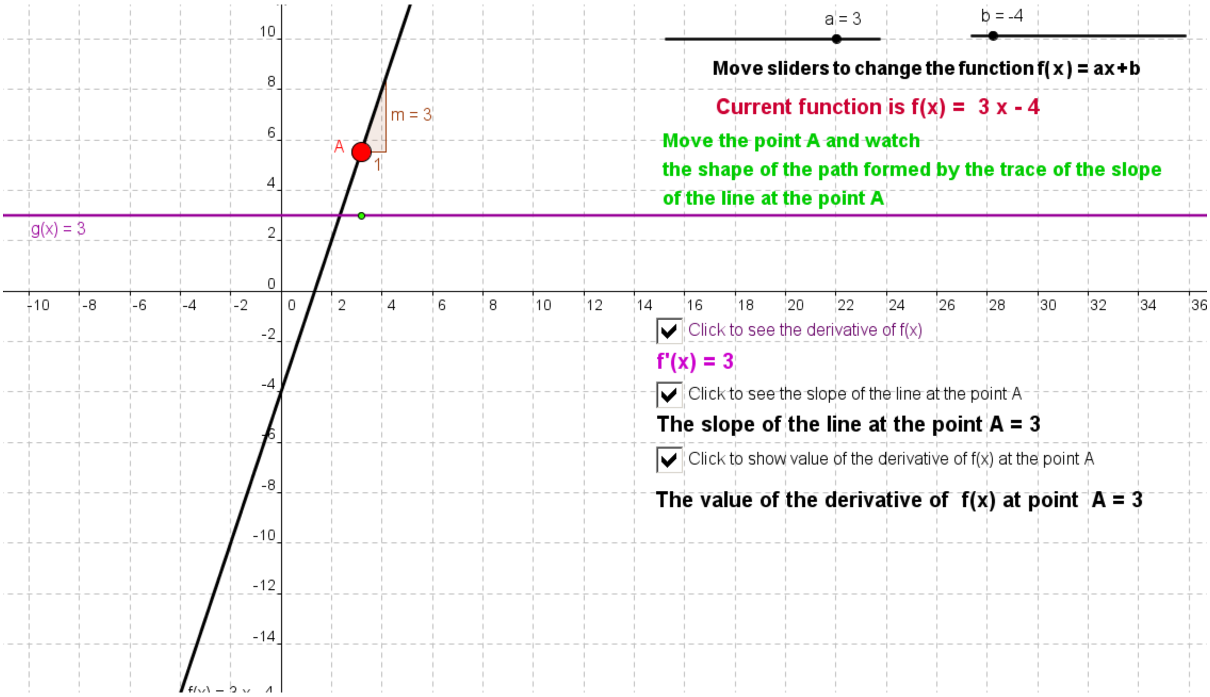
10. From your work above, what can you conclude about the derivative of a constant? Explain your reasoning.

____

____

Student Activity: To investigate the Derivative of a Linear Function

Use in connection with the interactive file, 'Derivative of a Linear Function', on the Student's CD.



<u>Note: Each time the sliders are changed when using this interactive file, one needs to click the reset button at the top right hand side of the screen.</u>

1. In the interactive file, what path does the point A follow and what shape is the path followed by the point A?

   ____

2. As the point A moves, what happens to the slope of the line?

   ____

3. In the interactive file, as the point A moves describe the path followed by the trace of the slope.

   ____

4. Click the button and set the sliders to the following values: a = 3 and b = 1. Describe the graph of the function f(x).____
5. With the sliders at these values, move the point A and describe the path followed by the trace of the slope.

   ____

6. Click the check box on the interactive file and note the equation of the derivative of the function.

____

7. Click the checkboxes to show the slope of the line at the point A and the value of the derivative of the function at the point A. As A moves along the curve of the function f(x), what do you notice about these values?

____

8. What do you notice about the path followed by the trace of the slope of the line and the graph of the derivative of the function?

____

9. Change sliders a and b, and move the point A as before. Is the relationship between the path followed by the trace of the slope and the graph of the derivative of the function the same as in Q8 above?

   ____

   Repeat this process at least five times and check if the relationship exists in all these cases.

   ____

   ____

   ____

   ____

   ____

10. Given a linear function, what can you conclude about the graph of its derivative?

    ____

11. What can you conclude about the derivative of a linear function and the slope of the graph of the function?

    ____

12. By moving the sliders in the interactive file, what can you conclude about the derivative of f(x) = 3x−4?

    ____

13. Find the derivatives of the following functions. (Check your answers using the interactive file.)

a. $f(x) = 3x + 4$

____

b. $f(x) = 3x - 4$

____

c. $f(x) = -3x + 1$

____

d. $f(x) = 4 - 2x$

____

e. $f(x) = x$

____

14. What is the derivative of $f(x) = mx+c$?

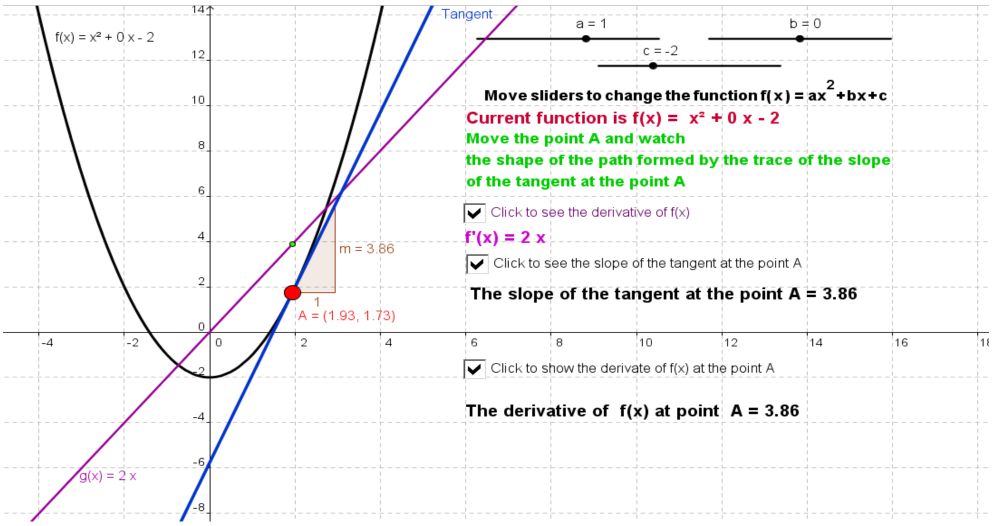
____

15. Draw the graphs of the function $f(x) = 2x+5$ and its derivative.

16. Draw the graphs of the function $f(x) = -2x + 5$ and its derivative.

Student Activity: To investigate the Derivative of a Quadratic Function

Use in connection with the interactive file, 'Derivative of a Quadratic Function', on the Student's CD.



Note: Each time the sliders are changed when using this interactive file, one needs to click the reset button at the top right hand side of the screen.

1. In the interactive file, describe shape of the path followed by the point A?

   ____

2. As the point A moves, what happens to the slope of the tangent to the curve at the point A?

   ____

3. In the interactive file, as the point A moves, describe the path followed by the trace of the slope of tangent to the curve at the point A

   ____

4. Click the button and set the sliders to the following values: a = 1, b = 4 and c = 1. Describe the graph of the resulting function.

   ____

5. With the sliders at the above values, move the point A and note the shape of the path formed by the trace of the slope of the tangent to the curve at the point A.

   ____

6. Click the check box on the interactive file and note the equation of the derivative of the function.

   ____

7. Click the checkboxes to show the slope of the tangent to the curve at the point A and the value of the derivative of the function at the point A. As A moves around the curve, what do you notice about these?

____

8. What do you notice the path followed by the trace of the slope of the tangent to the curve at the point A and the graph of the derivative of the function?

____

9. Change some or all of the sliders a, b, and c. and move the point A as before. Is the relationship between the path followed by the trace of the slope and the graph of the derivative of the function the same as in Q8 above?

____

Repeat this process at least five times and check if the relationship exists in all these cases.

____
____
____
____
____

10. Given a quadratic function, what type of function do you expect the derivative to be?

____

11. Given the slope of the tangent at a point, what can you conclude about the derivative of the function at that point?

____

12. By moving the sliders in the interactive file, determine what is the derivative of :
    i. x^2
    ii. $2x^2$
    iii. $3x^2$
    iv. $4x^2$

13. Given $f(x) = ax^2$, can you suggest a rule to find the derivative of this function. Verify your answer using the Tables and Formula booklet.

____

14. By moving the sliders in the interactive file, what can you conclude about the derivative of $f(x) = 3x^2 - 4x$?

____

15. Find the derivatives of the following functions. (Check your answers using the interactive file.)

   a. $f(x) = 2x^2 + 4x + 3$

   ____

   b. $f(x) = 2x^2 + 4x + 9$

   ____

   c. $f(x) = 2x^2 + 4x - 9$

   ____

   d. $f(x) = 2x^2 + 4x - 94$

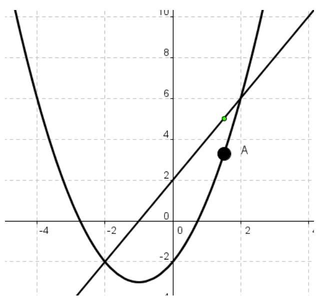
   ____

   e. $f(x) = x^2$

   ____

16. The diagram shows the graph of the function

$f(x) = x^2 + 2x - 2$ and its derivative. What is the equation of the line?

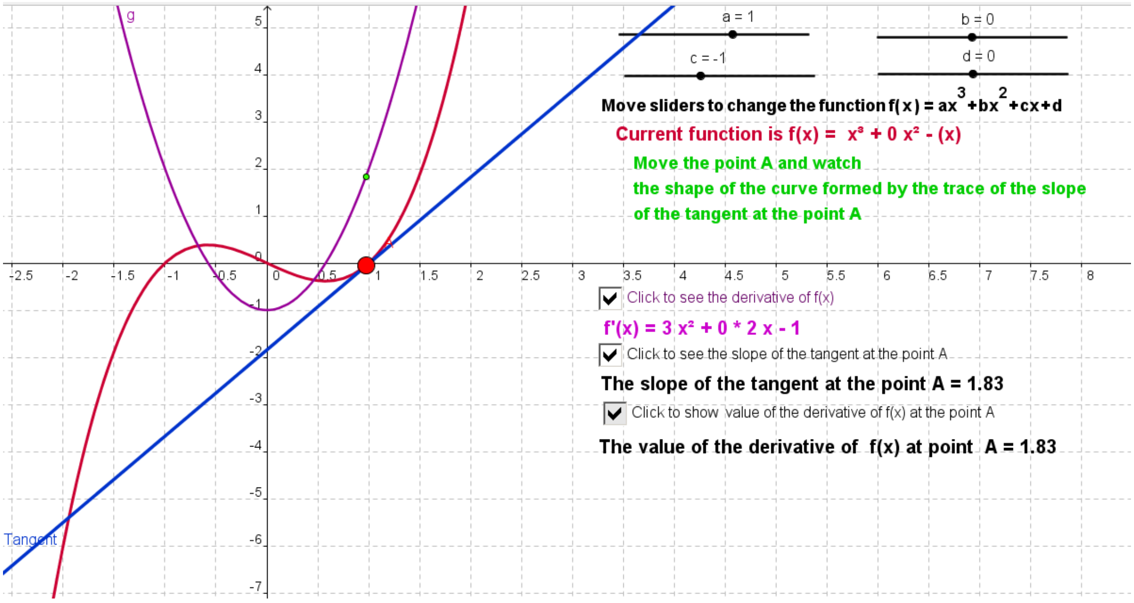


17. What is the derivative of $f(x) = ax^2 + bx + c$?

____

Student Activity: To investigate the Derivative of a Cubic Function

Use in connection with the interactive file, 'Derivative of a Cubic Function', on the Student's CD.



Note: Each time the sliders are changed when using this interactive file, one needs to click the reset button at the top right hand side of the screen.

1. In the interactive file, describe the path followed by the point A?

   ____

2. A tangent is drawn to the curve at the point A. As the point A moves, describe what happens to the slope of the tangent. Why is this important?

   ____

   ____

3. As the point A moves, what is the shape of the curve formed by the trace of the slope of the tangent to the curve at the point A?

   ____

4. Click the button and set the sliders to the following values: a = 1, b = 1, c = -2 and d = 2. Describe the shape of the graph of the function.

   ____

   ____

5. Now move the point A and describe the shape of the curve formed by the trace of the slope of the tangent to the curve at the point A.

____

6. Click the check box on the interactive file to reveal the derivative of f(x). What type of expression is used to describe the derivative?

____

7. Click the remaining checkboxes to show the slope of the tangent at the point A and the value of the derivative at the point A. As A moves around the curve, describe what happens to these values?

____

8. Describe the path followed by the trace of the slope of the tangent to the curve at A as A moves around the curve. How does this compare with expression that represents the derivative of the function?

____

9. Change some or all of the sliders a, b, c and d. and move the point A as before. Is the relationship between the path followed by the trace of the slope of the tangent and the graph of the derivative of the function the same as in Q8 above?.

____

Repeat this process at least five times and check if the relationship exists in all these cases.

____
____
____
____
____

10. Given a cubic function, what can you conclude about the shape of the derivative of this curve?

____

11. What can you conclude about the value derivative of the function at a given point the slope of the tangent at that point?

____

12. Move the sliders in the interactive file and, in each case, describe the derivative of the following functions: (Note: 0*2x = 0)
    a. $f(x)=x^3$
    b. $f(x)=2x^3$
    c. $f(x)=4x^3$
    d. $f(x)=ax^3$
13. Find the derivatives of the following functions. (Check your answers using the interactive file.)
    a. $f(x) = x^3 + 4$

    ____

    b. $f(x) = x^3 + 4x$

    ____

    c. $f(x) = x^3 + x^2$

    ____

    d. $f(x) = x^3 + 2x^2$

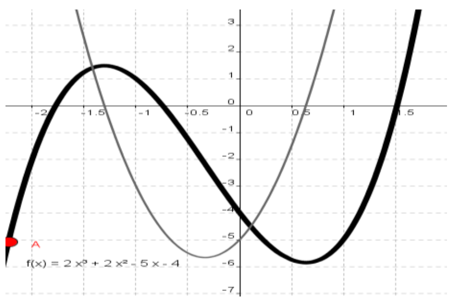
    ____

    e. $f(x) = x^3 + 2x^2 + 4x + 4$

    ____

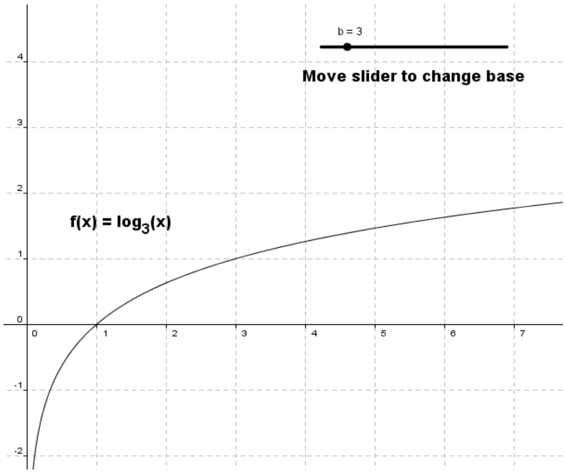
14. The diagram below shows the function $f(x) = 2x^3 + 2x^2 - 5x - 4$ and its derivative. What is the equation of this curve will be? Check your answer using the interactive file.

____



<u>Student Activity:</u> To investigate the graph of $\log_n x$

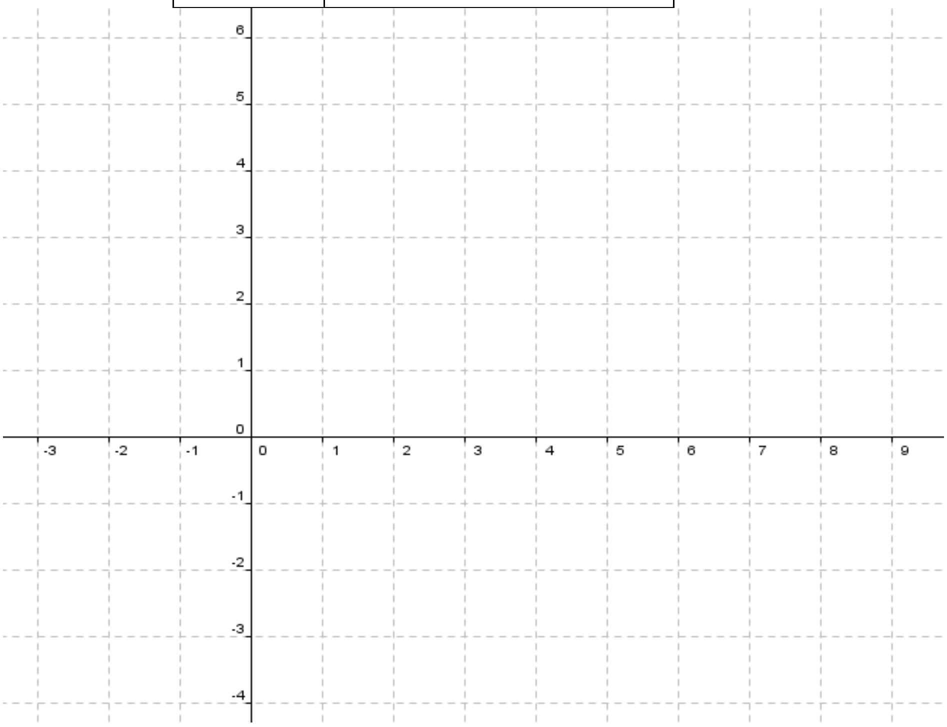
Use in connection with the interactive file, 'graph $\log_n x$', in the Student's CD.



1)

a) Complete the following table using your calculator and draw the graph f(x) = $\log_2 x$.

x	$\log_2 x = \dfrac{\log_{10} x}{\log_{10} 2}$
0	
1/8	
1/4	
1/2	
1	
4	
8	



b) Where does this graph cut the x-axis?

____

c) Determine from your graph an approximate value for $\log_2 7$.

____

d) Explain in your own words, why it is that the graph tends towards the y-axis as x tends towards zero.

____

____

e) Could the point (64, 6) lie on the graph $f(x) = \log_2 x$? Explain your answer; you may test on a calculator if necessary.

____

____

f) Could the point (56, 8) lie on the graph $f(x) = \log_2 x$? Explain your answer; you may test on a calculator if necessary.

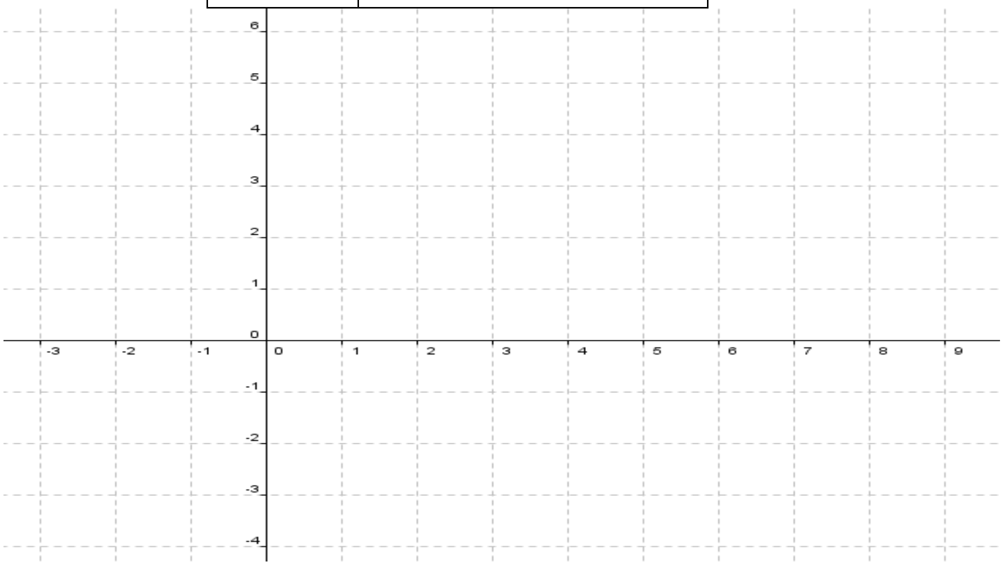
____

____

2)

a) Complete the following table using your calculator and draw the graph $f(x) = \log_3 x$.

x	$\log_3 x = \dfrac{\log_{10} x}{\log_{10} 3}$
0	
1/27	
1/9	
1/3	
1	
9	



b) Where does this graph cut the x-axis?

____

c) Determine from your graph an approximate value for $\log_3 10$.

____

d) Explain in your own words, why it is that the graph tends towards the y-axis as x tends towards zero.

____

____

e) Could the point (243, 5) lie on the graph $f(x) = \log_3 x$? Explain your answer; you may test on a calculator if necessary.

____

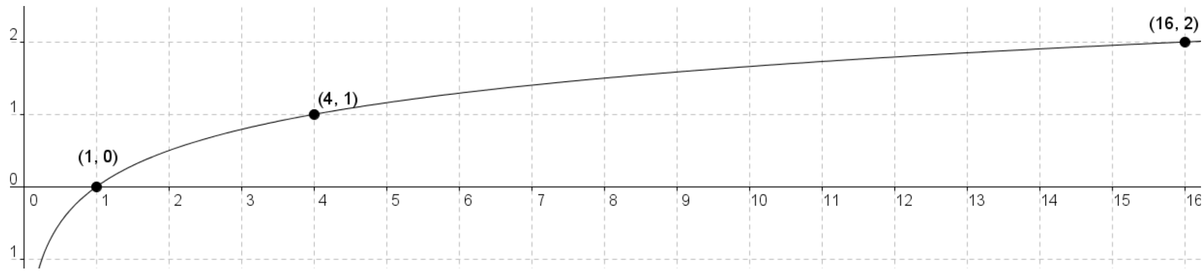
____

f) Could the point (56, 6) lie on the graph $f(x) = \log_3 x$? Explain your answer; you may test on a calculator if necessary.

____

____

3)



a. If you know this graph represents $f(x) = \log_b x$, use the interactive file to find what value b represents.

____

b. Verify algebraically, using indices, the answer you got for b above.

____

____

4) If $4 = \log_2 x$, calculate the numerical value of x.

____

____

5) Using the interactive file find approximate values for the following:

a) $\text{Log}_4 17$

____

____

____

b) $\log_2 10$

____

____

____

c) $\log_3 10$

____

____

____

d) $\log_5 19$.

____

____

____

6) Could the point (-2, 8) be found on the graph $f(x) = \log_b x$, for any possible value of b. Explain your answer.

____

____

7) List 4 points that would be found on the graph $f(x) = \log_4 x$.

____

8) Name 1 point that will always be on the curve $f(x) = \log_b x$, no matter what positive other than one value that b has. Explain why.

____

9) Describe 3 characteristics of the shape of the curve $f(x) = \log_b x$.

____

____

____

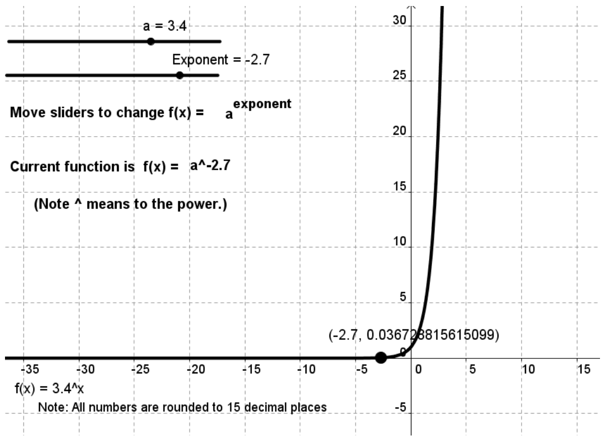
10) If $y = \log_b x$, write x in terms of b and y.

____

____

Student Activity: To investigate a^n

Use in connection with the interactive file, ‘Exponential Function’, on the student’s CD.

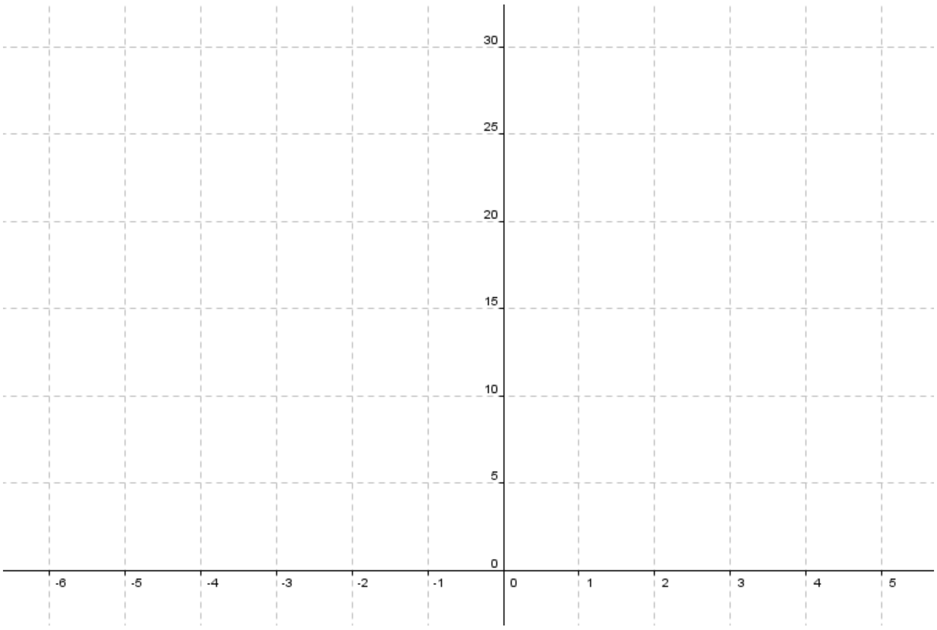


1.

a. Complete the following table:

x	2^x
5	
4	
3	
2	
1	
0	
-1	
-2	
-3	
-4	

b. Draw the graph of the data represented in the table above.



c. This graph is getting closer and closer to the x-axis. Will it ever touch it? Explain.

____

____

d. Is this an example of a linear, quadratic or exponential function? Explain your reason.

____

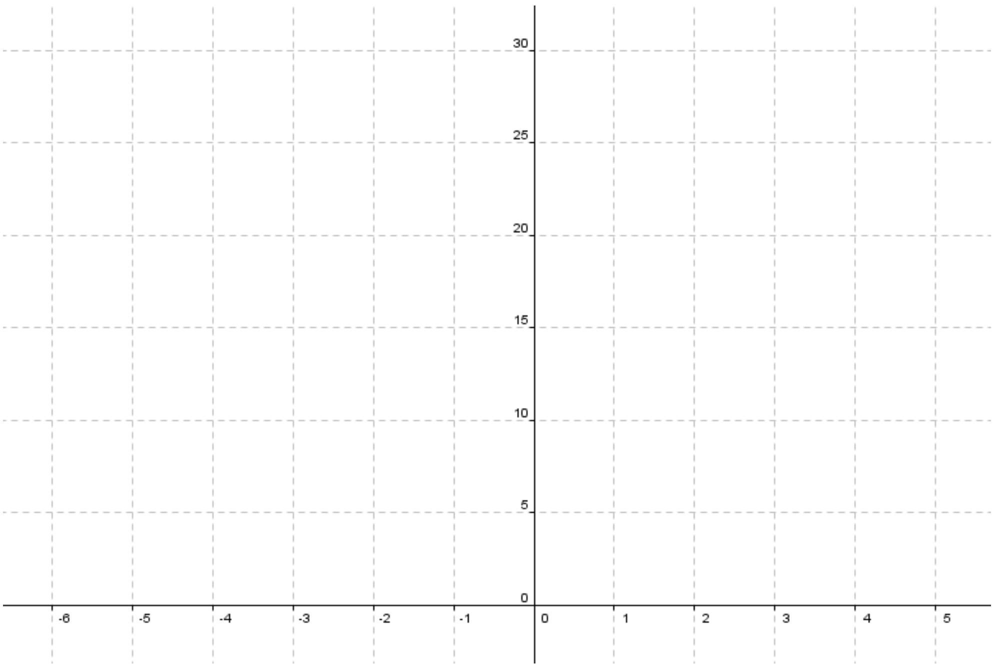
____

2.

a. Complete the following table:

x	3^x
3	
2	
1	
0	
-1	
-2	
-3	
-4	

b. Draw the graph of the data represented in the table above.



c. What do you notice about the graph for values of x less than one?

d. This graph is getting closer and closer to the x-axis. Will it ever touch it? Explain.

e. Is this an example of a linear, quadratic or exponential function? Explain your reason.

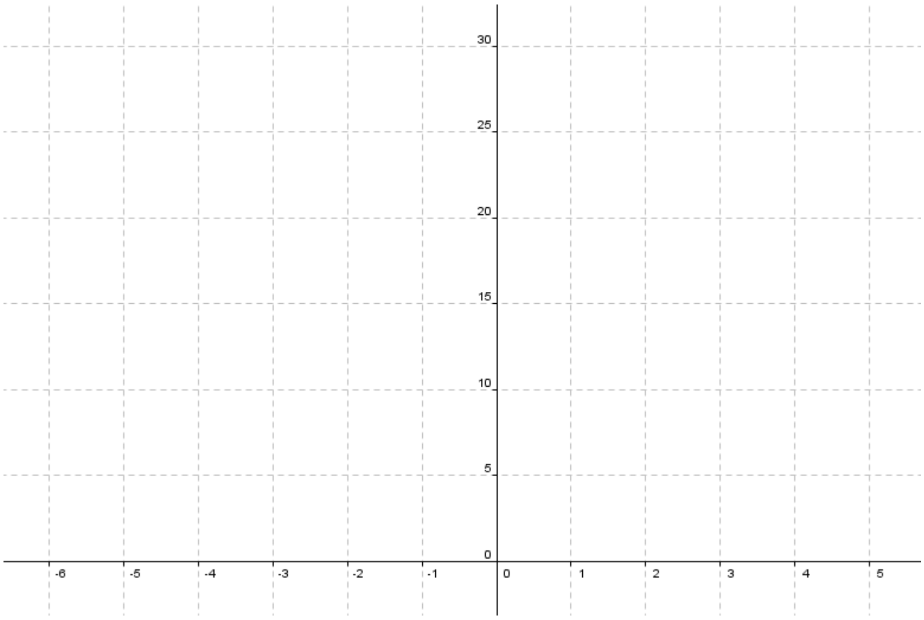
3. What will the shape of the graph a^x be, where $a \in N$ and $x \in R$?

4.

a. Complete the following table:

x	0.5^x
3	
2	
1	
0	
-1	
-2	
-3	
-4	

b. Draw the graph of the data represented in the table above.



c. When x is greater than zero what do you notice about the graph?

d. This graph is getting closer and closer to the x-axis. Will it ever touch it? Explain.

e. Is this an example of a linear, quadratic or exponential function? Explain your reason.

5. Using the interactive file describe what happens to the shape of the graph as 'a' varies from 1.1 to 5 while the exponent value remains unchanged. Explain this in terms of the rate of change of f(x).

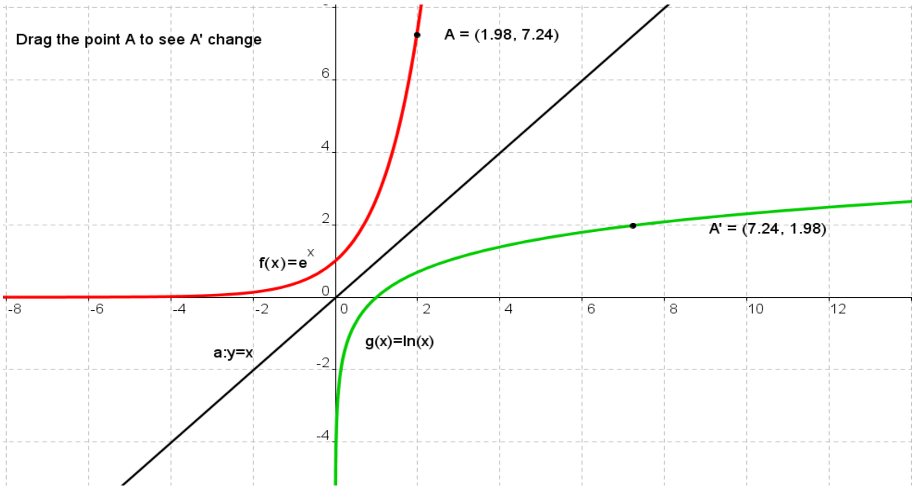
6. Using the interactive file describe what happens to the shape of the graph when 'a' equals 1? Explain why this happens in terms of the rate of change of f(x).

7. Using the interactive file describe what happens to the shape of the graph as 'a' goes from ·9 to ·1 while the exponent value remains unchanged. Explain this in terms of the rate of change of f(x).

Student Activity: To investigate f(x) = e^x and g(x) = ln(x)

Use in connection with the interactive file, 'e^x and ln(x)', on the student's CD.

Note Ln(x) = $\log_e x$.



1. Use a calculator to find an approximate value for e correct to 3 decimal places.

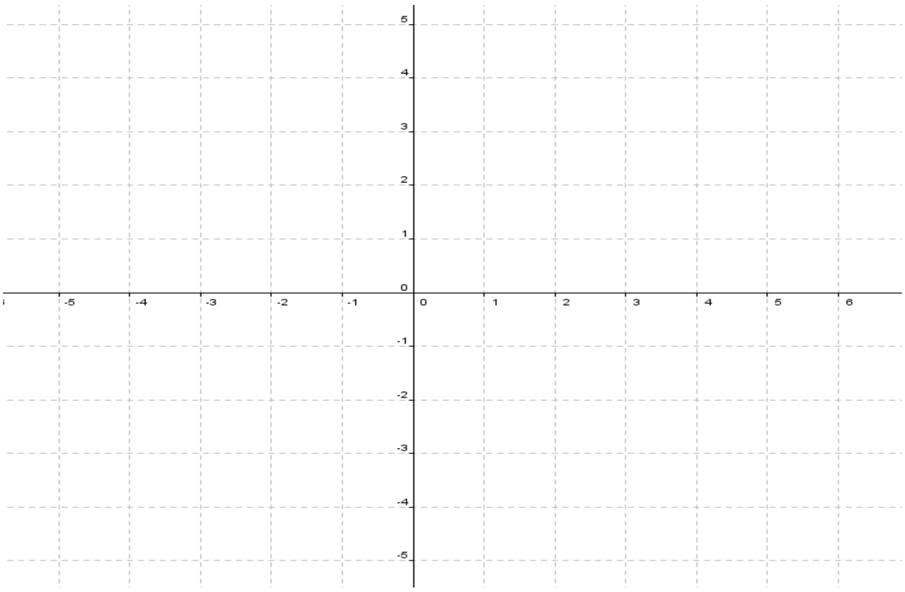
2.

e. Complete the following table giving answers correct to 3 decimal places.

x	$y = e^x$	ln(y)
3		
2		
1.5		
1		
0.5		
0		
-0.5		
-1		
-1.5		

f. What is the relationship between ln(y) and x in the above table?

g. On the same axis and scale draw the graphs of f(x) =e^x and $f^{-1}(x)$ using the data provided in the table above.



h. Using the interactive file complete the following table for any 4 values of A and the corresponding values of A’ and state what pattern you notice.

A (x, y)	A’(x, y)

____

i. What do you notice about the shapes of these graphs in relation to each other?

____

j. Given $e^{1.34} = 3.82$, what will ln(3.82) equal?

____

k. Given ln(0.33) = −1.1, what is e $e^{-1.1}$?

____

l. What line is e^x reflected in to give ln(x)?

____

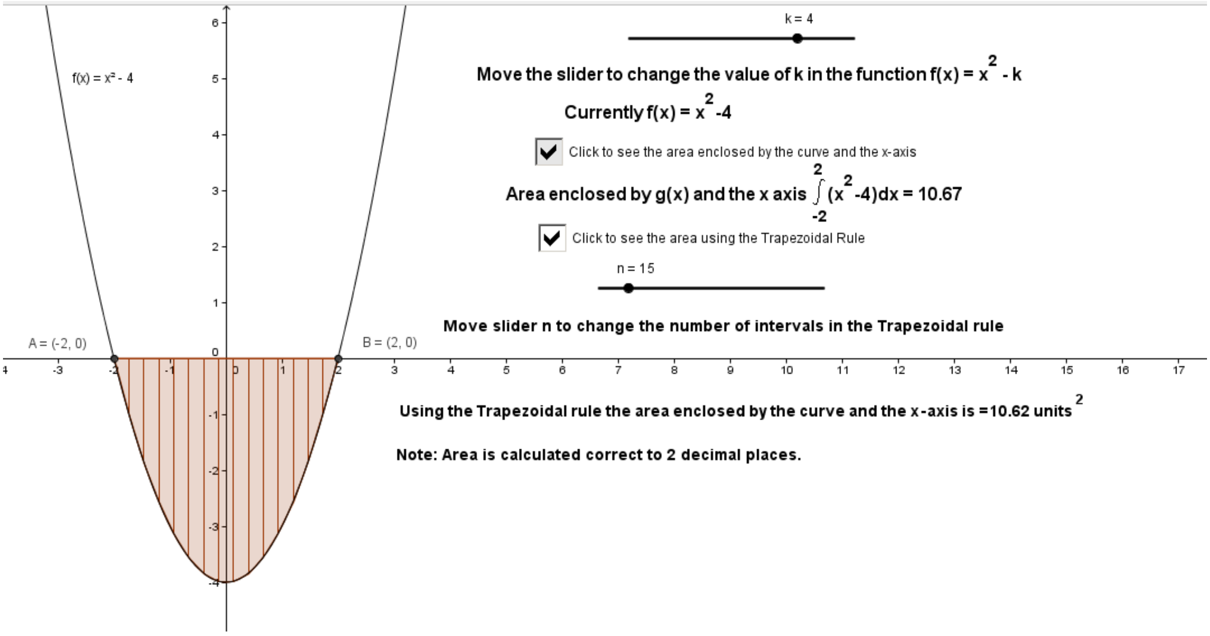
m. What conclusion have you arrived at with regard to the relationship between the function e^x and ln(x)?

____

____

Student Activity: To investigate the relationship between integration of a function and the area enclosed by the curve that represents the function and the x-axis

Use in connection with the interactive file, 'Integration and Area 1', on the Student's CD.



Note: It is understood students will have already covered the skill of integration and understand that the integral in an interval is equal to the area enclosed by the curve representing the function and the x-axis before commencing this lesson.

1.

   a. In the interactive file, find the points A and B, where the curve of the function $f(x) = x^2 - 4$ cuts the x-axis?

   ____

   b. Calculate $\int_a^b (x^2 - 4)\, dx$, where a = the x co-ordinate of the point A and b is equal to the x co-ordinate of the point B.

   ____

   ____

   c. Hence, write down is the area enclosed by the x-axis and the curve representing the function $f(x) = x^2 - 4$ in the interval [A, B]? Note: Area enclosed by the x-axis and the curve representing the function is the absolute value of the integral.

   ____

d. Using the Trapezoidal Rule in the interactive file, what is the area enclosed by the x-axis and the curve representing the function $f(x) = x^2 - 4$ in the interval [A, B]?

___

e. What is the effect of increasing the slider n?

___

f. When n=100, what do you notice about the area found by integration and the area found using the Trapezoidal Rule?

___

___

g. Use the integral of $f(x) = x^2 - 4$ to determine if the y-axis bisects the area enclosed by the x-axis and the curve of the function $f(x) = x^2 - 4$ in the interval [A, B]?

___

___

2. Given a = −2 and b = 2, what is the difference between $\int_b^a (x^2 - 4)\,dx$ and $\int_a^b (x^2 - 4)\,dx$? ? Note changing the limits won't ALWAYS give a positive outcome, it will have the reverse effect in some instances, hence we find the absolute value of the integral

___

___

___

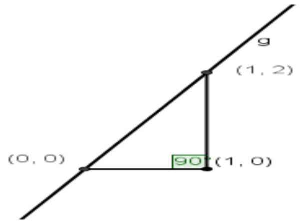
3. Calculate $\int_{-1}^{1} (x^2 - 1)\,dx$ and hence determine the area enclosed by the curve representing the function $f(x) = x^2 - 1$ and the x-axis in the interval [−1, 1]. Check your answer using the interactive file.

___

___

___

4.



a. Find the area of the triangle represented in the diagram using length of the base by the height.

____

b. Find the area of the triangle shown in the diagram using co-ordinate geometry.

____

c. Find the area of the triangle shown in the diagram using trigonometry.

____

d. Find the equation of the line g containing the points (0, 0) and (1, 2).

____

e. Find the area of the triangle shown in the diagram using $\int_0^1 g(x)dx$.

____

____

f. What do you notice about all the solutions above?

____

5.

a. Factorize $x^2 - 4x + 3$.

____

b. Draw a rough sketch of the function $f(x) = x^2 - 4x + 3$.

c. Using integration, find the area enclosed by the x-axis and the curve that represents the function $f(x) = x^2 - 4x + 3$.

____

____

6.

a. Factorize $x^2 - 4x - 5$.

____

b. Where does the graph of the function $f(x) = x^2 - 4x - 5$ cross the y-axis?

____

c. Draw a rough sketch of the function $f(x) = x^2 - 4x - 5$.

d. Using integration, find the area enclosed by the x-axis and the curve that represents the function $f(x) = x^2 - 4x - 5$.

____

____

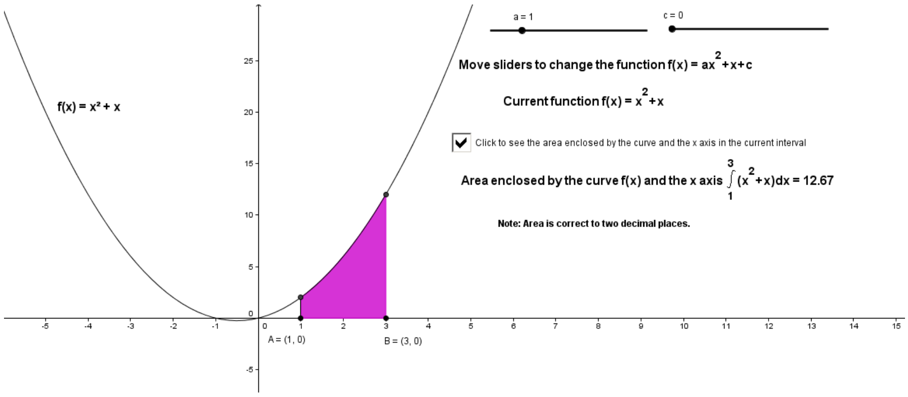
7. Given that the area enclosed by the x-axis and the curve that represents the function $f(x) = x + 4$ in the interval [0, a] is 10, find possible values of a.

____

____

Student Activity: To investigate the relationship between integration of a function and the area enclosed by the curve representing the function and the x-axis or the y-axis

Use in connection with the interactive file, 'Integration and Area 2', on the Student's CD.



<u>It is recommended that in all instances students draw a sketch of the function in question.</u>

1. Calculate $\int_1^3 (x^2 + x)dx$. Check your results using the interactive file.

____

____

2. What does the solution to $\int_1^3 (x^2 + x)dx$ represent?

____

3.
   a. Move the sliders in the interactive file to show the graph of the function $f(x) = x^2 + x + 1$. Move the point A to (-3, 0) and the point B to (3, 0). What value is now given for the area enclosed by the curve of the function $f(x) = x^2 + x + 1$and the x-axis in the interval [-3, 3]?

   ____

   b. Calculate the $\int_{-3}^{3} (x^2 + x + 1)dx$.

   ____

   ____

   c. Hence what is the area enclosed by the curve that represents the function $f(x) = x^2 + x + 1$ and the x-axis in the interval [-3, 3].

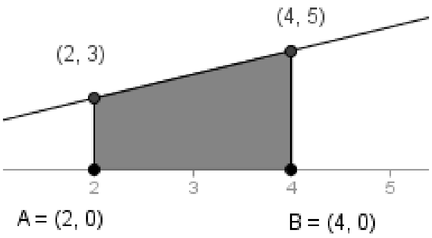
____

4.

a. Find the equation of the line between (2, 3) and (4, 5) and using integration, find the area of the shaded region in the diagram.

____

____



b. Verify your answer.

____

____

5. Given that the area enclosed by the x-axis and the curve that represents the function $f(x) = x^2 + x + 4$ in the interval $[0, b]$ is $12\frac{2}{3}$ and $b \in \mathbb{N}$, find b.

____

____

____

____

6. Find the area of the region bounded by the curve that represents $f(x) = 2x^2 + x + 1$ and the x-axis in the interval $[-4, 0]$.

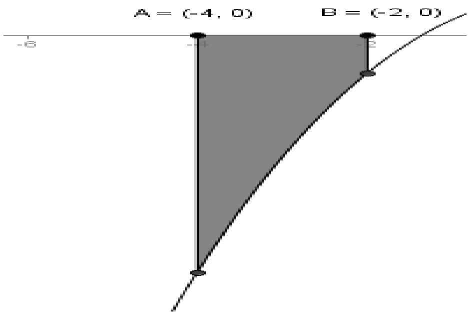
____

____

____

____

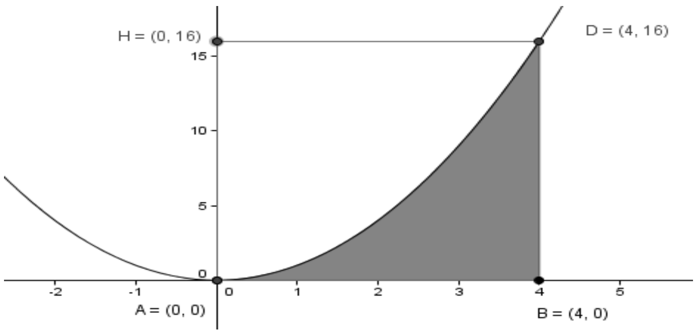
7. Given that the curve represented in the diagram represents the function $f(x) = -2x^2 + x + 5$, find the area of the shaded section. (Note: Area is always positive.)



8. Find the area enclosed by the lines $x = 1$, $x = 4$ and $y = x^2$.

9. Find the area enclosed by the lines $x = 0$, $x = 3$ and $y = x^2 + 4$.

10. The curve in the diagram below represents the function $f(x) = x^2$. (Note: In the diagram the x and y axes are not in the ratio 1:1.)



   a. Find the area enclosed by the curve that represents the function $f(x) = x^2$ and the x-axis in the interval x equals [0, 4].

   b. Find the area of the rectangle ABDH.

c. Find the area enclosed by the curve that represents the function $f(x) = x^2$ and the y-axis in the interval [0, 16]?

____

____

d. In the function represented in the diagram show that x=±root(y).

____

____

e. Find the integral from 0 to 16 of root y (positive root).

____

____

f. Why do we use the positive root only?

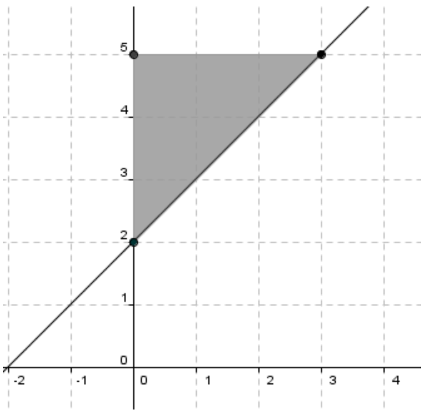
____

____

g. What do you notice about the answers to part c. and part e.?

____

h.



Using the procedure used in parts d. to f., find the area enclosed by the curve that represents the function $f(x) = x + 2$ and the y-axis in the interval x equals 0 to 3.

____

____

11. If the $\int_a^b f(x)dx$ is equal to the area enclosed by the curve of the function that represents f(x) and the x-axis, what does the $\int_{f(a)}^{f(b)} x\ dy$ represent?

____

____

12. By integrating with respect to y, find the area enclosed by the curve $y = \sqrt{x-1}$ and the y-axis in the region x=1 to x=5.

____

____

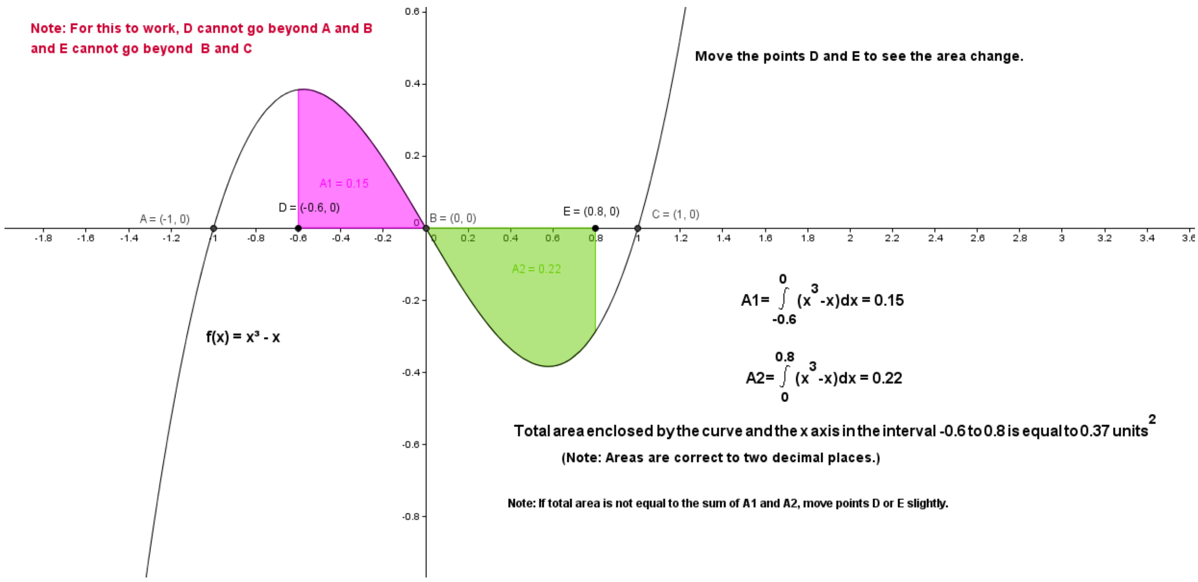
____

13. Complete the following: $\int_a^b y\ dx$ defines the area enclosed by the function f(x) = y and the ...... axis.

14. Complete the following: $\int_{f(a)}^{f(b)} x\ dy$ defines the area enclosed by the function f(x) = y and the ...... axis.

Student Activity: To investigate the relationship between integration of a function and the area enclosed by the curve representing the function and the x axis

Use in connection with the interactive file, 'Integration and Area 3', on the Student's CD.



It is recommended that in all instances students draw a sketch of the function in question.

1.

a. Calculate $\int_{-0.6}^{0} (x^3 - x)dx$.

____

____

____

b. What is the area enclosed by the graph of the function $f(x) = x^3 - x$ and the x axis in the interval $[-0.6, 0]$?

____

c. Calculate $\int_{0}^{0.8} (x^3 - x)dx$.

____

____

d. As area is always positive, what is the total area enclosed by the graph of the function $f(x) = x^3 - x$ and the x axis in the interval $[-0.6, 0.8]$?

____

____

e. Why does $\int_{-0.6}^{0.8} (x^3 - x)dx$ not equal to the total area enclosed by the graph of the function $f(x)=x^3-x$ and the x axis in the interval $[-0.6, 0.8]$

____

____

2.

a. Draw a rough sketch of the function $f(x) = x(x - 3)(x - 4)$.

b. Find the area enclosed by the curve representing the function $f(x) = x(x - 3)(x - 4)$ and the x axis in the interval $[0,4]$. Show calculations.

____

____

____

____

____

3. Find the area enclosed by the graph of the function $f(x) = x(x-4)$ and the x axis in the interval $[-1, 3]$.

____

____

____

____

4.

a. In order to calculate the area enclosed by the graph of the function $f(x) = x^3 + 3x^2 - x - 3$ and the x axis in the interval $[-3, 1]$, why is it not sufficient to calculate $\int_{-3}^{1} (x^3 + 3x^2 - x - 3)\, dx$ to represent the total area?

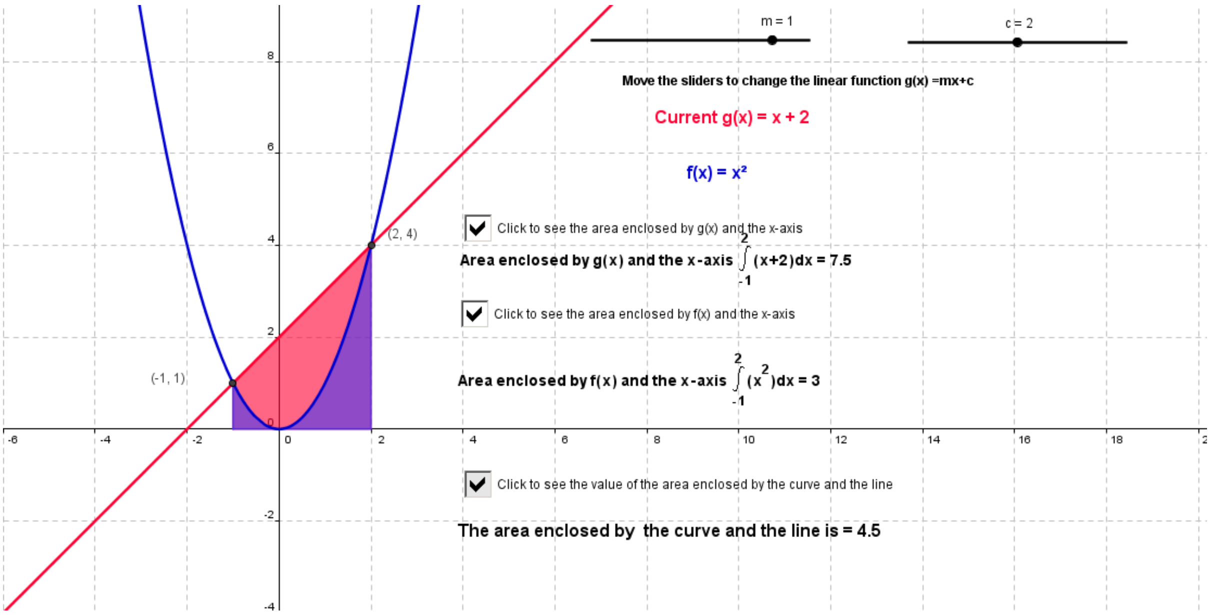
____

____

b. Calculate the area enclosed by the graph of the function
$f(x) = x^3 + 3x^2 - x - 3$

Student Activity: To investigate the relationship between integration of a function and the area enclosed by the curve representing the function and a line that intersects the curve

Use in connection with the interactive file, 'Integration and Area 4', on the Student's CD.



<u>**It is recommended that in all instances students draw a sketch of the function in question.**</u>

1.

a. Calculate $\int_{-1}^{2} x^2 dx$. Show your calculations.

____

____

b. Hence, write down the area enclosed by the curve that represents the function f(x) = x^2 and the x-axis in the interval [-1, 2]?

____

c. Calculate. $\int_{-1}^{2} (x+2)dx$. Show your calculations.

____

____

d. Hence, write down the area enclosed by the line f(x) = x +2 and the x-axis in the interval [-1, 2]?

____

e. Calculate $\int_{-1}^{2}(x+2)dx - \int_{-1}^{2}x^2dx$.

____

____

f. Find the points of intersection of $f(x) = x^2$ and $g(x) = x + 2$.

____

____

____

g. What is the area of the region enclosed by the line $g(x) = x + 2$ and the curve $f(x) = x^2$. Check your answer using the interactive file.

____

h. If you wish to find the area enclosed by a line g(x) and the graph of the function f(x), what extra information is required apart from calculating the integral of both f(x) and g(x)?

____

____

2. Find the area enclosed by graphs of the functions $f(x) = x^2$ and $g(x) = x$ correct to two decimal places.

____

____

____

3. Sketch the curve of $f(x) = -x^2$ and the line $g(x) = x+3$. Find the area enclosed by the curve and the line, correct to two decimal places.

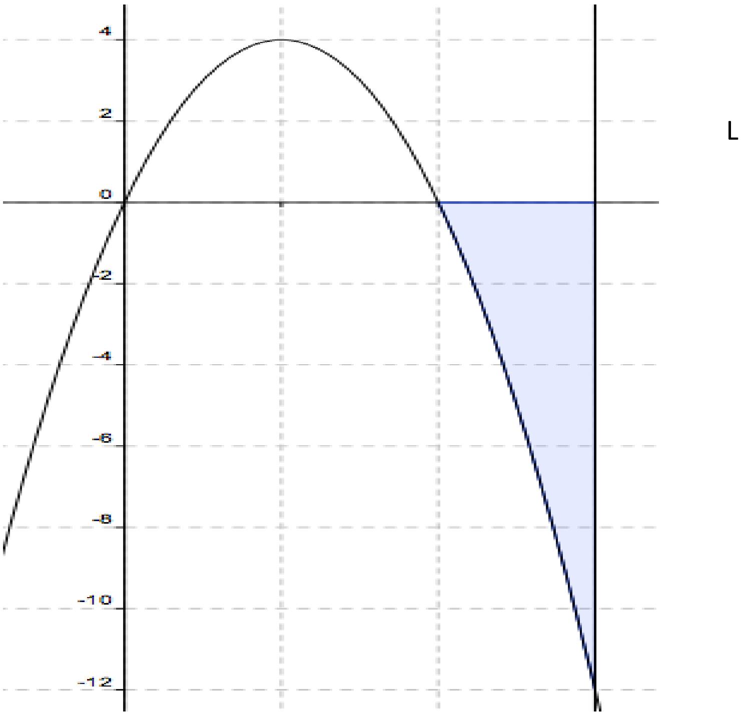
____

____

____

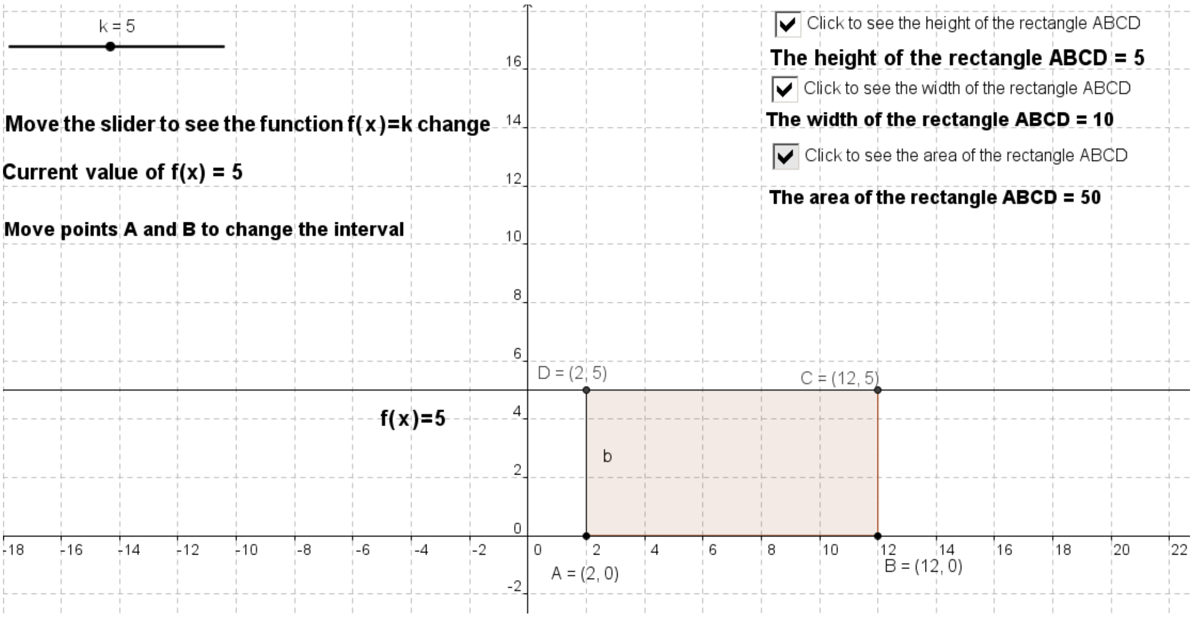
____

4. The diagram shows a part of the graph of the function. If the shaded areas are equal find the equation of the vertical line (L)



Student Activity: To investigate the Average Value of a Constant Function

Use in connection with the interactive file, 'Average Value 1', on the Student's CD.



1. In the interactive file, move the slider *k*, so that f(x) = 5 and move the points, A to (2, 0) and B to (12, 0).
   a. Find the height of the rectangle ABCD.

   ____

   b. Find the width of the rectangle ABCD.

   ____

   c. Find the area of the rectangle ABCD.

   ____

   ____

   d. Using integration, find the area between f(x) = 5 and the x-axis in the interval [2, 12].

   ____

   ____

   e. What do you notice about the area of the rectangle ABCD and the area between the function f(x) = 5 and the x-axis?

   ____

   f. What is the average value of f(x) = 5 in the interval [2, 12]? Hint: Check the values of f(x) for different values of x.

   ____

2.

a. What is the average value of the function f(x) = k in the interval [A, B]?

b. Let *a* be equal to the x co-ordinate of A and *b* be equal to the x co-ordinate of B. Write the area of the rectangle ABCD in the interactive file in terms of the average value of the function f(x) = k, *a* and *b*.

c. Write the area of ABCD in the interactive file in terms of the integral of f(x) = k, *a* the x co-ordinate of A and *b* the x co-ordinate of B.

d. Given that the answers to **b.** and **c.** both give the area of the rectangle ABCD, when f(x) = k and the interval is [A, B], derive a formula for the average value of f(x) = k in the interval [A, B]?

3. Find the integral of f(x) = 8 in the interval [2, 7]. Hence find the average value of the function f(x) = 8 in the interval [2, 7].

4. Find the average value of the function f(x) = 5 in the interval [1, 9] by two different methods. Show your calculations.

5. Find the average value of the function f(x) = 5 in the interval [1, 12] by two different methods. Show your calculations.

6. Find the average value of the function f(x) = k in the interval [1, 12] by two different methods. Show your calculations.

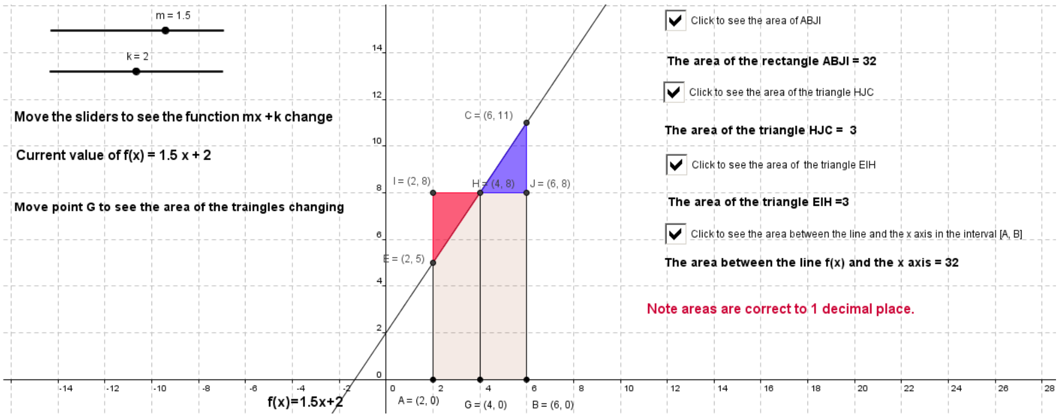
7. Given that the average value of the function f(x) = k in the interval [1, 5] is equal to 12, find k. Show your calculations.

8. Given that the average value of the function f(x) = k in the interval [2, 10] is equal to 12, find k. Show your calculations.

9. Explain in your own words what is meant by the average value of the function f(x) = k.

Student Activity: To investigate the Average Value of a Linear Function

Use in connection with the interactive file, 'Average Value 2', on the Student's CD.



1. In the interactive file, move the sliders *m* and *k* so that f(x) = 2x + 2 and move the points G to (3, 0), A to (2,0) and B to (6,0).
   a. What is the area of the rectangle ABJI?

   ____

   ____

   b. What is the area of the triangle HJC?

   ____

   ____

   c. What is the area of the triangle EIH?

   ____

   ____

   d. Is the area between the line f(x) = 2x + 2 and the x-axis in the interval [2, 6] equal to the area of the rectangle ABJI? Explain why this is so.

   ____

   ____

2. Keeping f(x) = 2x + 2, point A at (2, 0) and point B at (6, 0), move the point G to (4, 0).
   a. What is the area of the rectangle ABJI?

   ____

   b. What is the area of the triangle HJC?

   ____

c. What is the current area of the triangle EIH?

____

d. Is the area between the line f(x) = 2x + 2 and the x-axis in the interval [2, 6] equal to the current area of the rectangle ABJI? Explain why this is so?

____

e. Find the average value of the function f(x)= 2x+2 in the interval [2,6] using f(2) and f(6) only.

____

f. Find the average value of the function f(x)= 2x+2 in the interval [2,6] using f(2), f(3), f(4), f(5) and f(6).

____

g. How many values can $x \in R$ have in the interval [2, 6]? What would be the average value using all these values?

____

h. Express the area between the line f(x) = 2x + 2 and the x-axis in the interval [2, 6] using integration.

____

i. Find $\int_{2}^{6}(2x+2)dx$

____

____

j. Hence, find the area between the line f(x) = 2x + 2 and the x-axis in the interval [2, 6].

____

k. Did the answer to part j equal the answer to part a? Why does this tell us?

____

l. Find the area of the rectangle ABJI in terms of the x co-ordinate of A, the x co-ordinate of B and the y co-ordinate of H, when the area of the triangles HJC and EIH are equal.

____

____

m. Did your answer in part l equal the answer to part j?

____

n. Do you agree that the average value of the function in the interval [2, 6] is the y co-ordinate of the point H when the area of the triangles HJC and EIH are equal?

____

o. Using part l of this question write the average value of the function f(x) = 2x+2 in the interval [2, 6] in terms of $\int_2^6 (2x+2)dx$.

____

____

____

3.

a. For any linear function f(x)=mx+c, the area between the graph of the function and the x-axis in the interval [A, B] is equal to the area of the rectangle ABJI when the height of the rectangle is the **average value of the function f(x)** in the interval [A, B]. Write the area between the line and the x-axis in terms of a = x(A), the x co-ordinate of A, b = x(B), the x co-ordinate of B and the average value of the function in that interval.

____

____

b. Give an expression for the area between graph of the function and the x-axis in terms of the integral of f(x), in the interval [A, B] equal to. [Use a = x(A), the x co-ordinate of A and b = x(B), the x co-ordinate of B.]

____

____

c. At the point where the two answers above are equal, what is the average value of the function in the interval [A, B] in terms of a, b and the integral of f(x)?

____

4. Use the formula for the average value of a function $= \frac{1}{b-a}\int_a^b f(x)dx$, to find the average value of the function f(x) = x + 1 in the interval [0, 3]?
Check this using the interactive file.

____

____

5. Find the average value of the function f(x) = x+2 in the interval [0, 3].

____

____

6. Given that the average value of the function f(x) = *a*x is equal to 15 in the interval [2, 8], find the value of *a*.

____

____

____

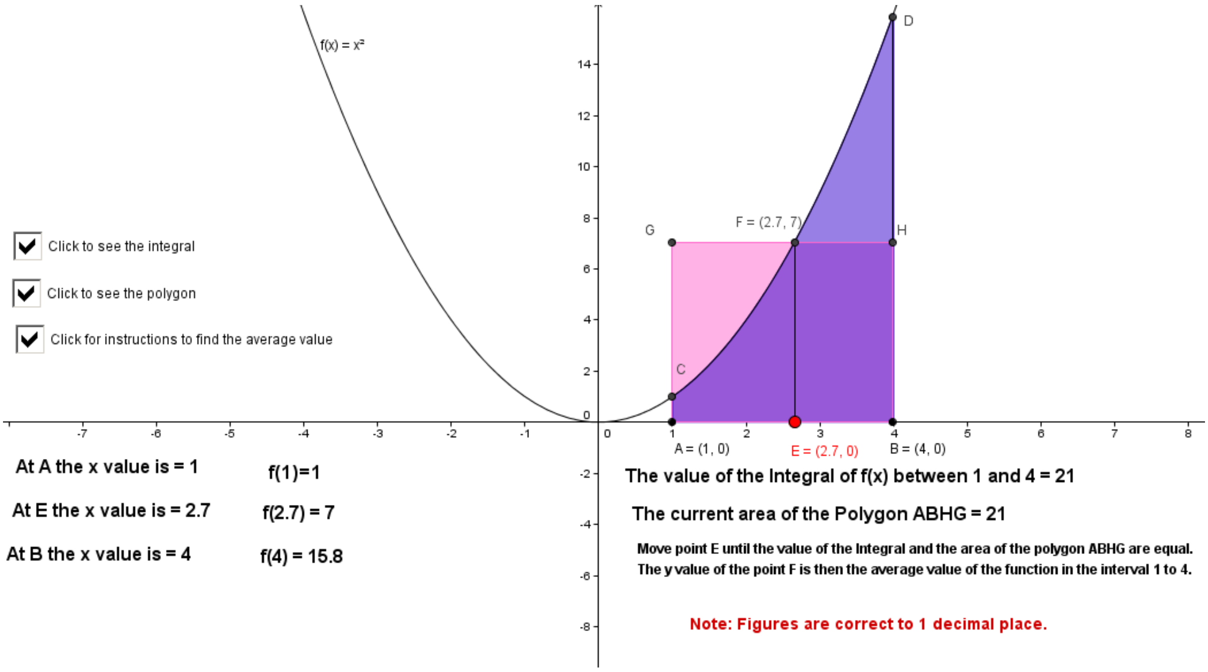
7. Given that the average value of a linear function f(x) = 2x+ 2 is 12 in the interval [0, *b*], find the value of *b*.

____

____

<u>Student Activity</u>: To investigate the Average Value of a Function 3

Use in connection with the interactive file, 'Average Value 3', on the Student's CD.



1. Click all the boxes in the interactive file. Move the point E to (2, 0). What is the area of the polygon ABHG? Is the area between the curve $f(x) = x^2$ and the x-axis in the interval [1, 4] greater than or less than 11.9. Explain why this is the case.

____

____

____

2. Move the point E in the interactive file to (2.5, 0). What is the area of the polygon ABHG now? Is the area of the curve between $f(x) = x^2$ and the x-axis in the interval [1, 4] greater than or less than 18.8. Explain why this is the case.

____

____

____

3. Move the point E in the interactive file to (3, 0). What is the area of the polygon ABHG now? Is the area between the curve $f(x) = x^2$ and the x-axis in the interval [1, 4] greater than or less than 26.8. Explain why this is the case.

____

____

4. By moving the point E in the interactive file, what is the approximate y value of the point F when the area of the polygon ABHG is equal to the area between the curve $f(x) = x^2$ and the x-axis in the interval [A, B].

____

5. Using the y value of the point F from question 4 above, what is the area of the polygon ABHG in terms of a = x(A) and b = x(B) and? Where a = x(A) is the x co-ordinate of the point A and b = x(B) the x co-ordinate of the point B. Don't simplify.

____

____

6. When the area of the polygon ABHG is equal to the area between the curve $f(x) = x^2$ and the x-axis in the interval [A, B], what is the relationship between a = x(A), b = x(B), the y value of the point F and $\int_a^b x^2 dx$?

____

____

____

7. Considering the values f(1), f(2), f(3) and f(4), find an estimate of the average value of the function $f(x) = x^2$ in the interval [1,4]. Why is this only an estimate of the average value?

____

____

____

8. Considering the values of f(1), f(1.5), f(2), f(2.5), f(3), f(3.5) and f(4), find an estimate of the average value of the function $f(x) = x^2$ in the interval [1,4]. Why is this only an estimate of the average value?

____

____

9. Considering the values of f(1), f(1.25), f(1.5),f(1.75) f(2), f(2.25), f(2.5), f(2.75), f(3), f(3.25), f(3.5), f(3.75) and f(4), find an estimate of the average value of the function $f(x) = x^2$ in the interval [1,4]. Why is this only an estimate of the average value?

____

____

____

10. Which of the above three answers do you think is the most accurate for the average value of the function $f(x) = x^2$ in the interval [1, 4]? Explain your choice.

____

11. Under what circumstances would the method used in questions 7, 8 and 9 to find the average value of the function $f(x) = x^2$ give the correct answer?

____

12. How many values can $x \in R$ have in the interval [1, 4]?

____

13. Note: Earlier you established that the y value at the point F, the average value of the function $f(x) = x^2$ in the interval [a, b]=

$$\frac{1}{b-a}\int_a^b x^2 dx .$$

   a. Hence, from the interactive file, what do you consider is the average value of the function $f(x) = x^2$ in the interval [1, 4]?

____

   b. Calculate $\frac{1}{4-1}\int_1^4 x^2 dx$. Are your answers in parts a and b equal?

____

____

____

14.

a. Find the average value of the function $f(x) = x^2$ in the interval[2, 4].

____

____

____

____

____

____

b. Would you expect the average value of the function $f(x) = x^2$ in this interval

[2, 4] used in part a to be greater than or less than the average value of the same function in the interval between [1, 4]? Explain your answer.

____

____

15. Find the average value of the function $f(x) = 4x^2+3x+2$ in the interval [1, 3].

____

____

16. Find the average value of the function $f(x) = 3x^2$ in the interval [-2, 2].

____

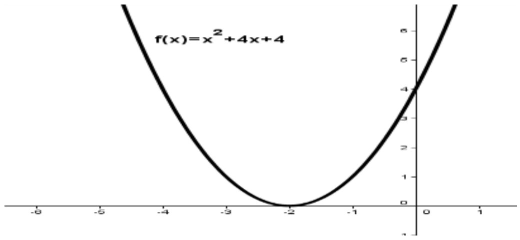
____

17. The temperature T (in °C) recorded during a day obeyed the equation followed the curve $T = 0.001t^4 - 0.280t^2 + 25$ where t is the number of hours from noon ($-12 \le t \le 12$). What was the average temperature during the day? (Note: Twelve hours before and twelve hours after noon.)

____

____

18. Find the average value of the function $f(x) = x^2 + 4x + 4$ represented in the diagram below in the interval [-4, 0].



19. Find the average value of the function $g(x) = x^3$ in the interval [0, 5].

___

___

___

20. Describe, in your own words, what is meant by the average value of a function.

___

___

21. The distance (s) travelled by a body in t seconds from rest is given by $s = 5t + 6t^2$
    a. Find the average distance travelled in the first 4 seconds.

    ___

    ___

22. Find the average distance travelled between the second and sixth second.

___

___

___

Account for the difference.

___

___

___